Fundamenta Informaticae 108 (2011) 119–149
DOI 10.3233/FI-2011-416
IOS Press

A Modal Interface Theory for Component-based Design [*]

Jean-Baptiste Raclet[†]
INRIA Grenoble - Rhônes-Alpes, France
Jean-Baptiste.Raclet@irit.fr

Eric Badouel
INRIA/IRISA Rennes, France
eric.badouel@irisa.fr

Albert Benveniste
INRIA/IRISA Rennes, France
albert.benveniste@irisa.fr

Benoît Caillaud
INRIA/IRISA Rennes, France
benoit.caillaud@irisa.fr

Axel Legay
INRIA/IRISA Rennes, France
axel.legay@irisa.fr

Roberto Passerone
University of Trento, Italy
roberto.passerone@unitn.it

Abstract. This paper presents the *modal interface* theory, a unification of *interface automata* and *modal specifications*, two radically dissimilar models for interface theories. Interface automata is a game-based model, which allows the designer to express assumptions on the environment and which uses an optimistic view of composition: *two components can be composed if there is an environment where they can work together*. Modal specifications are a language theoretic account of a fragment of the modal mu-calculus logic with a rich composition algebra which meets certain methodological requirements but which does not allow the environment and the component to be distinguished. The present paper contributes a more thorough unification of the two theories by correcting a first attempt in this direction by Larsen et al., drawing a complete picture of the modal interface algebra, and pushing the comparison between interface automata, modal automata and modal interfaces even further.

The work reported here is based on earlier work presented in [41] and [42].

Keywords: Component-based System, Compositional Reasoning, Interface Theory, Interface Automata, Modal Specifications.

[*]This work was funded in part by the European IP-SPEEDS project number 033471 and the European STREP-COMBEST project number 215543.
[†]Address for correspondence: INRIA Grenoble - Rhônes-Alpes, France

1. Introduction

Nowadays, systems are tremendously large and complex, resulting from the assembling of several components. These many components are in general designed by teams, working independently but with a common agreement on what the interface of each component should be. As a consequence, the study of mathematical foundations that allow designers to reason at the abstract level of interfaces is a very active research area. According to our understanding of industrial needs (see [5] for a discussion), an interface theory is at least subject to the following requirements:

1. *Satisfaction and satisfiability are decidable.* Interfaces should be seen as specifications whose models are its possible implementations. It should thus be decidable whether an interface admits an implementation and whether a given component implements a given interface.

2. *Refinement entails substitutability.* Refinement allows one to replace, in any context, an interface by a more detailed version of it. Refinement should entail substitutability of interface implementations, meaning that every implementation satisfying a refinement also satisfies the larger interface. For the sake of controlling design complexity, it is desirable to be able to decide whether there exists an interface that refines two different interfaces. This is called *shared refinement* [22]. In many situations, we are looking for the *greatest lower bound*, i.e., the shared refinement that could be refined by any other shared refinement.

3. *Interfaces are closed under conjunction.* Large systems are concurrently developed for their different *aspects* or *viewpoints* by different teams using different frameworks and tools. Examples of such aspects include the functional aspect, the safety or reliability aspect, the timing aspect. Each of these aspects requires specific frameworks and tools for their analysis and design. Yet, they are not totally independent but rather interact. The issue of dealing with multiple aspects or multiple viewpoints is thus essential. This implies that several introductions are associated with a same system, sub-system, or component, namely (at least) one per viewpoint. These introductions are to be interpreted in a conjunctive way. The need for supporting conjunctive introductions also follows from the current practice in which early requirement capture relies on Doors or even Excel sheets collecting many individual requirements. The latter typically consist of English text, semi-formal languages whose sentences are translatable into predefined behavioral patterns, or even graphical scenario languages.

4. *Composition supports independent design.* The interface theory should also provide a combination operator on interfaces, reflecting the standard composition of implementations by, e.g., parallel product. This operation must be associative and commutative to guarantee independence in the development. Depending on the model, a notion of compatibility for composition may also be considered, i.e., there can be cases where two systems cannot be composed.

5. *Interface quotient supports incremental design and component reuse.* A quotienting operation, dual to composition is crucial to perform incremental design. Consider a desired global specification and the specification of a preexisting component; the quotient specification describes the part of the global specification that remains to be implemented.

6. *A verification procedure*. In addition to the fact that an interface already represents a set of properties, one should be able to verify if an interface satisfies a set of requirements written in some specification language.

7. *Encompassing interfaces with dissimilar alphabets.* Complex systems are built by combining subsystems possessing dissimilar alphabets for referencing ports and variables. It is thus important to properly handle those different alphabets when combining interfaces.

Building good interface theories has been the subject of intensive studies (see, e.g., [29, 20, 11, 23, 25, 18, 21]). In this paper we will concentrate on two models: (1) *interface automata* [20] and (2) *modal specifications* [30]. Interface automata is a game-based variation of input/output automata which deals with open systems, their refinement and composition, and puts the emphasis on interface compatibility. Modal specifications is a language-theoretic account of a fragment of the modal mu-calculus logic [24] which admits a richer composition algebra with product, conjunction and residuation operators.

Modal specifications correspond to *deterministic* modal automata [30], i.e., automata whose transitions are typed with *may* and *must* modalities. A modal specification thus represents a set of models; informally, a must transition is available in every component that implements the modal specification, while a may transition needs not be. The components that implement modal specifications are prefix-closed languages, or equivalently deterministic automata/transition systems.

Satisfiability of modal specifications is decidable. Refinement between modal specifications coincides with model inclusion. Conjunction is effectively computed via a product-like construction. It can be shown that the conjunction of two modal specifications corresponds to their greatest common refinement. Combination of modal specifications, handling synchronization products *à la* Arnold and Nivat [4], and the dual quotient combinators can be efficiently handled in this setting [39, 40].

In interface automata [20], an interface is represented by an input/output automaton [34], i.e., an automaton whose transitions are labeled with *input* or *output* actions. The semantics of such an automaton is given by a two-player game: an *Input* player represents the environment, and an *Output* player represents the component itself. Interface automata do not encompass any notion of model, because one cannot distinguish between interfaces and implementations.

Refinement between interface automata corresponds to the alternating refinement relation between games [2], i.e., an interface refines another if its environment is more permissive whereas its component is more restrictive. Shared refinement is defined in an ad-hoc manner [22] for a particular class of interfaces [13]. Contrary to most interface theories, the game-based interpretation offers an *optimistic* treatment of composition: two interfaces can be composed if there exists at least one environment (i.e., one strategy for the Input player) in which they can interact together in a safe way (i.e., whatever the strategy of the Output player is). This is referred to as compatibility of interfaces. A quotient, which is the adjoint of the game-based composition, has been proposed in [10] for the deterministic case.

It is worth mentioning that, in existing work on interface automata and modal specifications, there is nothing about dissimilar alphabets. This is somehow surprising as it seems to be a quite natural question when performing operations that involve several components, e.g., conjunction, composition, and quotient. As we shall see in this paper, an explicit mechanism to handle dissimilar alphabets is not needed when considering interface automata, since conjunction is not discussed for this model. For the case of composition/quotient, instead, we shall see that the notion is implicitly encompassed in the definition of compatibility. Conjunction and quotient operators [30, 39, 40] that have been proposed for modal specifications do not take dissimilar alphabets into account. One thus needs to extend those operators to this more general setting. This is one of the subjects of this paper.

In conclusion, both models have advantages and disadvantages:

- Interface automata is a model that allows designers to make assumptions on the environment, which is mainly useful to derive a rich notion for composition with compatibility issues. In addition, the notion of dissimilar alphabets is not needed. Unfortunately, the model is incomplete as conjunction and shared refinement are not defined.

- Modal specification is a rich language-algebraic model on which most of the requirements for a good interface theory can be considered. Unfortunately, *may* and *must* modalities are not sufficient to derive a rich notion for composition including compatibility. Moreover, the notion of dissimilar alphabets is missing.

It is thus worth considering unifying the frameworks of interface automata and modal specifications. A first attempt was made by Larsen et al. [31, 36] who considered *modal interfaces*, which are modal specifications whose actions are also typed in *input* or *output* attributes. A modal interface can be viewed as simply a modal specification except for the composition operation for which the modalities are an additional complication. Refinement for modal interfaces is the same as refinement for modal specifications, while composition is the one from interface automata. Larsen et al. have shown that refinement for modal specifications is compatible with the composition operation for interface automata [31, 36]. The main problem with their results is that the composition operator is incorrect. Indeed, contrary to what is claimed by the authors, their composition operator is not monotone with respect to satisfaction. This fails to ensure that two *compatible* interfaces may be implemented separately. Moreover, requirements such as dissimilar alphabets, conjunction, and component reuse are not considered.

The present paper adds a new stone to the cathedral of results on interface theories by (1) proposing a new theory for dissimilar alphabets, (2) correcting the modal interface composition operator presented in [31, 36], (3) pushing the comparison between interface automata, modal automata and modal specifications and modal interfaces further, and (4) reasoning on architectural design for component-based systems.

The rest of the paper is organized as follows. In Sections 2 and 3 we recap the theory for modal specifications and interface automata, respectively. In Section 4, we present the complete theory for modal interfaces and correct the error in [31, 36]. Section 5 is dedicated to architectural design. Finally, in Section 6, we draw our conclusion and discuss future extensions for the model of modal interfaces.

2. Modal specifications

This section starts with an overview of existing results developed in [30, 39, 40] for modal specifications defined over a global alphabet (Sections 2.1, 2.2 and 2.3). We also propose a new methodology to encompass dissimilar alphabets (Section 2.4).

2.1. The Framework

Following our previous work [39, 40, 42], we will define modal specifications in term of languages, knowing that they can also be represented by *deterministic* automata whose transitions are typed with *may* and *must* modalities [30]. We start with the following definition.

Definition 2.1. (Modal specification)
A *modal specification* is a tuple $\mathcal{S} = (A, must, may)$, where A is a finite alphabet and

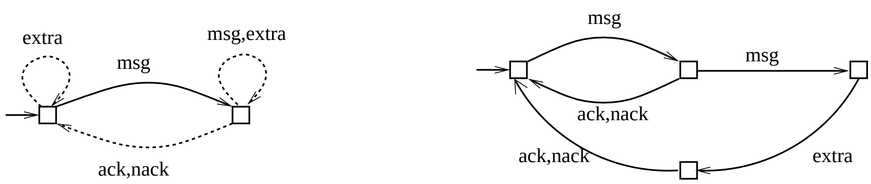
$$must, may : A^* \mapsto 2^A$$

are partial functions satisfying the following *consistency* condition:

$$must(u) \quad \subseteq \quad may(u). \tag{1}$$

If $a \in may(u)$, then a is *allowed* after the trace u whereas $a \in must(u)$ indicates that a is *required* after u. By negation, $a \notin may(u)$ means that a is *disallowed* after u. The latter is often written $a \in mustnot(u)$. Condition (1) naturally imposes that every required action is also allowed. We shall sometimes write $may_{\mathcal{S}_i}$ and $must_{\mathcal{S}_i}$ (or may_i and $must_i$ for short) to refer to the entities involved in the definition of $\mathcal{S}_i$.

Modal specifications that generate regular languages can be represented by *deterministic modal automata*, i.e., deterministic finite-word automata with two types of transitions: solid transitions if the action is required in the source state and dashed transitions if it is allowed but not required. The concept is illustrated with the example.



(a) Functional spec. *Fun* (b) A simple automaton

Figure 1. The modal specification *Fun* accepts the automaton

Example 2.1. Consider a producer whose alphabet of actions includes *msg* for when the producer sends a message as well as two kinds of acknowledgment for transmission: *ack* in case of success and *nack* in case of failure. Assume also the existence of an action *extra* which occurs when extra resources are requested to dispatch a message.

A functional specification *Fun* for the producer is given in Figure 1(a). It specifies that a *msg* may be sent again. Moreover every *msg* may be acknowledged. Additionally, the producer may request extra resources at any moment.

When composing specifications, discrepancies between the modal information carried out by the specifications may appear. We then consider *pseudo-modal specifications* (also called *mixed transition systems* in [16]), denoted ${}^p\mathcal{S}$; they are triples satisfying Definition 2.1 with the exception of (1). For

${}^{p}\mathcal{S}$ a pseudo-modal specification, a word $u \in A^*$ is called *consistently specified* in ${}^{p}\mathcal{S}$ if it satisfies (1) and *inconsistent* otherwise; modal specifications correspond exactly to the subclass of *consistent* pseudo-modal specifications, that is pseudo-modal specifications such that every $u \in A^*$ is consistently specified.

For ${}^{p}\mathcal{S} = (A, must, may)$ a pseudo-modal specification, the *support* of ${}^{p}\mathcal{S}$ is the least *prefix-closed* language $\mathcal{L}_{{}^{p}\mathcal{S}}$ such that

(i) $\epsilon \in \mathcal{L}_{{}^{p}\mathcal{S}}$, where ϵ denotes the empty word; and

(ii) $u \in \mathcal{L}_{{}^{p}\mathcal{S}}$ and $a \in may(u)$ imply $u.a \in \mathcal{L}_{{}^{p}\mathcal{S}}$.

2.2. Implementation, refinement and consistency

In this section, we study the concepts of *implementation*, *refinement* and *consistency*. We start with implementation, which is also called *model*.

Definition 2.2. (implementation)
A prefix-closed language $\mathcal{I} \subseteq A^*$ is an *implementation* (or model) of a pseudo-modal specification ${}^{p}\mathcal{S} = (A, must, may)$, denoted by $\mathcal{I} \models {}^{p}\mathcal{S}$, if

$$\forall u \in \mathcal{I} \quad \Rightarrow \quad must(u) \subseteq \mathcal{I}_u \subseteq may(u)$$

where $\mathcal{I}_u$ is the set of actions $a \in A$ such that $u.a \in \mathcal{I}$.

Example 2.2. A model for the specification given in Figure 1(a) is presented in Figure 1(b). It indicates that every message will be acknowledged either positively or negatively. Moreover, an extra resource is requested if the message has to be re-emitted.

Lemma 2.1. Let $\mathcal{I} \subseteq A^*$ be a prefix-closed language and ${}^{p}\mathcal{S}$ a pseudo-modal specification over A. If $\mathcal{I} \models {}^{p}\mathcal{S}$, then $\mathcal{I} \subseteq \mathcal{L}_{{}^{p}\mathcal{S}}$ holds and every word of $\mathcal{I}$ is consistently specified in ${}^{p}\mathcal{S}$.

