Course "Efficient Boolean Reasoning"
TEST

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1

Let φ be a generic Boolean formula, and let $\varphi_1 \stackrel{\text{def}}{=} CNF(\varphi)$, s.c. $CNF()$ is the "classic" CNF conversion (i.e., that using DeMorgan's rules). Let $|\varphi|$ and $|\varphi_1|$ denote the size of φ and φ_1 respectively.

For each of the following sentences, say if it is true or false.

(a) $|\varphi_1|$ is in worst-case polynomial in size wrt. $|\varphi|$.
(b) φ_1 has the same number of distinct Boolean variables as φ has.
(c) A model for φ_1 (if any) is also a model for φ, and vice versa.
(d) φ_1 is valid if and only if φ is valid.

2

Consider the following CNF formula:

$$
\begin{array}{l}
(\neg A_1 \vee \neg A_2 \vee \neg A_3) \wedge \\
(\ \ A_4 \vee \neg A_7 \vee \neg A_5) \wedge \\
(\neg A_2 \vee \neg A_3 \vee \neg A_6) \wedge \\
(\neg A_8 \vee \neg A_1 \vee \neg A_3) \wedge \\
(\neg A_2 \vee \neg A_3 \vee \neg A_8) \wedge \\
(\neg A_6 \vee \neg A_5 \vee \neg A_8) \wedge \\
(\ \ A_8 \vee \neg A_3 \vee \neg A_7) \wedge \\
(\neg A_6 \vee \neg A_5 \vee \neg A_2) \wedge \\
(\ \ A_8 \vee \neg A_1 \vee \neg A_3) \wedge \\
(\neg A_3 \vee \neg A_4 \vee \neg A_3) \wedge \\
(\ \ A_7 \vee \neg A_2 \vee \neg A_1) \wedge \\
(\ \ A_1 \vee \neg A_2 \vee \neg A_3) \wedge \\
(\neg A_4 \vee \neg A_5 \vee \neg A_2) \wedge \\
(\neg A_8 \vee \neg A_7 \vee \neg A_1) \wedge \\
(\neg A_5 \vee \neg A_4 \vee \neg A_7) \wedge \\
(\neg A_4 \vee \neg A_2 \vee \neg A_5) \wedge \\
(\ \ A_3 \vee \neg A_6 \vee \neg A_7) \wedge \\
(\ \ A_3 \vee \neg A_4 \vee \neg A_2) \wedge \\
(\ \ A_6 \vee \neg A_2 \vee \neg A_8) \wedge \\
(\ \ A_1 \vee \neg A_2 \vee \neg A_6) \wedge \\
(\ \ A_2 \vee \neg A_3 \vee \neg A_4) \wedge \\
(\ \ A_5 \vee \neg A_6 \vee \neg A_2) \wedge \\
(\neg A_5 \vee \neg A_3 \vee \neg A_4)
\end{array}
$$

Decide *quickly* if it is satisfiable or not, and briefly explain why.

3

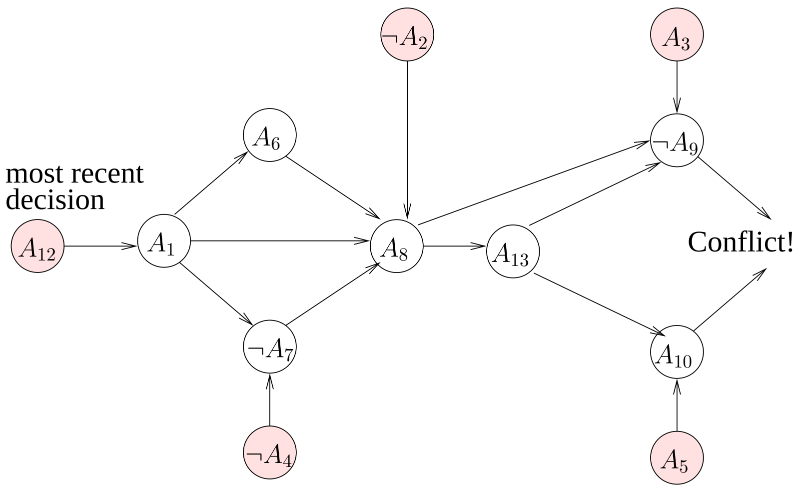
Using the basic DPLL algorithm, decide whether the following formula is satisfiable or not. (Write the search tree.)