The concept of *thorough refinement* follows immediately from Definition 2.2 by comparing, through set inclusion, the sets of implementations associated to two specifications.

Definition 2.3. (thorough refinement)
There exists a thorough refinement between specification ${}^{p}\mathcal{S}_1$ and specification ${}^{p}\mathcal{S}_2$ if and only if any model of ${}^{p}\mathcal{S}_1$ is also a model of ${}^{p}\mathcal{S}_2$.

Thorough refinement has been extensively studied in [32] and compared to the more syntactic notion of *modal refinement* that is recalled hereafter.

Definition 2.4. (modal refinement)
Let ${}^{p}\mathcal{S}_1$ and ${}^{p}\mathcal{S}_2$ be two pseudo-modal specifications. The specification ${}^{p}\mathcal{S}_1$ *refines* ${}^{p}\mathcal{S}_2$, written ${}^{p}\mathcal{S}_1 \leq {}^{p}\mathcal{S}_2$, if and only if, for all $u \in \mathcal{L}_1$, $may_1(u) \subseteq may_2(u)$ and $must_1(u) \supseteq must_2(u)$.

It is easy to see that modal refinement is a preorder relation that implies inclusion of supports:

$$^{p}\mathcal{S}_1 \leq {}^{p}\mathcal{S}_2 \implies \mathcal{L}_{^p\mathcal{S}_1} \subseteq \mathcal{L}_{^p\mathcal{S}_2}$$

Any two modal specifications $\mathcal{S}_1$ and $\mathcal{S}_2$ such that $\mathcal{S}_1 \leq \mathcal{S}_2 \leq \mathcal{S}_1$ have equal supports $\mathcal{L} = \mathcal{L}_{\mathcal{S}_1} = \mathcal{L}_{\mathcal{S}_2}$ and for all $u \in \mathcal{L}$, $may_1(u) = may_2(u)$ and $must_1(u) = must_2(u)$. Said differently, equivalent modal specifications differ only outside of their support. A unique representative $\mathcal{S} = (A, must, may)$ of equivalence classes of modal specifications is defined by assuming that for all $u \notin \mathcal{L}_\mathcal{S}$, $must(u) = \emptyset$ and $may(u) = A$. In the sequel, only modal specifications satisfying this property are considered. Under this assumption, modal refinement is a partial order relation on modal specifications.

In [39, 40, 6], it is shown that modal refinement for modal specifications is sound *and complete*, i.e., it is equivalent to thorough refinement[1]. For *nondeterministic* modal specifications, checking thorough refinement is PSPACE-hard [3] (and also EXPTIME). As modal refinement is P-complete, a faster decision procedure exists in the deterministic case.

The following result relates implementations to consistency, for a pseudo-modal specification.

Theorem 2.1. (consistency [39, 40])
Let $^p\mathcal{S}$ be a pseudo-modal specification. Either $^p\mathcal{S}$ possesses no implementation, or there exists a largest (for refinement order) modal specification $\rho(^p\mathcal{S})$ having the same alphabet of actions and such that $\rho(^p\mathcal{S}) \leq {}^p\mathcal{S}$. In addition, $\rho(^p\mathcal{S})$ possesses the same set of implementations as $^p\mathcal{S}$.

The modal specification $\rho(^p\mathcal{S})$ is called the *pruning* of $^p\mathcal{S}$. It is obtained from $^p\mathcal{S}$ through the following steps:

1. Start from R_0, a copy of $^p\mathcal{S}$;

2. Let U_0 be the set of words inconsistently specified in R_0, meaning that $u \in U_0$ does not satisfy condition (1). For each $u \in U_0$, set $may_{R_0}(u) = A$ and $must_{R_0}(u) = \emptyset$. Then, for each word $v \in A^*$ such that $v.a = u$ for some $u \in U_0$ and $a \in A$, remove a from $may_{R_0}(v)$. Performing these two operations yields a pseudo-modal specification R_1 such that U_0 is consistently specified in R_1. Since we have only removed inconsistently specified words from $\mathcal{L}_{R_0}$, by Lemma 2.1, R_1 and R_0 possess identical sets of implementations.

3. Observe that, if $a \in must_{R_1}(v)$, then v becomes inconsistently specified in R_1. So we repeat the above step on R_1, by considering U_1, the set of words u inconsistently specified in R_1. Let $\Delta_1 \subseteq U_0 \times U_1$ be the relation consisting of the pairs (u, v) such that $v.a = u$ for some a and v is inconsistently specified in R_1. Note that v is a strict prefix of u.

4. Repeating this, we get a sequence of triples $(R_k, U_k, \Delta_k)_{k \geq 0}$ such that 1) $\bigcup_{m \leq k} U_m$ is consistently specified in R_{k+1}, and 2) $may_{R_{k+1}}(v) \subseteq may_{R_k}(v)$ for each v, with strict inclusion whenever $v.a = u$ for some $u \in U_k$, and 3) $\Delta_{k+1} \subseteq U_k \times U_{k+1}$ is the relation consisting of the pairs (u, v) such that $v.a = u$ for some a and v is inconsistently specified in R_{k+1} — again, v is a strict prefix of u.

[1] Completeness of modal refinement does not hold for nondeterministic modal automata [32]. It holds in our case since we work with specifications (for which determinism is hardwired).

5. Call *chain* a sequence $u_0, u_1, \ldots$ of words such that $(u_k, u_{k+1}) \in \Delta_{k+1}$ for every $k \geq 0$. Since u_{k+1} is a strict prefix of u_k, every chain is of length at most $|u_0|$. Thus, every inconsistently specified word of ${}^p\mathcal{S}$ is removed after finitely many steps of the above algorithm. This proves that the procedure eventually converges. The limit $\rho({}^p\mathcal{S})$ is consistent and is given by:

$$may(u) = \bigcap_k may_{R_k}(u)$$

$$must(u) = \begin{cases} must_{{}^p\mathcal{S}}(u) & \text{if } must_{{}^p\mathcal{S}}(u) \subseteq may(u) \\ \emptyset & \text{otherwise} \end{cases}$$

The above procedure terminates in finitely many steps if the support of the pseudo-modal specification is regular which is, in particular, the case of pseudo-modal specifications originated from a *deterministic pseudo-modal automaton*. This procedure also entails a sufficient condition for the satisfiability problem: a pseudo-modal specification admits a model if and only if there is no word $u \in \mathcal{L}_{{}^p\mathcal{S}}$ such that u is inconsistent and for all prefixes v of u if $u = v.a.v'$ then $a \in must(v)$. Hence this problem is NLOGSPACE-complete; it is PSPACE-hard for nondeterministic pseudo-modal specifications [3].

2.3. Operations on modal specifications

Greatest Lower Bound: The set of all pseudo-modal specifications equipped with modal refinement $\leq$ is a lattice. We denote by ${}^p\mathcal{S}_1 \;\&\; {}^p\mathcal{S}_2$ the *Greatest Lower Bound* (GLB) of ${}^p\mathcal{S}_1$ and ${}^p\mathcal{S}_2$ defined over the same alphabet. The GLB ${}^p\mathcal{S}_1 \;\&\; {}^p\mathcal{S}_2$ can be computed as

$$\begin{aligned} may(u) &= may_1(u) \cap may_2(u) \\ must(u) &= must_1(u) \cup must_2(u) \end{aligned} \tag{2}$$

Observe that, even if ${}^p\mathcal{S}_1$ and ${}^p\mathcal{S}_2$ satisfy (1), it is not guaranteed that ${}^p\mathcal{S}_1 \;\&\; {}^p\mathcal{S}_2$ does too. Hence, by using Theorem 2.1, for $\mathcal{S}_1$ and $\mathcal{S}_2$ two modal specifications, we define $\mathcal{S}_1 \wedge \mathcal{S}_2$ as being the (uniquely defined) modal specification

$$\mathcal{S}_1 \wedge \mathcal{S}_2 = \rho(\mathcal{S}_1 \;\&\; \mathcal{S}_2). \tag{3}$$

GLB satisfies the following key property, which relates it to logic formulas:

Theorem 2.2. (conjunctive interfaces [39, 40])
Let $\mathcal{I}$ be a prefix-closed language and $\mathcal{S}_1$ and $\mathcal{S}_2$ be modal specifications. Then,

$$\mathcal{I} \models \mathcal{S}_1 \wedge \mathcal{S}_2 \;\Leftrightarrow\; \mathcal{I} \models \mathcal{S}_1 \text{ and } \mathcal{I} \models \mathcal{S}_2$$

The following holds regarding supports: $\mathcal{L}_{\mathcal{S}_1 \wedge \mathcal{S}_2} \subseteq \mathcal{L}_{\mathcal{S}_1} \cap \mathcal{L}_{\mathcal{S}_2}$, with equality if and only if no pruning is needed, i.e., $\mathcal{S}_1 \wedge \mathcal{S}_2 = \mathcal{S}_1 \;\&\; \mathcal{S}_2$.

Composition: Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two modal specifications over the same alphabet. Their *composition* $\mathcal{S}_1 \otimes \mathcal{S}_2$ is defined by

$$\begin{aligned} may(u) &= may_1(u) \cap may_2(u) \\ must(u) &= must_1(u) \cap must_2(u) \end{aligned} \tag{4}$$

The following theorem shows that composition ensures substitutability.

Theorem 2.3. (substitutability in composition [39, 40])
Let $\mathcal{I}_1, \mathcal{I}_2$ be two prefix-closed languages and $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}'_1$ and $\mathcal{S}'_2$ be modal specifications:

1. If $\mathcal{S}'_1 \leq \mathcal{S}_1$ and $\mathcal{S}'_2 \leq \mathcal{S}_2$, then $\mathcal{S}'_1 \otimes \mathcal{S}'_2 \leq \mathcal{S}_1 \otimes \mathcal{S}_2$.
2. If $\mathcal{I}_1 \models \mathcal{S}_1$ and $\mathcal{I}_2 \models \mathcal{S}_2$, then $\mathcal{I}_1 \times \mathcal{I}_2 \models \mathcal{S}_1 \otimes \mathcal{S}_2$, where $\mathcal{I}_1 \times \mathcal{I}_2 = \mathcal{I}_1 \cap \mathcal{I}_2$.
3. The following holds regarding supports: $\mathcal{L}_{\mathcal{S}_1 \otimes \mathcal{S}_2} = \mathcal{L}_{\mathcal{S}_1} \cap \mathcal{L}_{\mathcal{S}_2}$.

Residuation: We now discuss the *residuation* operation which was introduced in [39, 40]. We will show that this operation is the adjoint of composition. For $\mathcal{S}_1$ and $\mathcal{S}_2$ two modal specifications, we first define their *pseudo-quotient* $\mathcal{S}_1 /\!\!/ \mathcal{S}_2$ according to the following disjunctive and exhaustive cases:

$$
\begin{array}{rll}
a \in may(u) \cap must(u) & \text{if} & a \in must_1(u) \\
& \text{and} & a \in must_2(u) \\
a \in must(u) \setminus may(u) & \text{if} & a \in must_1(u) \\
& \text{and} & a \notin must_2(u) \\
a \in may(u) \setminus must(u) & \text{if} & a \in may_1(u) \\
& \text{and} & a \notin must_1(u) \\
a \in may(u) \setminus must(u) & \text{if} & a \notin may_1(u) \\
& \text{and} & a \notin may_2(u) \\
a \notin may(u) \cup must(u) & \text{if} & a \notin may_1(u) \\
& \text{and} & a \in may_2(u)
\end{array}
$$

Observe that, due to the second case, $\mathcal{S}_1 /\!\!/ \mathcal{S}_2$ is not consistent. Having defined $\mathcal{S}_1 /\!\!/ \mathcal{S}_2$, using the pruning operation of Theorem 2.1, we can now set

$$\mathcal{S}_1 / \mathcal{S}_2 = \rho(\mathcal{S}_1 /\!\!/ \mathcal{S}_2). \tag{5}$$

Any prefix-closed language $\mathcal{I} \subseteq A^*$ can be viewed as a modal specification whose must set coincides with its may set: $\forall u \in A^*, must(u) = may(u) = \mathcal{I}_u$. Using this embedding, the quotient of two prefix-closed languages can be defined. Observe that, because of the fourth rule, the quotient of two languages is a modal specification that is not necessarily a language.

We now show that the quotient operation is indeed the adjoint of the composition operation:

Theorem 2.4. (residuation [39, 40])
Let $\mathcal{I}_1, \mathcal{I}_2$ be prefix-closed languages and $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}$ be modal specifications. Then,

1. $\mathcal{S}_1 \otimes \mathcal{S}_2 \leq \mathcal{S}$ if and only if $\mathcal{S}_2 \leq \mathcal{S} / \mathcal{S}_1$
2. $\forall \mathcal{I}_1 : [\mathcal{I}_1 \models \mathcal{S}_1 \Rightarrow \mathcal{I}_1 \times \mathcal{I}_2 \models \mathcal{S}]$ iff $\mathcal{I}_2 \models \mathcal{S} / \mathcal{S}_1$.

Example 2.3. Quotient and conjunction are illustrated in Figure 2. Suppose one aims at realizing a system whose behavior is given by the left-hand side specification: every message must be acknowledged

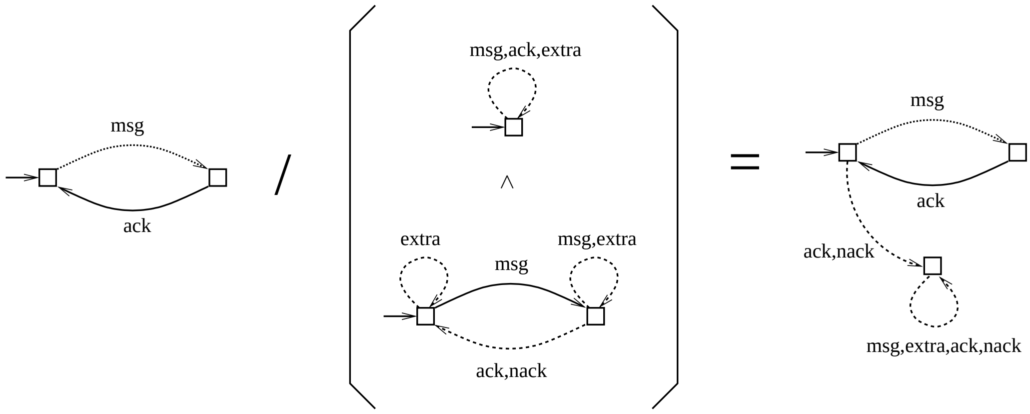


Figure 2. Quotient for top-down design

positively. For this purpose, a preexisting component conforming to the middle-hand side specifications is available in the context; it implements the specification *Fun* of Figure 1(a) with the additional assumption that the communication channel never distributes a negative acknowledgment. Then, the product of the context with any implementation of the right-hand side specification is guaranteed to be an implementation of the desired behavior.

2.4. Dissimilar alphabets

Complex systems are built by composing and combining many subsystems or components. Clearly, those objects should possess their own local alphabet of ports and variables. Dealing with those local aspects when developing the fundamental services seems like a trivial notice but has deep technical consequences. As we shall see in this section, modalities appear as an elegant solution to address alphabet equalization with appropriate flexibility.

Let us first recall how alphabet equalization is performed for the shuffle product of languages. For w a word over some alphabet A, and $B \subseteq A$, let $\mathbf{pr}_B(w)$ denote the word over B obtained by erasing, from w, all symbols not belonging to B. For $\mathcal{L}$ a language over A and $B \subseteq A \subseteq C$, the *restriction* of $\mathcal{L}$ to B is the language $\mathcal{L}_{\downarrow B} = \{u \in B^* \mid u = \mathbf{pr}_B(w)\,, w \in \mathcal{L}\}$ and the *extension* of $\mathcal{L}$ to C is the language $\mathcal{L}_{\uparrow C} = \{u \in C^* \mid \mathbf{pr}_A(u) \in \mathcal{L}\}$. The *shuffle product* $\mathcal{L}_1 \times \mathcal{L}_2$ of the two languages $\mathcal{L}_1 \subseteq A_1^*$ and $\mathcal{L}_2 \subseteq A_2^*$ is then defined as

$$\mathcal{L}_1 \times \mathcal{L}_2 = (\mathcal{L}_1)_{\uparrow C} \cap (\mathcal{L}_2)_{\uparrow C}\,, \text{ where } C = A_1 \cup A_2.$$

The shuffle product uses inverse projection to equalize alphabets. The same holds for automata over dissimilar alphabets and their synchronous product.

Using modalities allows for a *neutral* procedure for equalizing alphabets. The principle is as follows.

Observe that, by (4),

$$
\begin{array}{c}
a \in must_1(s) \ \text{ and } \ a \in \mathbf{whatever}_2(s) \\
\Downarrow \\
a \in \mathbf{whatever}(s)
\end{array}
\tag{6}
$$

holds if the two interfaces are combined using parallel composition (here, $\mathbf{whatever}$ denotes an arbitrary modality). Similarly, by (2),

$$
\begin{array}{c}
a \in may_1(s) \ \text{ and } \ a \in \mathbf{whatever}_2(s) \\
\Downarrow \\
a \in \mathbf{whatever}(s)
\end{array}
\tag{7}
$$

holds if the two interfaces are combined using conjunction. The observation above reveals our solution: alphabet extension is performed by setting the *specific* modalities for extended traces, specifically

- may in case of the conjunction $\wedge$;
- $must$ in case of the parallel composition $\otimes$.

These two types of alphabet extensions are called *weak* and *strong*. This is a key contribution of our work as it will provide us with a very elegant way of dealing with dissimilar alphabets.

Definition 2.5. (weak and strong extensions)
Let ${}^p\mathcal{S} = (A, must_{{}^p\mathcal{S}}, may_{{}^p\mathcal{S}})$ be a pseudo-modal specification and let $C \supseteq A$.

1. The *weak extension* of ${}^p\mathcal{S}$ to C is the pseudo-modal specification ${}^p\mathcal{S}_{\Uparrow C} = (C, must, may)$ such that $\forall v \in C^*$:
$$\begin{cases} must(v) &= must_{{}^p\mathcal{S}}(\mathbf{pr}_A(v)) \\ may(v) &= may_{{}^p\mathcal{S}}(\mathbf{pr}_A(v)) \cup (C \setminus A) \end{cases}$$

2. The *strong extension* of ${}^p\mathcal{S}$ to C is the pseudo-modal specification ${}^p\mathcal{S}_{\uparrow C} = (C, must, may)$ such that $\forall v \in C^*$:
$$\begin{cases} must(v) &= must_{{}^p\mathcal{S}}(\mathbf{pr}_A(v)) \cup (C \setminus A) \\ may(v) &= may_{{}^p\mathcal{S}}(\mathbf{pr}_A(v)) \cup (C \setminus A) \end{cases}$$

Regarding supports, the following equalities hold: $\mathcal{L}_{(\mathcal{S}_{\Uparrow C})} = \mathcal{L}_{(\mathcal{S}_{\uparrow C})} = (\mathcal{L}_\mathcal{S})_{\uparrow C}$. We are now ready to extend the operations of Sections 2.2 and 2.3 to the case of dissimilar alphabets.

Definition 2.6. Let ${}^p\mathcal{S}, {}^p\mathcal{S}_i$ and $\mathcal{S}_i$ be pseudo-modal or modal specifications over alphabets A, A_i for $i = 1, 2$, respectively. The relations and operations of Section 2.2 are redefined as follows:

$$
\begin{array}{ll}
[\text{weak implementation; } C \supseteq A] & \\
& \mathcal{I} \subseteq C^* \models_w {}^p\mathcal{S} \ \text{ iff } \ \mathcal{I} \models {}^p\mathcal{S}_{\Uparrow C} \\
[\text{strong implementation; } C \supseteq A] & \\
& \mathcal{I} \subseteq C^* \models_s {}^p\mathcal{S} \ \text{ iff } \ \mathcal{I} \models {}^p\mathcal{S}_{\uparrow C}
\end{array}
$$

$$
\begin{aligned}
&[\text{weak refinement; } A_2 \supseteq A_1] \\
&\qquad {}^p\mathcal{S}_2 \leq_w {}^p\mathcal{S}_1 \;\text{ iff }\; {}^p\mathcal{S}_2 \leq {}^p\mathcal{S}_{1\Uparrow A_{\mathcal{S}_2}} \\
&[\text{strong refinement; } A_2 \supseteq A_1] \\
&\qquad {}^p\mathcal{S}_2 \leq_s {}^p\mathcal{S}_1 \;\text{ iff }\; {}^p\mathcal{S}_2 \leq {}^p\mathcal{S}_{1\uparrow A_{\mathcal{S}_2}}
\end{aligned}
$$

$$
\begin{aligned}
&[\text{operators; } C = A_1 \cup A_2] \\
&\qquad \mathcal{S}_1 \wedge \mathcal{S}_2 = \mathcal{S}_{1\Uparrow C} \wedge \mathcal{S}_{2\Uparrow C} \\
&\qquad \mathcal{S}_1 \otimes \mathcal{S}_2 = \mathcal{S}_{1\uparrow C} \otimes \mathcal{S}_{2\uparrow C} \\
&\qquad \mathcal{S}_1 \,/\, \mathcal{S}_2 = \mathcal{S}_{1\Uparrow C} \,/\, \mathcal{S}_{2\uparrow C}
\end{aligned}
$$

Note the careful use of weak and strong extensions in the different operations. The results of Sections 2.2 and 2.3 are slightly weakened as indicated next.

Theorem 2.5. Let $\mathcal{S}$, $\mathcal{S}_i$ and $\mathcal{S}'_i$ be modal specifications defined over A, A_i and A'_i respectively, for $i = 1, 2$.

1. Weak and strong implementation / refinement relations are related as follows:

$$\models_s \subseteq \models_w \quad \text{and} \quad \leq_s \subseteq \leq_w$$

2. Weak and strong modal refinement are both sound and complete w.r.t. weak and strong thorough refinement, respectively:

$$
\begin{aligned}
\mathcal{S}_2 \leq_w \mathcal{S}_1 \quad &\Leftrightarrow \quad \{\mathcal{I} \mid \mathcal{I} \models_w \mathcal{S}_2\} \subseteq \{\mathcal{I} \mid \mathcal{I} \models_w \mathcal{S}_1\} \\
\mathcal{S}_2 \leq_s \mathcal{S}_1 \quad &\Leftrightarrow \quad \{\mathcal{I} \mid \mathcal{I} \models_s \mathcal{S}_2\} \subseteq \{\mathcal{I} \mid \mathcal{I} \models_s \mathcal{S}_1\}
\end{aligned}
$$

3. The following holds regarding conjunction:

$$\mathcal{I} \models_w \mathcal{S}_1 \wedge \mathcal{S}_2 \;\Leftrightarrow\; \mathcal{I} \models_w \mathcal{S}_1 \text{ and } \mathcal{I} \models_w \mathcal{S}_2$$

4. Theorem 2.3 still holds when alphabets are different, provided that *strong* refinement and implementation are used — it is actually false if weak refinement or implementation are used:

   - If $\mathcal{S}'_1 \leq_s \mathcal{S}_1$ and $\mathcal{S}'_2 \leq_s \mathcal{S}_2$, then $\mathcal{S}'_1 \otimes \mathcal{S}'_2 \leq_s \mathcal{S}_1 \otimes \mathcal{S}_2$;
   - If $\mathcal{I}_1 \models_s \mathcal{S}_1$ and $\mathcal{I}_2 \models_s \mathcal{S}_2$, then $\mathcal{I}_1 \times \mathcal{I}_2 \models_s \mathcal{S}_1 \otimes \mathcal{S}_2$;
   - $\mathcal{S}'_1 \leq_w \mathcal{S}_1$ and $\mathcal{S}'_2 \leq_w \mathcal{S}_2$ in general do *not* imply that $\mathcal{S}'_1 \otimes \mathcal{S}'_2 \leq_w \mathcal{S}_1 \otimes \mathcal{S}_2$;
   - $\mathcal{I}_1 \models_w \mathcal{S}_1$ and $\mathcal{I}_2 \models_w \mathcal{S}_2$ in general do *not* imply that $\mathcal{I}_1 \times \mathcal{I}_2 \models_w \mathcal{S}_1 \otimes \mathcal{S}_2$.

5. Relations between the quotient and the composition operators are preserved provided additional assumptions on alphabets:

$$\left.\begin{array}{r}\mathcal{S}_2 \leq_s \mathcal{S}/\mathcal{S}_1 \\ A_1 \subseteq A\end{array}\right\} \Rightarrow \mathcal{S}_1 \otimes \mathcal{S}_2 \leq_s \mathcal{S}$$

$$\left.\begin{array}{r}\mathcal{S}_1 \otimes \mathcal{S}_2 \leq_s \mathcal{S} \\ A_2 \supseteq A \cup A_1\end{array}\right\} \Rightarrow \mathcal{S}_2 \leq_s \mathcal{S}/\mathcal{S}_1$$

$$\left.\begin{array}{r}\mathcal{I}_1 \models_s \mathcal{S}_1 \text{ and } \mathcal{I}_2 \models_s \mathcal{S}/\mathcal{S}_1 \\ A_1 \subseteq A\end{array}\right\} \Rightarrow \mathcal{I}_1 \times \mathcal{I}_2 \models_s \mathcal{S}$$

$$\left.\begin{array}{r}\forall \mathcal{I}_1 : \mathcal{I}_1 \models_s \mathcal{S}_1 \ \Rightarrow\ \mathcal{I}_1 \times \mathcal{I}_2 \models_s \mathcal{S} \\ \text{and } A_{\mathcal{I}_2} \supseteq A \cup A_1\end{array}\right\} \Rightarrow \mathcal{I}_2 \models_s \mathcal{S}/\mathcal{S}_1$$

Observe that the last sub-statement of statement 5 refines Theorem 2.4.

Proof:
The detailed proof of this theorem can be found in [43]. We only give here the counterexamples for the third and the fourth bullets of statement 4. First, the following counterexample shows that *composition is not monotonic wrt to the weak refinement when alphabets are different*. Consider the three modal specifications:

- $\mathcal{S}_1$ with $A_1 = \{a\}$ and $may_1(\epsilon) = must_1(\epsilon) = \emptyset$;
- $\mathcal{S}'_1$ with $A'_1 = \{a, b\}$ and $may'_1(\epsilon) = \{b\}$ and $must'_1(\epsilon) = \emptyset$;
- $\mathcal{S}_2$ with $A_{\mathcal{S}_2} = \{b\}$ and $may_2(\epsilon) = must_2(\epsilon) = \{b\}$.

Then $\mathcal{S} = \mathcal{S}_1 \otimes \mathcal{S}_2$ is defined over $\{a, b\}$ and $may(\epsilon) = must(\epsilon) = \{b\}$; and, $\mathcal{S}' = \mathcal{S}'_1 \otimes \mathcal{S}_2$ is defined over $\{a, b\}$ and $may'(\epsilon) = \{b\}$ and $must'(\epsilon) = \emptyset$. Thus we have: $\mathcal{S}'_1 \leq_w \mathcal{S}_1$ and $\mathcal{S}'_1 \otimes \mathcal{S}_2 \not\leq_w \mathcal{S}_1 \otimes \mathcal{S}_2$.

Now, this counter-example shows that $\mathcal{I}_1 \models_w \mathcal{S}_1$ *and* $\mathcal{I}_2 \models_w \mathcal{S}_2$ *do not imply* $\mathcal{I}_1 \times \mathcal{I}_2 \models_w \mathcal{S}_1 \otimes \mathcal{S}_2$:

- $\mathcal{S}_1$ with $A_1 = \{a\}$ and $may_1(\epsilon) = must_1(\epsilon) = \emptyset$; $\mathcal{I}_1$ with $A_{\mathcal{I}_1} = \{a, b\}$ and $\mathcal{I}_1 = \{\epsilon\}$;
- $\mathcal{S}_2$ with $A_2 = \{b\}$ and $may_2(\epsilon) = must_2(\epsilon) = \{b\}$; $\mathcal{I}_2$ with $A_{\mathcal{I}_2} = \{b\}$ and $\mathcal{I}_2 = \{\epsilon, b\}$.