$$
\begin{array}{rcrcr}
 & & & & (\neg A_1) \wedge \\
 & & (\ A_1 & \vee & \neg A_2) \wedge \\
(\ A_1 & \vee & A_2 & \vee & A_3) \wedge \\
(\ A_4 & \vee & \neg A_3 & \vee & A_6) \wedge \\
(\ A_4 & \vee & \neg A_3 & \vee & \neg A_6) \wedge \\
(\neg A_3 & \vee & \neg A_4 & \vee & A_7) \wedge \\
(\neg A_3 & \vee & \neg A_4 & \vee & \neg A_7)
\end{array}
$$

(Literal-selection criteria to your choice.)

4

Consider the following implication graph:



A_{12} being the most recent decision literal. Write the conflict clauses generated by

1. the last UIP conflict analysis technique
2. the 1st UIP conflict analysis technique

5

Consider the following Boolean formulas:

$$\begin{array}{rl} \varphi_1 \stackrel{\text{def}}{=} & (\ \ A_1 \vee \ \ A_2) \ \wedge \\ & (\neg A_1 \vee \ \ A_2) \ \wedge \\ & (\ \ A_3 \vee \ \ A_4) \ \wedge \\ & (\neg A_3 \vee \ \ A_4) \end{array}$$

$$\begin{array}{rl} \varphi_2 \stackrel{\text{def}}{=} & (\neg A_2 \vee \ \ A_5) \ \wedge \\ & (\neg A_4 \vee \neg A_5) \end{array}$$

which are such that $\varphi_1 \wedge \varphi_2 \models \bot$. For each of the following formulas, say if it is a Craig interpolant for (φ_1, φ_2) or not.

(a)

$$\begin{array}{l} (\ \ A_1 \vee \ \ A_2) \ \wedge \\ (\neg A_1 \vee \ \ A_2) \ \wedge \\ (\ \ A_4) \end{array}$$

(b)

$$(\ \ A_2)$$

(c)

$$(\ \ A_2 \wedge \ \ A_4)$$

6

Consider the following formula in the theory $\mathcal{LRA}$ of linear arithmetic on the Rationals.

$$
\begin{array}{rl}
\varphi = & \{(v_1 - v_2 \leq 3) \ \vee A_2\} \quad \wedge \\
& \{\neg(2v_3 + v_4 \geq 5) \ \vee \underline{\neg(v_1 - v_3 \leq 6)} \ \vee \neg A_1\} \ \ \wedge \\
& \{A_1 \vee (v_1 - v_2 \leq 3)\} \quad \wedge \\
& \{\underline{(v_2 - v_4 \leq 6)} \ \vee (v_5 = 5 - 3v_4) \vee \neg A_1\} \ \ \wedge \\
& \{\underline{\neg(v_2 - v_3 > 2)} \ \vee A_1\} \wedge \\
& \{\underline{\neg A_2} \vee (v_1 - v_5 \leq 1)\} \wedge \\
& \{A_1 \vee \underline{(v_3 = v_5 + 6)} \ \vee A_2\}
\end{array}
$$

and consider the partial truth assignment μ given by the underlined literals above:

$$\{\neg(v_2 - v_3 > 2), \neg A_2, \neg(v_1 - v_3 \leq 6), (v_2 - v_4 \leq 6), (v_3 = v_5 + 6)\}.$$

1. Does (the Boolean abstraction of) μ propositionally satisfy (the Boolean abstraction of) φ?
2. Is μ satisfiable in $\mathcal{LRA}$?
   (a) If no, find a minimal conflict set for μ and the corresponding conflict clause C.
   (b) If yes, show one unassigned literal which can be deduced from μ, and show the corresponding deduction clause C.