Then $\mathcal{I}_1 \models_w \mathcal{S}_1$ and $\mathcal{I}_2 \models_w \mathcal{S}_2$. $\mathcal{I}_1 \times \mathcal{I}_2 = \{\emptyset\}$ and $may_{\mathcal{S}_1 \otimes \mathcal{S}_2}(\epsilon) = must_{\mathcal{S}_1 \otimes \mathcal{S}_2}(\epsilon) = \{b\}$ thus $\mathcal{I}_1 \times \mathcal{I}_2$ is not a weak implementation of $\mathcal{S}_1 \otimes \mathcal{S}_2$. □

Example 2.4. Consider now a second specification *Rel* for a producer in Figure 3(a) dealing with reliability: messages are negatively acknowledged until the system is reset. The specification *Fun* in Figure 1(a) and *Rel* are defined on different alphabets: the action *reset* is not part of the alphabet of *Fun* and similarly for *extra* in *Rel*. The conjunction of the two aspects is depicted in Figure 3(b); observe that the modalities of the transitions labeled by *reset* are directly inherited from those in *Rel*.

3. Interface automata

In [20], de Alfaro and Henzinger introduced *interface automata*, which are automata whose transitions are typed with *input* and *output* actions rather than with modalities. In this section, we briefly overview the theory of interface automata and refer the reader to [20, 17] for more details.

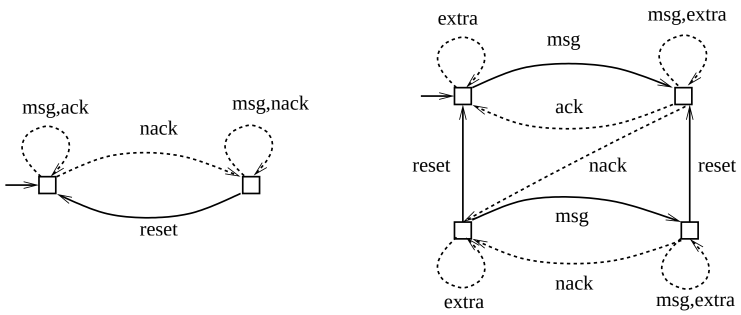
Definition 3.1. ([20])
An *interface automaton* is a tuple $\mathcal{P} = (X, x_0, A, \rightarrow)$, where X is the set of *states*, $x_0 \in X$ is the *initial state*, A is the alphabet of *actions*, and $\rightarrow \subseteq X \times A \times X$ is the transition relation.

We decompose $A = A? \cup A!$, where $A?$ is the set of inputs and $A!$ is the set of outputs. In the rest of the paper, we shall often use $a?$ to emphasize that $a \in A?$ and $a!$ for $a \in A!$. We will also use $x \stackrel{a}{\longrightarrow} y$ to emphasize that $(x, a, y) \in \rightarrow$. Observe that if we consider deterministic interface automata, then we can propose a language-based definition similar to the one we gave for modal specifications.

The semantics of an interface automaton is given by a two-player game between an *input* player that represents the environment (the moves are the input actions), and an *output* player that represents the component itself (the moves are the output actions). Input and output moves are in essence orthogonal to modalities. Interface automata are operational models that do not distinguish between an interface and one of its models. More precisely, the model of an interface automaton is any of its refinements. As a consequence, the notion of refinement coincides with the one of satisfaction. Moreover, any interface automaton is always satisfiable except if it is empty.

Remark 3.1. In interface automata, the distinction between inputs and outputs should not be interpreted as a function from the Inputs to the Outputs.

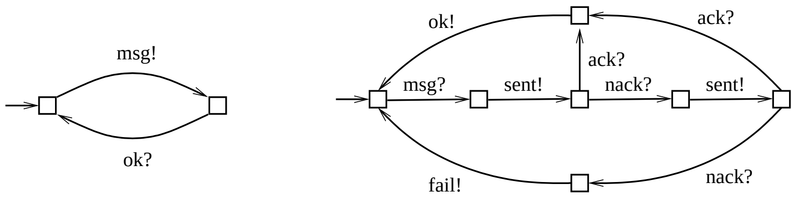
Example 3.1. Two interface automata are depicted in Figure 4 (this example is adapted from [20]). The client *Cl* in Figure 4(a) is defined over the alphabet $\{ok?, fail?\} \cup \{msg!\}$. The action *fail?* never occurs which encodes the assumption that the environment of the client never transmits a *fail* to the client. The server *Serv* in Figure 4(b) is defined over the alphabet $\{msg?, ack?, nack?\} \cup \{sent!, ok!, fail!\}$; when *msg* is invoked, the server tries to send the message and resends it if the first transmission fails. If both transmissions fail, the component reports failure (*fail!*), otherwise it reports success (*ok!*).



(a) Reliability spec. *Rel* (b) Conjunction *Fun* $\wedge$ *Rel*

Figure 3. Conjunction of specifications on dissimilar alphabets

Alternatively, properties of interfaces are described in game-based logics, such as ATL or ATL*[1] whose complexities are PSPACE and PTIME-complete, respectively. Refinement between interface automata corresponds to the alternating refinement relation between games [2], i.e., an interface refines another one if its environment is more permissive whereas its component is more restrictive. This problem is known to be PTIME-complete. There is no notion of component reuse and shared refinement is defined in an ad-hoc manner [22].



(a) Client *Cl*

(b) Server *Serv*

Figure 4. Two interface automata to be composed

Remark 3.2. Contrary to input/output automata, interface automata are generally not input-enabled[2]. Refinement of input/output automata corresponds to simulation between traces. If the model was not input-enabled, then a refinement could accept less inputs than its abstraction. The game-based approach allows us to avoid such a situation even when the system is not input enabled.

The main advantage of the game-based approach appears in the definition of composition and *compatibility* between interface automata. Following [17], two interface automata are *composable* if they have disjoint sets of output actions, compose by synchronizing shared actions and interleave asynchronously all other actions.

Definition 3.2. (Product of interface automata [20])
Let $\mathcal{P}_1 = (X_1, x_{01}, A_1, \rightarrow_1)$ and $\mathcal{P}_2 = (X_2, x_{02}, A_2, \rightarrow_2)$ be two interface automata. The product over $\mathcal{P}_1$ and $\mathcal{P}_2$ is an interface automaton $\mathcal{P}_1 \times \mathcal{P}_2 = (X, x_0, A, \rightarrow)$, where

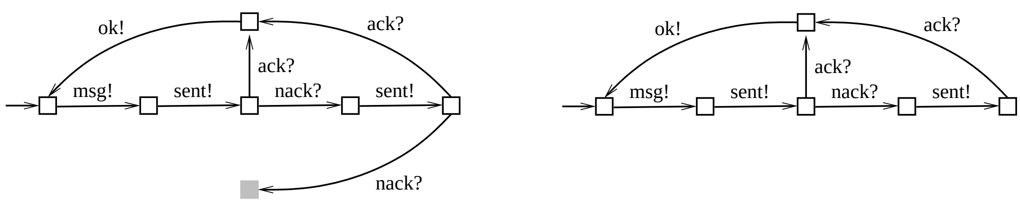
- $X = X_0 \times X_1$;
- $x_0 = x_{01} \times x_{02}$;
- $A = A_1 \cup A_2$, and $A? = (A_1? \cup A_2?) \setminus ((A_1? \cap A_2!) \cup (A_2? \cap A_1!))$, and $A! = A_1! \cup A_2!$;
- $\rightarrow$ is defined as follows:
  - For each action $a \in A$ such that $a \notin A_1 \cap A_2$, there exists a transition $(x_1, y_1) \xrightarrow{a} (x_2, y_2)$ iff there exists $(x_1) \xrightarrow{a}_1 (x_2)$ and $y_1 = y_2$ or $(y_1) \xrightarrow{a}_2 (y_2)$ and $x_1 = x_2$.
  - For each action $a \in A_1? \cap A_2?$, there exists a transition $(x_1, y_1) \xrightarrow{a?} (x_2, y_2)$ iff there exists $(x_1) \xrightarrow{a?}_1 (x_2)$ and $(y_1) \xrightarrow{a?}_2 (y_2)$.

[2]Recall that a system is input-enabled if it can react to any input action in any moment.

– For each $a \in (A_1? \cap A_2!) \cup (A_2? \cap A_1!)$, there exists a transition $(x_1, y_1) \xrightarrow{a!} (x_2, y_2)$ iff there exists $(x_1) \xrightarrow{a}_1 (x_2)$ and $(y_1) \xrightarrow{a}_2 (y_2)$.

Since interface automata are not necessarily input-enabled (which is what allows the automaton to make assumptions on the environment), the product $\mathcal{P}_1 \times \mathcal{P}_2$ of two interface automata $\mathcal{P}_1$ and $\mathcal{P}_2$ may have *illegal states* where one of the automata may produce an output action that is also in the input alphabet of the other automaton, but is not accepted at this state. In most of existing models for interface theories that are based on an input/output setting, the interfaces would be declared to be *incompatible*. This is a pessimistic approach that can be avoided by exploiting the game-based semantics. Indeed, the game semantics supports an optimistic approach:

> "Two interfaces can be composed and are compatible if there is at least one environment where they can work together (i.e., where they can avoid the illegal states)."



(a) Product *Cl* × *Serv* (b) Composition *Cl* || *Serv*

Figure 5. Compatibility of *Serv* and *Cl*

Deciding whether there exists an environment where the two interfaces can work together is equivalent to checking whether the environment in the product of the interfaces has a strategy to always avoid illegal states. This can be viewed as a reachability game whose complexity is linear [20]. The set of states from which the environment has a strategy to avoid the illegal states whatever the component does can be recursively computed as follows.

Let $\mathit{Illegal}(\mathcal{P}_1, \mathcal{P}_2)$ be the subset of pairs $(x_1, x_2) \in X_1 \times X_2$ such that there exists

either an action $a \in A_1! \cap A_2?$	with	$x_1 \xrightarrow{a!}_1$
	but not	$x_2 \xrightarrow{a?}_2$
or an action $a \in A_2! \cap A_1?$	with	$x_2 \xrightarrow{a!}_2$
	but not	$x_1 \xrightarrow{a?}_1$

where $x \xrightarrow{a}$ means that $x \xrightarrow{a} y$ for some state y. If illegal states exist in the product $\mathcal{P}_1 \times \mathcal{P}_2$, there may still exist refinements of the product without illegal state. Those refinements specify how the resulting product should be restricted in order to guarantee that illegal states cannot be reached. As proved in [20], there is one such largest refinement which can be obtained by backward pruning $\mathcal{P}_1 \times \mathcal{P}_2$ as follows. For $Y \subseteq X$, the set of states of $\mathcal{P}_1 \times \mathcal{P}_2$, let $pre_!(Y)$ be the subset $Z \subseteq X$ of states z such that $z \xrightarrow{a!} y$ for some $y \in Y$ and $a! \in A!$ (an output action of the product). Let $pre_!^0(Y) = Y$ and, for $k \geq 0$,

$pre_!^{k+1}(Y) = pre_!\big(pre_!^k(Y)\big)$ and let $pre_!^*(Y) = \bigcup_k pre_!^k(Y)$.

The desired pruning consists in:

- Removing $pre_!^*(\mathit{Illegal}(\mathcal{P}_1, \mathcal{P}_2))$ from X, and
- Removing transitions to states in $pre_!^*(\mathit{Illegal}(\mathcal{P}_1, \mathcal{P}_2))$, and
- Removing unreachable states.

The result of applying the pruning to $\mathcal{P}_1 \times \mathcal{P}_2$ is denoted by $\mathcal{P}_1 \parallel \mathcal{P}_2$ and is called the *composition* of the two interface automata. $\mathcal{P}_1$ and $\mathcal{P}_2$ are called *compatible* if applying the pruning leaves the initial state [20]. We now recall the two following theorems from [20] that show that interface automata support independent design and substitutability.

Theorem 3.1. ([20])
The composition operation for interface automata is associative and commutative.

Theorem 3.2. ([20])
Let $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ be three interface automata. If $\mathcal{P}_2$ refines $\mathcal{P}_1$ and the set of shared actions of $\mathcal{P}_2 \parallel \mathcal{P}_3$ is included in the set of shared actions of $\mathcal{P}_1 \parallel \mathcal{P}_3$, then $\mathcal{P}_2 \parallel \mathcal{P}_3$ refines $\mathcal{P}_1 \parallel \mathcal{P}_3$.

Example 3.2. The product of the interface automata in Figure 4 is represented in Figure 5(a). The gray state is illegal as the server wants to report a failure (*fail!*) which is not accepted as an input by the client. The result of applying the pruning operation is then depicted in Figure 5(b).

Bhaduri has proposed a quotient operation that is the adjoint of the composition operation [10]. This quotient, which is defined for the deterministic fragment only, is characterized in the following theorem. Let $\mathcal{P}^\perp$ be the interface $\mathcal{P}$ where input and output actions have been exchanged.

Theorem 3.3. ([20])
Consider two deterministic interface automata $\mathcal{P}_1$ and $\mathcal{P}_2$. If $\mathcal{P}_1$ and $\mathcal{P}_2^\perp$ are compatible, then there exists $\mathcal{P}$ such that

1. $\mathcal{P}_1 \parallel \mathcal{P} \leq \mathcal{P}_2$,
2. for each $\mathcal{P}'$ such that $\mathcal{P}_1 \parallel \mathcal{P}' \leq \mathcal{P}_2$, we have $\mathcal{P}' \leq \mathcal{P}$ and,
3. $\mathcal{P}$ is given by $(\mathcal{P}_1 \parallel \mathcal{P}_2^\perp)^\perp$.

The theorem above states that, contrary to the case of modal automata, the quotient for interface automata can be derived from the composition operation with a simple switch operation between input and output actions.

Remark 3.3. The operations between interface automata that have been defined so far do not require an explicit treatment of dissimilar alphabets as is the case for modal specifications. Indeed, it is implicitely handled with the help of the game-based approach. Conjunction is not defined for interface automata. For such an operation, we conjuncture that the game-based approach is not powerful enough for an implicit treatment.

4. On modal interfaces

We now present the full theory for *modal interfaces*. Modal interfaces is an extension of modal specifications where actions are also typed with *input* and *output*. This addition allows us to define notions of composition and compatibility for modal specifications in the spirit of interface automata.

The first account on compatibility for modal interfaces was proposed in [31, 36]. In this section, we propose a full interface theory for modal interfaces, which includes composition, product, conjunction, and component reuse via quotient. Moreover, we show that the composition operator proposed in [31, 36] is incorrect and we propose a correction.

We shall start our theory with the definition of *profiles* which are used to type actions of modal specifications with *input* and *output* modalities.

4.1. Profiles

Given an alphabet of actions A, a *profile* is a function $\pi : A \mapsto \{?, !\}$, labeling actions with the symbols ? (for *inputs*) or ! (for *outputs*). We write "$a?$" (respectively, $a!$) to express that "$\pi(a) = ?$" (respectively, $\pi(a) = !$). The set of $a \in A$ such that $\pi(a) = ?$ (respectively, $\pi(a) = !$) is denoted $A?$ (respectively $A!$). We shall sometimes write by abuse of notation, $\pi = (A?, A!)$.

We now discuss operations on profiles. We consider a profile $\pi_1 = (A_1?, A_1!)$ defined over A_1 and a profile $\pi_2 = (A_2?, A_2!)$ defined over A_2.

Refinement between profiles. Profile π_2 refines π_1, denoted $\pi_2 \leq \pi_1$, if and only if $A_2 \supseteq A_1$ and both profiles coincide on A_1: $\forall a \in A_1, \pi_2(a) = \pi_1(a)$.

Proposition 4.1. The refinement $\leq$ between profiles is transitive.

Product between profiles. The product between π_1 and π_2, denoted $\pi_1 \otimes \pi_2$ is defined if and only if $A_1! \cap A_2! = \emptyset$, and is equal to the profile $\pi = (A?, A!)$ over $A_1 \cup A_2$ such that:

$$\pi = \pi_1 \otimes \pi_2 : \begin{cases} A! & = & (A_1! & \cup & A_2!) \\ A? & = & (A_1? & \cup & A_2?) \setminus A! \end{cases}$$

Proposition 4.2. Let π_1', π_2', π_1, and π_2 be profiles. The product between profiles is monotonic with respect to refinement: if $\pi_1' \leq \pi_1$ and $\pi_2' \leq \pi_2$ and $\pi_1 \otimes \pi_2$ and $\pi_1' \otimes \pi_2'$ are defined, then $\pi_1' \otimes \pi_2' \leq \pi_1 \otimes \pi_2$.

Conjunction between profiles. The conjunction of π_1 and π_2, denoted $\pi_1 \wedge \pi_2$, is the greatest lower bound of the profiles, whenever it exists. More precisely, the conjunction of profiles π_1 and π_2 is defined if and only if both profiles coincide on their common alphabet: $\forall a \in A_1 \cap A_2, \pi_1(a) = \pi_2(a)$. Whenever defined, the conjunction $\pi_1 \wedge \pi_2$ coincides with π_1 for every symbol in A_1 and with π_2 for every symbol in A_2.

Proposition 4.3. Let π_1, π_2 and π be profiles. Then, $\pi \leq \pi_1 \wedge \pi_2$ if and only if $\pi \leq \pi_1$ and $\pi \leq \pi_2$.

Quotient between profiles. The *quotient* of π_1 and π_2, denoted π_1 / π_2, is defined as the adjoint of $\otimes$, if it exists, namely $\pi_1 / \pi_2 = \max\{\pi \mid \pi \otimes \pi_2 \leq \pi_1\}$. More precisely, π_1 / π_2 is defined when $A_1? \cap A_2! = \emptyset$, and is thus equal to the profile $\pi = (A?, A!)$ such that

$$\pi_1 / \pi_2 : \begin{cases} A! &= A_1! \setminus (A_1! \cap A_2!) \\ A? &= [(A_1? \cup A_2?) \setminus A!] \cup A_2! \end{cases}$$

Proposition 4.4. Let π, π_1 and π_2 be profiles defined over the A, A_1 and A_2 respectively:

- if $\pi_1 \otimes \pi_2 \leq \pi$ and $A_2 \supseteq A \cup A_1$, then $\pi_2 \leq \pi/\pi_1$;
- if $\pi_2 \leq \pi/\pi_1$ and $A_1 \subseteq A$, then $\pi_1 \otimes \pi_2 \leq \pi$.

4.2. The framework of modal interfaces

We now formally introduce modal interfaces that are modal specifications whose actions are also labeled with *input* and *output* attributes. We will consider the language representation in the spirit of [40, 39, 42], while Larsen et al. followed the automata-based representation (the two representations are equivalent).

Definition 4.1. (Modal interface)
A *modal interface* is a pair $\mathcal{C} = (\mathcal{S}, \pi)$, where $\mathcal{S}$ is a modal specification over the alphabet $A_{\mathcal{S}}$ and $\pi : A_{\mathcal{S}} \rightarrow \{?, !\}$ is a *profile*.

A model for a modal interface is a pair $(\mathcal{I}, \pi')$, where $\mathcal{I}$ is a prefix-closed language and π' is a profile for $\mathcal{I}$. We say that $(\mathcal{I}, \pi')$ *strongly implements* $(\mathcal{S}, \pi)$, written $(\mathcal{I}, \pi') \models_s (\mathcal{S}, \pi)$, if $\mathcal{I} \models_s \mathcal{S}$ and $\pi' \leq \pi$, and similarly for *weak implementation*. We say that $(\mathcal{S}_2, \pi_2) \leq_s (\mathcal{S}_1, \pi_1)$ if $\mathcal{S}_2 \leq_s \mathcal{S}_1$ and $\pi_2 \leq \pi_1$, and analogously for weak refinement $\leq_w$. The *composition* of two models is the pair that results from the shuffle product $\times$ of their prefix-closed languages and of the product of their profiles.

4.3. Operations on modal interfaces

Operations on modal specifications directly extend to operations on modal interfaces. We have the following definition.

Definition 4.2. Consider two modal interfaces $\mathcal{C}_1 = (\mathcal{S}_1, \pi_1)$ and $\mathcal{C}_2 = (\mathcal{S}_2, \pi_2)$, and let $\star \in \{\wedge, \otimes, /\}$. If $\pi_1 \star \pi_2$ is defined, then

$$\mathcal{C}_1 \star \mathcal{C}_2 = (\mathcal{S}_1 \star \mathcal{S}_2, \pi_1 \star \pi_2).$$

The following theorem states that all the characteristic properties of modal specifications directly extend to modal interfaces.

Theorem 4.1. Propositions stated in Theorem 2.5 extend to modal interfaces.

We now recap the translation from interface automata to modal interfaces, which will help us make the link between modalities and input or output actions.

4.4. From interface automata to modal interfaces

We recap the translation from interface automata to modal automata that has been proposed in [31]. In this section, we extend this translation to modal specifications, the language-algebraic extension corresponding to modal automata.

We consider an interface automaton $\mathcal{P} = (X, x_0, A, \rightarrow)$. We assume $\mathcal{P}$ to be deterministic and we let $\mathcal{L}_\mathcal{P}$ denote the (prefix-closed) language defined by $\mathcal{P}$. The alphabet of $\mathcal{S}_\mathcal{P}$ is $A_{\mathcal{S}_\mathcal{P}} = A$ and modalities are defined for all $u \in A^*_\mathcal{P}$:

$$\begin{array}{lll} a? \in must_{\mathcal{S}_\mathcal{P}}(u) & \text{if} & u.a? \in \mathcal{L}_\mathcal{P} \\ a! \in may_{\mathcal{S}_\mathcal{P}}(u) \setminus must_{\mathcal{S}_\mathcal{P}}(u) & \text{if} & u.a! \in \mathcal{L}_\mathcal{P} \\ a? \in may_{\mathcal{S}_\mathcal{P}}(u) \setminus must_{\mathcal{S}_\mathcal{P}}(u) & \text{if} & u \in \mathcal{L}_\mathcal{P} \\ & \text{and} & u.a? \notin \mathcal{L}_\mathcal{P} \\ a! \notin may_{\mathcal{S}_\mathcal{P}}(u) & \text{if} & u \in \mathcal{L}_\mathcal{P} \\ & \text{and} & u.a! \notin \mathcal{L}_\mathcal{P} \\ a \in may_{\mathcal{S}_\mathcal{P}}(u) \setminus must_{\mathcal{S}_\mathcal{P}}(u) & \text{if} & u \notin \mathcal{L}_\mathcal{P}. \end{array} \tag{8}$$

Theorem 1 of [31] shows that, with the above correspondence, alternating simulation for interface automata and modal refinement for modal interfaces coincide. Regarding supports, we have:

$$\mathcal{L}_{\mathcal{S}_\mathcal{P}} = \mathcal{L}_\mathcal{P} \cup \{u.a?.v \mid u \in \mathcal{L}_\mathcal{P}, u.a? \notin \mathcal{L}_\mathcal{P}, v \in A^*_\mathcal{P}\}\,. \tag{9}$$

It is worth making some comments about this translation, given by formulas (8,9). Regarding formula (9), the supporting language $\mathcal{L}_{\mathcal{S}_\mathcal{P}}$ allows the environment to violate the constraints set on it by the interface automaton $\mathcal{P}$. When this happens—formally, the environment exits the alternating simulation relation—the component considers that the assumptions under which it was supposed to perform are violated, so it allows itself breaching its own promises and can perform anything afterward. One could also see the violation of assumptions as an exception. Then, $\mathcal{L}_{\mathcal{S}_\mathcal{P}}$ states no particular exception handling since everything is possible. Specifying exception handling then amounts to refining this modal interface.

Formula (8) refines (9) by specifying obligations. Case 1 expresses that the component *must* accept from the environment any input within the assumptions. Case 2 indicates that the component behaves according to best effort regarding its own outputs actions. Finally, cases 3 and 4 express that the violation by the environment of the assumptions made by the component are seen as an exception, and that exception handling is unspecified and not mandatory. This embedding is illustrated in Figure 6 for the case of the Client of Figure 4(a).

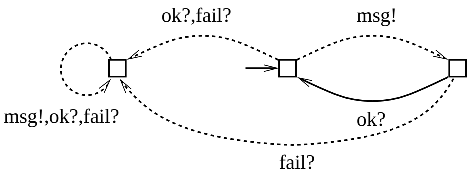


Figure 6. Embedding of the interface automaton Cl from Figure 4(a) into a modal interface

4.5. On compatibility for modal interfaces

In this section, we take advantage of profiles to define a notion of composition for modal interfaces including the compatibility issue introduced for interface automata. We shall recap the solution proposed in [31, 36], then we shall show a counter example to Theorem 10 in [31] and then propose our correction.

4.5.1. The composition and the bug in Theorem 10 of [31]

We now consider the notion of compatibility of two modal interfaces $\mathcal{C}_1 = (\mathcal{S}_1, \pi_1)$ and $\mathcal{C}_2 = (\mathcal{S}_2, \pi_2)$ with $\mathcal{S}_1$ defined over A_1 and $\mathcal{S}_2$ defined over A_2. We assume that $\mathcal{C}_1$ and $\mathcal{C}_2$ do not share common output actions (which is the composability requirement similar to the one for interface automata). We first compute the product between $\mathcal{C}_1$ and $\mathcal{C}_2$ following Definition 4.3. We then define $Illegal(\mathcal{C}_1, \mathcal{C}_2)$ to be the subset of words u belonging to the support of $\mathcal{C}_1 \otimes \mathcal{C}_2$, such that one interface *may* produce an output that *may not* be accepted as an input by the other interface:

$$\begin{array}{rl} \text{either} & \text{an action } a \in A_1! \cap A_2? \\ & \text{with } a \in may_1(u_1) \setminus must_2(u_2) \\ \text{or} & \text{an action } a \in A_2! \cap A_1? \\ & \text{with } a \in may_2(u_2) \setminus must_1(u_1), \end{array} \tag{10}$$

where $u_1 = \mathbf{pr}_{A_1}(u)$ and similarly $u_2 = \mathbf{pr}_{A_2}(u)$. In order to get rid of illegal runs, we must first consider the words v having a suffix v' such that $v.v'$ is illegal and v' is a sequence of outputs; this way, no environment can prevent v' to occur from v. For U a set of words of modal interface $\mathcal{C}$, let $pre_!(U)$ be the set

$$pre_!(U) \quad = \quad \{v \in \mathcal{L}_{\mathcal{C}} \mid \exists a! \in may(v), v.a! \in U\}$$

Let $pre_!^0(U) = U$, and, for $k \geq 0$, $pre_!^{k+1}(U) = pre_!\big(pre_!^k(U)\big)$. Finally, let $pre_!^*(U) = \bigcup_k pre_!^k(U)$. The composition of two modal interfaces is obtained from their product by removing words in $pre_!^*(U)$, following the approach outlined for interface automata. Two modal interfaces are compatible if the pruning with the illegal words do not remove the empty word. The composition between $\mathcal{C}_1$ and $\mathcal{C}_2$ is denoted $\mathcal{C}_1 \| \mathcal{C}_2$.

Theorem 10 in [31, 36] states that

> "*(Independent Implementability). For any two composable modal interfaces $\mathcal{C}_1, \mathcal{C}_2$ and two implementations $(\mathcal{I}_1, \pi_1)$ and $(\mathcal{I}_2, \pi_2)$. If $(\mathcal{I}_1, \pi_1) \leq \mathcal{C}_1$ and $(\mathcal{I}_2, \pi_2) \leq \mathcal{C}_2$, then it holds that $(\mathcal{I}_1, \pi_1) \times (\mathcal{I}_2, \pi_2) \leq \mathcal{C}_1 \| \mathcal{C}_2$.*"

The following example[3] shows that Theorem 10 in [31, 36] is wrong.

Example 4.1. Figure 7 depicts two modal interfaces $\mathcal{C}_1$ and $\mathcal{C}_2$; $\mathcal{I}_1$ and $\mathcal{I}_2$ are implementations of $\mathcal{C}_1$ and $\mathcal{C}_2$, respectively. Alphabets are indicated for each modal interface. Parallel composition according to [31] is named $[\mathcal{C}_1 \| \mathcal{C}_2]_0$. Word $c?.a!$ is illegal since in the state reached after this run: $\mathcal{C}_1$ may offer $b!$ whereas $\mathcal{C}_2$ may (in fact will) not accept it. However, $c?.a!$ is in the product of the two implementations.

[3] This example is due to discussions of the authors with Barbara Jobstmann and Laurent Doyen.