7

Consider the following $\mathcal{LRA}$ formula φ.

$$
\begin{array}{rcrcrc}
((-x+y>-1)) & & & & & \wedge \\
((x+y\geq -3) & \vee & \neg(-x+y>-1)) & & & \wedge \\
(\neg(x+y\geq -3) & \vee & \neg(x<-2) & \vee & (y<-1)) & \wedge \\
(\neg(-x+y>-1) & \vee & (x<-2) & \vee & (y<-1)) & \\
((x+y\geq -3) & \vee & \neg(5y-4z>1) & \vee & \neg(3v-5x>7)) & \wedge \\
((-x+y>-1) & \vee & (3v-5x>7) & \vee & \neg(5y-4z>1)) & \wedge
\end{array}
$$

(a) Write the Boolean Abstraction of φ.

(b) Using the standard lazy SMT approach (literal-decision order, techniques and strategies to your choice), decide if φ is satisfiable in $\mathcal{LRA}$, plotting the corresponding search tree and producing the $\mathcal{T}$-lemmas involved.

8

Let $\mathcal{LA}(\mathbb{Q})$ be the logic of linear arithmetic over the rationals and $\mathcal{EUF}$ be the logic of equality and uninterpreted functions, and consider the following pure formula φ in the combined logic $\mathcal{LA}(\mathbb{Q}) \cup \mathcal{EUF}$:

$$(h = 3.0) \wedge (k = -2.0) \wedge (x = 1.0) \wedge (y = h + k) \wedge \quad (1)$$

$$(z = f(x)) \wedge (w = f(y)) \wedge \neg(g(z) = g(w)) \quad (2)$$

Say which variables are interface variables, list the interface equalities for this formula, and decide whether this formulas is $\mathcal{LA}(\mathbb{Q}) \cup \mathcal{EUF}$-satisfiable or not, using either Nelson-Oppen or Delayed Theory Combination.

9

Consider the following set of clauses φ in the theory of linear arithmetic on the Integers $\mathcal{LIA}$.

$$\left\{\begin{array}{l}(\neg(x=0) \vee \neg(x=1)),\\ (\neg(x=0) \vee \quad (x=1)),\\ (\quad (x=0) \vee \neg(x=1)),\\ (\quad (x=0) \vee \quad (x=1))\end{array}\right\}$$

Say which of the following sets is a $\mathcal{LIA}$-unsatisfiable core of φ and which is not. For each one, explain why.

(a)

$$\left\{\begin{array}{l}(\neg(x=0) \vee \quad (x=1)),\\ (\quad (x=0) \vee \neg(x=1)),\\ (\quad (x=0) \vee \quad (x=1))\end{array}\right\}$$

(b)

$$\left\{\begin{array}{l}(\neg(x=0) \vee \neg(x=1)),\\ (\quad (x=0) \vee \neg(x=1)),\\ (\quad (x=0) \vee \quad (x=1))\end{array}\right\}$$

(c)

$$\left\{\begin{array}{l}(\neg(x=0) \vee \quad (x=1)),\\ (\quad (x=0) \vee \neg(x=1)),\\ (\quad (x=0) \vee \quad (x=1)),\\ ((x=y))\end{array}\right\}$$

10

Consider the following formulas in difference logic ($\mathcal{DL}$):

$$\begin{array}{rll}\varphi_1 \stackrel{\text{def}}{=} & (x_4 - x_5 \leq -2) & \wedge \\ & (x_5 - x_6 \leq -4) & \wedge \\ & (x_1 - x_2 \leq 3) & \wedge \\ & (x_2 - x_3 \leq 1) & \end{array}$$

$$\begin{array}{rll}\varphi_2 \stackrel{\text{def}}{=} & (x_6 - x_1 \leq 0) & \wedge \\ & (x_3 - x_4 \leq 1) & \end{array}$$

which are such that $\varphi_1 \wedge \varphi_2 \models_{\mathcal{DL}} \bot$. For each of the following formulas, say if it is a Craig interpolant in $\mathcal{DL}$ for (φ_1, φ_2), and explain why.

(a) $(x_1 - x_2 + x_4 - x_6 \leq -3)$

(b) $(x_1 - x_3 \leq 4)$

(c) $(x_1 - x_3 \leq 4) \quad \wedge$
$(x_4 - x_6 \leq -6)$