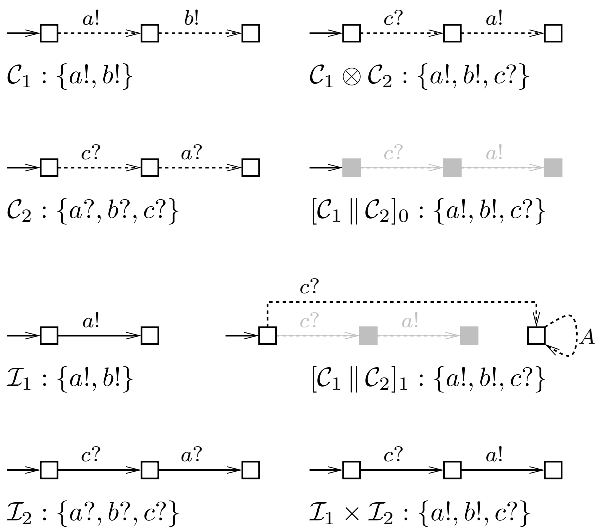


Figure 7. Counterexample regarding compatibility. Grey-shaded states are to be removed.

4.5.2. The correction

Call *exception* any word in $\mathcal{L}_{\mathcal{C}_1 \otimes \mathcal{C}_2}$ from which the environment has no strategy to prevent the occurrence of an illegal word, meaning that an illegal word can be obtained from the exception by following only output actions.

Definition 4.3. (compatibility)
The *exception language* of modal interfaces $\mathcal{C}_1$ and $\mathcal{C}_2$ is the language $\mathcal{E}_{\mathcal{C}_1 \,\|\, \mathcal{C}_2} = pre^*_!(Illegal(\mathcal{C}_1, \mathcal{C}_2))$. Modal interfaces $\mathcal{C}_1$ and $\mathcal{C}_2$ are said to be *compatible* if and only if the empty word ϵ is not in $\mathcal{E}_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}$.

Definition 4.4. (parallel composition)
Given two modal interfaces $\mathcal{C}_1$ and $\mathcal{C}_2$, the *relaxation* of $\mathcal{C}_1 \otimes \mathcal{C}_2$ is obtained by applying the following pseudo-algorithm to $\mathcal{C}_1 \otimes \mathcal{C}_2$:

```
for all v in L_{C1⊗C2} do
    for all a in A do
        if v ∉ E_{C1||C2} and v.a ∈ E_{C1||C2} then
            for all w in A* do
                must(v.a.w) := ∅
                may(v.a.w) := A
            end for
        end if
    end for
end for
```

If $\mathcal{C}_1$ and $\mathcal{C}_2$ are compatible, the relaxation of $\mathcal{C}_1 \otimes \mathcal{C}_2$ is called the *parallel composition* of $\mathcal{C}_1$ and $\mathcal{C}_2$, denoted by $\mathcal{C}_1 \,\|\, \mathcal{C}_2$. Whenever $\mathcal{C}_1$ and $\mathcal{C}_2$ are incompatible, the parallel composition $\mathcal{C}_1 \,\|\, \mathcal{C}_2$ is defined as the inconsistent modal specification $\perp$.

If the environment performs an action $a?$ to which the "**if** ... **then** ..." statement applies, then illegal words may exist for certain pairs $(\mathcal{I}_1, \mathcal{I}_2)$ of strong implementations of $\mathcal{C}_1$ and $\mathcal{C}_2$. If this occurs, then $\mathcal{C}_1 \,\|\, \mathcal{C}_2$ relaxes all constraints on the future of the corresponding runs — Nothing is forbidden, nothing is mandatory: the system has reached a "universal" state. This parallels the pruning rule combined with alternating simulation, in the context of interface automata.

Example 4.2. We now show that our relaxation allows us to correct the counter example stated in Figure 7. We observe that our relaxation procedure yields $[\mathcal{C}_1 \,\|\, \mathcal{C}_2]_1$, with $A = \{a!, b!, c?\}$, which has $\mathcal{I}_1 \times \mathcal{I}_2$ as an implementation.

Associativity of the parallel composition operator is one of the key requirements of an interface framework, since it enables independent design of sub-systems. Unlike in [31, 36], where associativity is only mentioned, we can now state the following theorem:

Theorem 4.2. The parallel composition operator is commutative and associative.

Proof:
Commutativity of $\|$ immediately holds by definition. We now consider associativity. Let three modal interfaces $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$. We characterize the set of illegal words in $((\mathcal{C}_1 \,\|\, \mathcal{C}_2) \,\|\, \mathcal{C}_3)$ and then prove that rearranging the parentheses will not change this set.

In the sequel we shall write A_i, $must_i$, may_i the elements of $\mathcal{C}_i$ (with $i = 1, 2, 3$) and A_{ij}, $must_{ij}$, may_{ij} the elements of $\mathcal{C}_i \otimes \mathcal{C}_j$ (for $(i, j) \in \{1, 2, 3\} \times \{1, 2, 3\}$, such that $i \neq j$). We shall also write u_i for $\mathbf{pr}_{A_i}(u)$ and u_{ij} for $\mathbf{pr}_{A_i \cup A_j}(u)$.

Observe first that, by definition, $\otimes$ is associative. Moreover, a word u is illegal in $\mathcal{C}_1 \otimes \mathcal{C}_2$ iff

$$\big(may_1^!(u_1) \setminus must_2^?(u_2)\big) \cup \big(may_2^!(u_2) \setminus must_1^?(u_1)\big) \quad \neq \emptyset \tag{11}$$

where $u_i = \mathbf{pr}_{A_i}(u)$ and $may_1^!(u_1) = may_1(u_1) \cap A_1!$ and similarly for other cases. Then, in building $\mathcal{C}_1 \,\|\, \mathcal{C}_2$ from $\mathcal{C}_1 \otimes \mathcal{C}_2$, relaxation of Def. 4.4 applies to every word $v \in \mathcal{L}_{\mathcal{C}_1 \otimes \mathcal{C}_2}$ such that

$$\exists b? \in may_{12}(v) \quad : \quad v.b? \in \mathbf{I} \tag{12}$$

Consequently, every word in $\mathcal{L}_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}$:

$$\text{either belongs itself to } \mathcal{L}_{\mathcal{C}_1 \otimes \mathcal{C}_2}\text{, or has a strict prefix } v \in \mathcal{L}_{\mathcal{C}_1 \otimes \mathcal{C}_2} \text{ satisfying (12).} \tag{13}$$

Observe that (12) rewrites as

$$\exists b? \in may_{12}(v), \exists w \in (A_1! \cup A_2!)^* \;\Rightarrow\; v.b?.w \text{ satisfies (11).} \tag{14}$$

Apply this to the pair $(\mathcal{C}_1 \,\|\, \mathcal{C}_2, \mathcal{C}_3)$: word u is illegal in $(\mathcal{C}_1 \,\|\, \mathcal{C}_2) \otimes \mathcal{C}_3$ iff

$$\big(may_{12}^!(u_{12}) \setminus must_3^?(u_3)\big) \cup \big(may_3^!(u_3) \setminus must_{12}^?(u_{12})\big) \quad \neq \emptyset \tag{15}$$

where A_{12} is the alphabet of $\mathcal{C}_1 \,\|\, \mathcal{C}_2$. Let U be the set of all such u's, and set $\mathbf{I} = pre_!^*(U)$. Then, in building $(\mathcal{C}_1 \,\|\, \mathcal{C}_2) \,\|\, \mathcal{C}_3$ from $(\mathcal{C}_1 \,\|\, \mathcal{C}_2) \otimes \mathcal{C}_3$, relaxation applies to every word $v \in \mathcal{L}_{(\mathcal{C}_1 \,\|\, \mathcal{C}_2) \otimes \mathcal{C}_3}$ satisfying

(12). Consequently, every word in $\mathcal{L}_{(\mathcal{C}_1 \,\|\, \mathcal{C}_2) \,\|\, \mathcal{C}_3}$ satisfies (13) with $\mathcal{C}_1 \otimes \mathcal{C}_2 \otimes \mathcal{C}_3$ instead of $\mathcal{C}_1 \otimes \mathcal{C}_2$. Finally, (12) rewrites as:

$$\exists b? \in may(v), \exists w \in (A_1! \cup A_2! \cup A_3!)^* \;\Rightarrow\; v.b?.w \text{ satisfies (15).} \tag{16}$$

Let us further analyse (15). Two cases must be considered:

1. u_{12} has reached a universal state of $\mathcal{C}_1 \,\|\, \mathcal{C}_2$: in this case, by (13), u_{12} has a strict prefix $\hat{u}_{12} \in \mathcal{L}_{\mathcal{C}_1 \otimes \mathcal{C}_2}$ satisfying (12), meaning that $\hat{u}_{12}.b?$ may for some $b?$, by subsequently performing only output actions, reach a deadlock in the product of the pair $(\mathcal{C}_1, \mathcal{C}_2)$.

2. u_{12} has not reached a universal state of $\mathcal{C}_1 \,\|\, \mathcal{C}_2$: in this case, $u_{12} \in \mathcal{L}_{\mathcal{C}_1 \otimes \mathcal{C}_2}$ and

$$\begin{aligned} may^!_{12}(u_{12}) &= may^!_1(u_1) \cup may^!_2(u_2) \\ must^?_{12}(u_{12}) &= \big(must^?_1(u_1) \cap must^?_2(u_2)\big) \cup (A_1? \setminus A_2) \cup (A_2? \setminus A_1) \end{aligned}$$

Hence the non-emptiness of (15) is equivalent to u_{12} causing a deadlock in the pair $(\mathcal{C}_1, \mathcal{C}_3)$ or the pair $(\mathcal{C}_2, \mathcal{C}_3)$.

Let us summarize how the two conditions (12) were rewritten:

$$\exists b? \in may(v), \exists w \in (A_1! \cup A_2! \cup A_3!)^* \;\Rightarrow\; u = v.b?.w \text{ satisfies the following condition :}$$

- There exists a pair $(i, j) \in \{1, 2, 3\} \times \{1, 2, 3\}$, such that $i \neq j$ and $u_{ij} = \mathbf{pr}_{A_i}(u)$ possesses a prefix $\hat{u}_{ij}.b?$ that may, for some $b?$ and by subsequently performing only output actions, reach a deadlock in the pair $(\mathcal{C}_i, \mathcal{C}_j)$.

The bottom line is that the condition and (12) is indeed symmetric with respect to the considered three modal interfaces. This proves the associativity of $\|$. ☐

As for interface automata (Theorem 4 in [20]), strong refinement preserves compatibility, assuming that the refined modal interface does not introduce new shared actions.

Lemma 4.1. Consider three modal interfaces $\mathcal{C}_i$, $i = 1...3$, such that $\mathcal{C}_2 \leq_s \mathcal{C}_1$ and $A_2 \cap A_3 \subseteq A_1 \cap A_3$.

- $\mathbf{pr}_{A_1 \cup A_3}(\mathit{Illegal}(\mathcal{C}_2, \mathcal{C}_3))$ is included in $\mathit{Illegal}(\mathcal{C}_1, \mathcal{C}_3)$;
- $\mathbf{pr}_{A_1 \cup A_3}\big(\mathcal{E}_{\mathcal{C}_2 \,\|\, \mathcal{C}_3}\big)$ is included in $\mathcal{E}_{\mathcal{C}_1 \,\|\, \mathcal{C}_3}$.

Proof:
Consider an illegal word $u \in \mathit{Illegal}(\mathcal{C}_2, \mathcal{C}_3)$ for $\mathcal{C}_2 \otimes \mathcal{C}_3$. This means that there exists an action $a \in A_2 \cap A_3$ such that (i) either a is an output of $\mathcal{C}_2$ and an input of $\mathcal{C}_3$, such that $a \in may_2(\mathbf{pr}_{A_2}(u))$ and $a \notin must_3(\mathbf{pr}_{A_3}(u))$, or (ii) a is an input of $\mathcal{C}_2$ and an output of $\mathcal{C}_3$, such that $a \notin must_2(\mathbf{pr}_{A_2}(u))$ and $a \in may_3(\mathbf{pr}_{A_3}(u))$.

By Definition 2.5, u is also in $\mathcal{L}_{\mathcal{C}_1 \otimes \mathcal{C}_{3 \uparrow A2 \cup A3}}$. By Definition 2.5, $u' = \mathbf{pr}_{A_1 \cup A_3}(u)$ belongs to $\mathcal{L}_{\mathcal{C}_1 \otimes \mathcal{C}_3}$. Since it is assumed that $A_2 \cap A_3 \subseteq A_1 \cap A_3$, action a belongs to $A_1 \cap A_3$. By Definition 2.4, either a is an output of $\mathcal{C}_1$ and an input of $\mathcal{C}_3$, such that $a \in may_1(\mathbf{pr}_{A_1}(u'))$ and $a \notin must_3(\mathbf{pr}_{A_3}(u'))$, or (ii) a is an input of $\mathcal{C}_1$ and an output of $\mathcal{C}_3$, such that $a \notin must_1(\mathbf{pr}_{A_1}(u'))$ and $a \in may_3(\mathbf{pr}_{A_3}(u'))$. Meaning that $u' \in \mathit{Illegal}(\mathcal{C}_1, \mathcal{C}_3)$, which proves the first point of the lemma.

Consider now the second point. Recall that $A_1! \cup A_3!$ is included in $A_2! \cup A_3!$. Hence, the set of actions $\mathbf{pr}_{A_1 \cup A_3}(pre^*_!(\mathit{Illegal}(\mathcal{C}_2, \mathcal{C}_3)))$ is included in $pre^*_!\big(\mathbf{pr}_{A_1 \cup A_3}(\mathit{Illegal}(\mathcal{C}_2, \mathcal{C}_3))\big)$, which is in turn included in $pre^*_!(\mathit{Illegal}(\mathcal{C}_1, \mathcal{C}_3))$, thanks to the first point of the lemma. □

Corollary 4.1. (compatibility preservation)
Given any three modal interfaces $\mathcal{C}_i$, $i = 1...3$, such that $\mathcal{C}_2 \leq_s \mathcal{C}_1$ and $A_1 \cap A_3 \supseteq A_2 \cap A_3$. $\mathcal{C}_1$ compatible with $\mathcal{C}_3$ implies that $\mathcal{C}_2$ and $\mathcal{C}_3$ are also compatible.

Proof:
This is an immediate consequence of the previous Lemma 4.1. Assume $\mathcal{C}_2$ and $\mathcal{C}_3$ incompatible, meaning that $\epsilon \in \mathcal{E}_{\mathcal{C}_2 \,\|\, \mathcal{C}_3}$. By Lemma 4.1, $\epsilon = \mathbf{pr}_{A_1 \cup A_3}(\epsilon) \in \mathcal{E}_{\mathcal{C}_1 \,\|\, \mathcal{C}_3}$. Hence $\mathcal{C}_1$ and $\mathcal{C}_3$ are also incompatible. □

Contrary to interface automata for which $\mathcal{C}_1 \,\|\, \mathcal{C}_2$ is a refinement of $\mathcal{C}_1 \otimes \mathcal{C}_2$ [20], relaxation of modal interfaces amounts to computing an abstraction of the product:

Lemma 4.2. Given two modal interfaces $\mathcal{C}_1$ and $\mathcal{C}_2$:

$$\mathcal{C}_1 \otimes \mathcal{C}_2 \leq \mathcal{C}_1 \,\|\, \mathcal{C}_2$$

Proof:
Two cases are possible:

- if $u \in \mathcal{L}_{\mathcal{C}_1 \otimes \mathcal{C}_2} \setminus \mathcal{E}_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}$ then $must_{\mathcal{C}_1 \otimes \mathcal{C}_2}(u) = must_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}(u)$ and $may_{\mathcal{C}_1 \otimes \mathcal{C}_2}(u) = may_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}(u)$;
- if $u \in \mathcal{E}_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}$ then $u \in \mathcal{L}_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}$ and $must_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}(u) = \emptyset$ and $may_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}(u) = A$.

Thus, $must_{\mathcal{C}_1 \otimes \mathcal{C}_2}(u) \supseteq must_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}(u)$ and $may_{\mathcal{C}_1 \otimes \mathcal{C}_2}(u) \subseteq may_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}(u)$. □

Theorem 10 stated in [31, 36] now holds for the parallel composition operator.

Theorem 4.3. (independent implementability)
For any two modal interfaces $\mathcal{C}_1, \mathcal{C}_2$ and two implementations $(\mathcal{I}_1, \pi_1)$, $(\mathcal{I}_2, \pi_2)$ such that $(\mathcal{I}_1, \pi_1) \models_s \mathcal{C}_1$ and $(\mathcal{I}_2, \pi_2) \models_s \mathcal{C}_2$, it holds that $(\mathcal{I}_1, \pi_1) \times (\mathcal{I}_2, \pi_2) \models_s \mathcal{C}_1 \,\|\, \mathcal{C}_2$.

Proof:
If $(\mathcal{I}_1, \pi_1) \models_s \mathcal{C}_1$ and $(\mathcal{I}_2, \pi_2) \models_s \mathcal{C}_2$, then, by Theorem 4.1, $(\mathcal{I}_1, \pi_1) \times (\mathcal{I}_2, \pi_2) \models_s \mathcal{C}_1 \otimes \mathcal{C}_2$. By Lemma 4.2 and by the generalization of Theorem 1 in Theorem 4.1: $(\mathcal{I}_1, \pi_1) \times (\mathcal{I}_2, \pi_2) \models_s \mathcal{C}_1 \,\|\, \mathcal{C}_2$. □

5. Methodological considerations

While the framework we propose adds significant flexibility to design flows, it also raises some methodological issues that we discuss now.

As previously remarked, a designer may want to specify the aspects of the system via different interfaces (called *viewpoints*). In this situation, she may wonder whether the same system would be obtained by implementing all the viewpoints in a single component, or alternatively, as several components where each of them implementing some of the viewpoints. This question amounts relating the operation of product/composition to the one of conjunction. We have the following result.

Theorem 5.1. Let $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$ be three modal interfaces. Then,

1. $\mathcal{C}_1 \otimes (\mathcal{C}_2 \wedge \mathcal{C}_3) \leq (\mathcal{C}_1 \otimes \mathcal{C}_2) \wedge (\mathcal{C}_1 \otimes \mathcal{C}_3)$;
2. $\mathcal{C}_1 \,\|\, (\mathcal{C}_2 \wedge \mathcal{C}_3) \leq (\mathcal{C}_1 \,\|\, \mathcal{C}_2) \wedge (\mathcal{C}_1 \,\|\, \mathcal{C}_3)$;
3. The reverse refinements in points 1 and 2 do not hold.

Proof:
Recall that may_i, $must_i$ and A_i denote the elements of $\mathcal{C}_i$ and let $A = A_1 \cup A_2 \cup A_3$.

Proof of statement 1. By definition of the GLB of modal specifications and by Theorem 2.1, we have: $\rho(\mathcal{C}_2 \ \& \ \mathcal{C}_3) \leq \mathcal{C}_2 \ \& \ \mathcal{C}_3$. Note that the definition of composition for modal specification can be immediatly extended to *pseudo*-modal specifications with preservation of Proposition 2.3. As a result:

$$\mathcal{C}_1 \otimes \rho(\mathcal{C}_2 \ \& \ \mathcal{C}_3) \leq \mathcal{C}_1 \otimes (\mathcal{C}_2 \ \& \ \mathcal{C}_3)$$

We then can easily prove that that $\otimes$ distributes over $\&$. Thus:

$$\mathcal{C}_1 \otimes \rho(\mathcal{C}_2 \ \& \ \mathcal{C}_3) \leq (\mathcal{C}_1 \otimes \mathcal{C}_2) \ \& \ (\mathcal{C}_1 \otimes \mathcal{C}_3)$$

Recall that $\rho({}^p\mathcal{S})$ is the largest modal specification (for refinement order) such that $\rho({}^p\mathcal{S}) \leq {}^p\mathcal{S}$. Thus:

$$\mathcal{C}_1 \otimes \rho(\mathcal{C}_2 \ \& \ \mathcal{C}_3) \leq \rho((\mathcal{C}_1 \otimes \mathcal{C}_2) \ \& \ (\mathcal{C}_1 \otimes \mathcal{C}_3)).$$

That is: $\mathcal{C}_1 \otimes (\mathcal{C}_2 \wedge \mathcal{C}_3) \leq (\mathcal{C}_1 \otimes \mathcal{C}_2) \wedge (\mathcal{C}_1 \otimes \mathcal{C}_3)$.

Proof of statement 2. Let $u \in \mathcal{L}_{\mathcal{C}_1 \,\|\, (\mathcal{C}_2 \wedge \mathcal{C}_3)}$ then:

- either $u \in \mathcal{L}_{\mathcal{C}_1 \otimes (\mathcal{C}_2 \wedge \mathcal{C}_3)}$;
- or, u has a strict prefix $v \in \mathcal{L}_{\mathcal{C}_1 \otimes (\mathcal{C}_2 \wedge \mathcal{C}_3)}$ such that $v.b? \in \mathit{Illegal}(\mathcal{C}_1, \mathcal{C}_2 \wedge \mathcal{C}_3)$ for some action $b?$.

In the first case, according to the point 1 of Theorem 5.1, every $a \in may_{\mathcal{C}_1 \otimes (\mathcal{C}_2 \wedge \mathcal{C}_3)}(u)$ also belongs to $may_{(\mathcal{C}_1 \otimes \mathcal{C}_2) \wedge ((\mathcal{C}_1 \otimes \mathcal{C}_3)}(u)$. By definition of the conjunction:

$$a \in [may_{\mathcal{C}_1 \otimes \mathcal{C}_2}(\mathbf{pr}_{A_1 \cup A_2}(u)) \cup (A \setminus (A_1 \cup A_2))] \cap [may_{\mathcal{C}_1 \otimes \mathcal{C}_3}(\mathbf{pr}_{A_1 \cup A_3}(u)) \cup (A \setminus (A_1 \cup A_3))]$$

Moreover, by Lemma 4.2, $\mathcal{C}_1 \otimes \mathcal{C}_i \leq \mathcal{C}_1 \,\|\, \mathcal{C}_i$ for $i = 1, 2$, thus:

$$a \in [may_{\mathcal{C}_1 \,\|\, \mathcal{C}_2}(\mathbf{pr}_{A_1 \cup A_2}(u)) \cup (A \setminus (A_1 \cup A_2))] \cap [may_{\mathcal{C}_1 \,\|\, \mathcal{C}_3}(\mathbf{pr}_{A_1 \cup A_3}(u)) \cup (A \setminus (A_1 \cup A_3))]$$

that is, $a \in may_{(\mathcal{C}_1 \,\|\, \mathcal{C}_2) \wedge ((\mathcal{C}_1 \,\|\, \mathcal{C}_3)}(u)$.

In the second case, let us first show that if $v.b? \in \mathit{Illegal}(\mathcal{C}_1, \mathcal{C}_2 \wedge \mathcal{C}_3)$ then $\mathbf{pr}_{A_1 \cup A_i}(v.b?) \in \mathit{Illegal}(\mathcal{C}_1, \mathcal{C}_i)$ for $i = 1, 2$. Let $may^!_i = may_i \cap A_i!$ and similarly for other cases. When $v.b? \in \mathit{Illegal}(\mathcal{C}_1, \mathcal{C}_2 \wedge \mathcal{C}_3)$ there exists $a \in A_1 \cap A_2 \cap A_3$ such that:

$$\begin{aligned} a \in \quad & \big(may^!_1(\mathbf{pr}_{A_1}(v.b?)) \setminus must^?_{\mathcal{C}_2 \wedge \mathcal{C}_3}(\mathbf{pr}_{A_2 \cup A_3}(v.b?))\big) \\ & \quad \cup \big(may^!_{\mathcal{C}_2 \wedge \mathcal{C}_3}(\mathbf{pr}_{A_2 \cup A_3}(v.b?)) \setminus must^?_{12}(u_1(\mathbf{pr}_{A_1}(v.b?)))\big) \end{aligned}$$

If $a \notin must^?_{\mathcal{C}_2 \wedge \mathcal{C}_3}(\mathbf{pr}_{A_2 \cup A_3}(v.b?))$ then $a \notin [must^?_{\mathcal{C}_2}(\mathbf{pr}_{A_2}(v.b?)) \cup must^?_{\mathcal{C}_3}(\mathbf{pr}_{A_3}(v.b?))]$; moreover if $a \in may^!_{\mathcal{C}_2 \wedge \mathcal{C}_3}(\mathbf{pr}_{A_2 \cup A_3}(v.b?))$, as $a \in A_1 \cap A_2 \cap A_3$, we have: $a \in may^!_{\mathcal{C}_2}(\mathbf{pr}_{A_2}(v.b?)) \cap may^!_{\mathcal{C}_3}(\mathbf{pr}_{A_3}(v.b?))$. As a result, $\mathbf{pr}_{A_1 \cup A_i}(v.b?)$ is illegal in $\mathcal{C}_1 \otimes \mathcal{C}_i$ for $i = 1, 2$ and u has reached a universal state in $(\mathcal{C}_1 \parallel \mathcal{C}_2) \wedge (\mathcal{C}_1 \parallel \mathcal{C}_3)$. In conclusion, $may_{\mathcal{C}_1 \otimes (\mathcal{C}_2 \wedge \mathcal{C}_3)}(u) \subseteq may_{(\mathcal{C}_1 \parallel \mathcal{C}_2) \wedge ((\mathcal{C}_1 \parallel \mathcal{C}_3)}(u)$.

Now if $a \in must_{(\mathcal{C}_1 \parallel \mathcal{C}_2) \wedge ((\mathcal{C}_1 \parallel \mathcal{C}_3)}(u)$ then by definition:

$$\begin{aligned} a \in \quad & [(must_1(\mathbf{pr}_{A_1}(u)) \cup (A_2 \setminus A_1)) \ \cap \ (must_2(\mathbf{pr}_{A_2}(u)) \cup (A_1 \setminus A_2))] \\ & \cup [(must_1(\mathbf{pr}_{A_1}(u)) \cup (A_3 \setminus A_1)) \ \cap \ (must_3(\mathbf{pr}_{A_3}(u)) \cup (A_1 \setminus A_3))] \end{aligned} \tag{17}$$

We have to prove $a \in must_{\mathcal{C}_1 \parallel (\mathcal{C}_2 \wedge \mathcal{C}_3)}(u)$, that is:

$$\begin{aligned} a \in \quad & [must_1(\mathbf{pr}_{A_1}(u)) \cup ((A_2 \cup A_3) \setminus A_1)] \\ & \cap \big[\big(must_2(\mathbf{pr}_{A_2}(u)) \cup must_3(\mathbf{pr}_{A_3}(u))\big) \cup (A_1 \setminus (A_2 \cup A_3))\big] \end{aligned} \tag{18}$$

If $a \notin must_1(\mathbf{pr}_{A_1}(u))$ then from Equation 17 we deduce:

$$a \in must_2(\mathbf{pr}_{A_2}(u)) \cap must_3(\mathbf{pr}_{A_3}(u)) \cap (A_2 \setminus A_1) \cap (A_3 \setminus A_1)$$

If $a \in must_1(\mathbf{pr}_{A_1}(u))$ then from Equation 17 we deduce:

$$a \in \big((must_2(\mathbf{pr}_{A_2}(u)) \cup (A_1 \setminus A_2)\big) \cap \big((must_3(\mathbf{pr}_{A_3}(u)) \cup (A_1 \setminus A_3))\big)$$

In the two situations, Equation 18 is true and thus $a \in must_{\mathcal{C}_1 \parallel (\mathcal{C}_2 \wedge \mathcal{C}_3)}(u)$.

Proof of statement 3 and 4. Consider the three following modal interfaces defined over the alphabet $\{a\}$ with the same profile $\pi(a) = ?$:

- $\mathcal{C}_1$ with $may_1(\epsilon) = \{a\}$, $may_1(a) = \{a\}$ and $may_1(aa) = \emptyset$;
- $\mathcal{C}_2$ with $may_2(\epsilon) = \{a\}$, $may_2(a) = \{a\} = \emptyset$;
- $\mathcal{C}_3$ with $may_3(\epsilon) = \{a\}$, $may_3(a) = must_3(a) = \{a\}$ and $may_1(aa) = \emptyset$;

Then $may_{(\mathcal{C}_1 \otimes \mathcal{C}_2) \wedge (\mathcal{C}_1 \otimes \mathcal{C}_3)}(\epsilon) = \{a\}$ whereas $may_{\mathcal{C}_1 \otimes (\mathcal{C}_2 \wedge \mathcal{C}_3)}(\epsilon) = \emptyset$. As a result:

$$(\mathcal{C}_1 \otimes \mathcal{C}_2) \wedge (\mathcal{C}_1 \otimes \mathcal{C}_3) \not\preceq \mathcal{C}_1 \otimes (\mathcal{C}_2 \wedge \mathcal{C}_3).$$

The same counterexample can be used to prove that: $(\mathcal{C}_1 \parallel \mathcal{C}_2) \wedge (\mathcal{C}_1 \parallel \mathcal{C}_3) \not\preceq \mathcal{C}_1 \parallel (\mathcal{C}_2 \wedge \mathcal{C}_3)$. $\square$

The interpretation of this theorem is as follows. We assume two components indexed by 1 and 2, with associated interfaces. The left hand side of equations in point 1 and 2 captures the design process in which (1) the two viewpoints for component 2 are first combined, and (2) the two components are combined; this is called a *component-centric* design process because it aims at specifying components completely before assembling them. The right hand side captures the design process in which viewpoints are first considered separately for all components, and then fused; this is called a *viewpoint-centric* design process. Theorem 5.1 expresses that viewpoint-centric design processes leave more room for implementations than component-centric ones.

6. Conclusion, related work and future work

This paper presents a *modal interface* framework, a unification of interface automata and modal specifications. It is a complete theory with a powerful composition algebra that includes operations such as conjunction (for requirements composition) and residuation (for component reuse but also assume/guarantee contract-based reasoning [42]). However, the core contributions of the paper are (1) a parallel composition operator that reflects a rich notion of compatibility between components, actually correcting the parallel composition proposed in [31, 36], and (2) a new theory that encompasses dissimilar alphabets.

Interface automata were first introduced as an extension of Input/Output automata with an optimistic approach for composition. Modal specifications have been proposed as an extension of process-algebraic theories [35, 30] which allows for a better distinction between successive implementations (see the introduction of [36] and [30] for some discussion). Modal interfaces are a model that mixes both I/O automata and modal specifications.

There are various other approaches for interface theories (see [5] for a survey). One of them is based on contracts [7, 36, 38, 27], that is a representation where one keeps an explicit distinction between assumptions on the environment and guarantees on behaviors of the system. A similar approach to ours has been developed in [33] for a *non-modal* process-algebraic framework in which a dedicated predicate is used to model inconsistent processes.

Interface automata and modal specifications are incomparable models as *must*, *may* and *input*, *output* have orthogonal meanings. Modal specification can be viewed as an abstraction of a set of closed systems[4] (as a modal specification does not allow a component and its environment to be distinguished). As a consequence, specification logics and verification procedures for this model [26, 28] are extensions of those defined for transition systems [37, 15]. Interface automata is a more "open" model (as it distinguishes between the component and its environment) and it is thus not surprising that specification logics and verification procedures for such a model correspond to those defined for reactive systems, e.g., ATL [1]. This paper did not focus on verification procedures, but we believe that this research direction is of importance and deserves further studies.

There are several possible directions for future research. A first step would be to implement all the concepts and operations presented in the paper and evaluate the resulting tool on concrete case studies. Extensions of modal specifications can be investigated, where states are described as valuations of a set of variables just as it has been the case for interface automata [13, 18]. One should also propose definitions of quotient and conjunction for interface automata.

[4] A closed system is a system that does not interact with an unknown environment. On the contrary, an open system is a system that continuously interacts with an unknown environment.

Another promising direction would be a timed extension of modal interfaces. In [21], de Alfaro et al. proposed *timed interface automata* that extend timed automata just as interface automata extend finite-word automata. The semantics of a timed interface automaton is given by a timed game [19, 12], which allows one to capture the *timed dimension* in composition, i.e., “what are the temporal ordering constraints on communication events between components? [21]”. Up to now, composition is the only operation that has been defined on timed interface automata. In [14], Chatain et al. have proposed a notion of refinement for timed games. However, monotonicity of parallel composition with respect to this refinement relation has not been investigated yet. In [9, 8], *timed modal specifications* are proposed. As modal specifications, timed modal specifications admit a rich composition algebra with product, conjunction and residuation operators. Thus, a natural direction for future research would be to unify timed interface automata and timed modal specifications. This would imply a translation from timed interface automata to timed modal specifications.

Acknowledgments

We are grateful to Barbara Jobstmann and Laurent Doyen who proposed the counter example given in Section 4.5.1 which proved that thhe construction in [31] was incorrect.

References

[1] R. Alur, T. A. Henzinger, and O. Kupferman. Alternating-time temporal logic. *Journal of the ACM*, 49(5):672–713, 2002.

[2] R. Alur, T. A. Henzinger, O. Kupferman, and M. Y. Vardi. Alternating refinement relations. In *Proc. of the 9th International Conference on Concurrency Theory (CONCUR'98)*, volume Lecture Notes in Computer Science 1466, pages 163–178. Springer, 1998.

[3] A. Antonik, M. Huth, K. G. Larsen, U. Nyman, and A. Wasowski. Complexity of decision problems for mixed and modal specifications. In *Proc. of the 11th International Conference on Foundations of Software Science and Computation Structures (FoSSaCS'08)*, volume Lecture Notes in Computer Science 4962, pages 112–126. Springer, 2008.

[4] A. Arnold and M. Nivat. Metric interpretations of infinite trees and semantics of non deterministic recursive programs. *Theoretical Computer Science*, 11, 1980.

[5] E. Badouel, A. Benveniste, B. Caillaud, T. A. Henzinger, A. Legay, and R. Passerone. Contract theories for embedded systems : A white paper. Research report, IRISA/INRIA Rennes, 2009.

[6] N. Benes, J. Kretínský, K. G. Larsen, and J. Srba. On determinism in modal transition systems. *Theoretical Computer Science*, 410(41):4026–4043, 2009.

[7] A. Benveniste, B. Caillaud, and R. Passerone. A generic model of contracts for embedded systems. Research report 6214, IRISA/INRIA Rennes, 2007.

[8] N. Bertrand, A. Legay, S. Pinchinat, and J.-B. Raclet. A compositional approach on modal specifications for timed systems. In *Proc. of the 11th International Conference on Formal Engineering Methods (ICFEM'09)*, volume Lecture Notes in Computer Science 5885, pages 679–697. Springer, 2009.

[9] N. Bertrand, S. Pinchinat, and J.-B. Raclet. Refinement and consistency of timed modal specifications. In *Proc. of the 3rd International Conference on Language and Automata Theory and Applications (LATA'09)*, volume Lecture Notes in Computer Science 5457, pages 152–163. Springer, 2009.

[10] P. Bhaduri. Synthesis of interface automata. In *Proc. of the 3rd Automated Technology for Verification and Analysis Conference (ATVA'05)*, volume Lecture Notes in Computer Science 3707, pages 338–353. Springer, 2005.

[11] S. Bliudze and J. Sifakis. A notion of glue expressiveness for component-based systems. In *Proc. of the 19th International Conference on Concurrency Theory (CONCUR'08)*, volume Lecture Notes in Computer Science 5201, pages 508–522. Springer, 2008.

[12] T. Brihaye, F. Laroussinie, N. Markey, and G. Oreiby. Timed concurrent game structures. In *Proc. of the 18th International Conference on Concurrency Theory (CONCUR'07)*, volume Lecture Notes in Computer Science 4703, pages 445–459. Springer, 2007.

[13] A. Chakrabarti, L. de Alfaro, T. A. Henzinger, and F. Y. C. Mang. Synchronous and bidirectional component interfaces. In *Proc. of the 14th International Conference on Computer Aided Verification (CAV'02)*, volume Lecture Notes in Computer Science 2404, pages 414–427. Springer, 2002.

[14] T. Chatain, A. David, and K. G. Larsen. Playing games with timed games. In *Proc. of the 3rd IFAC Conference on Analysis and Design of Hybrid Systems (ADHS'09)*, 2009.

[15] E. M. Clarke and E. A. Emerson. Design and synthesis of synchronization skeletons using branching-time temporal logic. In *Logic of Programs*, volume Lecture Notes in Computer Science 131, pages 52–71. Springer, 1981.

[16] D. Dams, R. Gerth, and O. Grumberg. Abstract interpretation of reactive systems. *ACM Trans. Program. Lang. Syst.*, 19(2):253–291, 1997.

[17] L. de Alfaro. Game models for open systems. In *Verification: Theory and Practice*, volume Lecture Notes in Computer Science 2772, pages 269–289. Springer, 2003.

[18] L. de Alfaro, L. Dias da Silva, M. Faella, A. Legay, P. Roy, and M. Sorea. Sociable interfaces. In *Proc. of the 5th International Workshop on Frontiers of Combining Systems (FroCos'05)*, volume Lecture Notes in Computer Science 3717, pages 81–105. Springer, 2005.

[19] L. de Alfaro, M. Faella, T. A. Henzinger, R. Majumdar, and M. Stoelinga. The element of surprise in timed games. In *Proc. of the 14th International Conference on Concurrency Theory (CONCUR'03)*, volume Lecture Notes in Computer Science 2761, pages 142–156. Springer, 2003.

[20] L. de Alfaro and T. A. Henzinger. Interface automata. In *Proc. of the 9th ACM SIGSOFT International Symposium on Foundations of Software Engineering (FSE'01)*, pages 109–120. ACM Press, 2001.

[21] L. de Alfaro, T. A. Henzinger, and M. Stoelinga. Timed interfaces. In *Proc. of the 2nd Workshop on Embedded Software (EMSOFT'02)*, volume Lecture Notes in Computer Science 2491, pages 108–122. Springer, 2002.

[22] L. Doyen, T. A. Henzinger, B. Jobstmann, and T. Petrov. Interface theories with component reuse. In *Proc. of the 8th International Conference on Embedded Software (EMSOFT'08)*, pages 79–88. ACM Press, 2008.

[23] J. Eker, J. W. Janneck, E. A. Lee, J. Liu, X. Liu, J. Ludvig, S. Neuendorffer, S. Sachs, and Y. Xiong. Taming heterogeneity - the Ptolemy approach. *Proc. of the IEEE*, 91(1):127–144, 2003.

[24] G. Feuillade and S. Pinchinat. Modal specifications for the control theory of discrete-event systems. *Discrete Event Dynamic Systems*, 17(2):181–205, 2007.

[25] C. Fournet, C. A. R. Hoare, S. K. Rajamani, and J. Rehof. Stuck-free conformance. In *Proc. of the 16th International Conference on Computer Aided Verification (CAV'04)*, volume Lecture Notes in Computer Science 3114, pages 242–254. Springer, 2004.

[26] P. Godefroid, M. Huth, and R. Jagadeesan. Abstraction-based model checking using modal transition systems. In *Proc. of the 12th International Conference on Concurrency Theory (CONCUR'01)*, volume Lecture Notes in Computer Science 2154, pages 426–440. Springer, 2001.

[27] G. Goessler and J.-B. Raclet. Modal contracts for component-based design. In *Proc. of the 7th IEEE International Conference on Software Engineering and Formal Methods (SEFM'09)*, pages 295–303. IEEE Computer Society Press, 2009.

[28] O. Grumberg, M. Lange, M. Leucker, and S. Shoham. Don't know in the μ-calculus. In *Proc. of the 6th International Conference on Verification, Model Checking, and Abstract Interpretation (VMCAI'05)*, volume Lecture Notes in Computer Science 3385, pages 233–249. Springer, 2005.

[29] T. A. Henzinger and J. Sifakis. The embedded systems design challenge. In *Proc. of the 14th International Symposium on Formal Methods (FM'06)*, volume Lecture Notes in Computer Science 4085, pages 1–15. Springer, 2006.

[30] K. G. Larsen. Modal specifications. In *Automatic Verification Methods for Finite State Systems*, volume Lecture Notes in Computer Science 407, pages 232–246. Springer, 1989.

[31] K. G. Larsen, U. Nyman, and A. Wasowski. Modal I/O automata for interface and product line theories. In *Proc. of the 16th European Symposium on Programming Languages and Systems (ESOP'07)*, volume Lecture Notes in Computer Science 4421, pages 64–79. Springer, 2007.

[32] K. G. Larsen, U. Nyman, and A. Wasowski. On modal refinement and consistency. In *Proc. of the 18th International Conference on Concurrency Theory (CONCUR'07)*, volume Lecture Notes in Computer Science 4703, pages 105–119. Springer, 2007.

[33] G. Lüttgen and W. Vogler. Conjunction on processes: Full abstraction via ready-tree semantics. *Theoretical Computer Science*, 373:19–40, 2007.

[34] N. Lynch and M. R. Tuttle. An introduction to Input/Output automata. *CWI-quarterly*, 2(3), 1989.

[35] R. Milner. A complete axiomatisation for observational congruence of finite-state behaviors. *Information and Computation*, 81(2):227–247, 1989.

[36] U. Nyman. *Modal Transition Systems as the Basis for Interface Theories and Product Lines*. PhD thesis, Aalborg University, Department of Computer Science, 2008.

[37] A. Pnueli. The temporal logic of programs. In *Proc. 18th Annual Symposium on Foundations of Computer Science (FOCS'77)*, pages 46–57, 1977.

[38] S. Quinton and S. Graf. Contract-based verification of hierarchical systems of components. In *Proc. of the 6th IEEE International Conference on Software Engineering and Formal Methods (SEFM'08)*, pages 377–381. IEEE Computer Society, 2008.

[39] J.-B. Raclet. *Quotient de spécifications pour la réutilisation de composants*. PhD thesis, Université de Rennes I, 2007. (In French).

[40] J.-B. Raclet. Residual for component specifications. In *Proc. of the 4th International Workshop on Formal Aspects of Component Software (FACS'07)*, volume Electronic Notes Theoretical Computer Science 215, pages 93–110, 2008.

[41] J.-B. Raclet, E. Badouel, A. Benveniste, B. Caillaud, A. Legay, and R. Passerone. Modal interfaces: Unifying interface automata and modal specifications. In *Proc. of the 9th International Conference on Embedded Software (EMSOFT'09)*, pages 87–96. ACM Press, 2009.

[42] J.-B. Raclet, E. Badouel, A. Benveniste, B. Caillaud, and R. Passerone. Why are modalities good for interface theories? In *Proc. of the 9th International Conference on Application of Concurrency to System Design (ACSD'09)*, pages 199–127. IEEE Computer Society Press, 2009.

[43] J.-B. Raclet, E. Badouel, A. Benveniste, B. Caillaud, and R. Passerone. Why are modalities good for Interface Theories? Research Report RR-6899, INRIA, 2009.