A Tag Contract Framework for Heterogeneous Systems

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Abstract. In the distributed development of modern IT systems, contracts play a vital role in ensuring interoperability of components and adherence to specifications. The design of embedded systems, however, is made more complex by the heterogeneous nature of components, which are often described using different models and interaction mechanisms. Composing such components is generally not well-defined, making design and verification difficult. Several frameworks, both operational and denotational, have been proposed to handle heterogeneity using a variety of approaches. However, the application of heterogeneous operational models to contract-based design has not yet been investigated. In this work, we adopt the operational mechanism of tag machines to represent heterogeneous systems and construct a full contract model. We introduce heterogeneous composition, refinement, dominance, and compatibility between contracts, altogether enabling a formalized and rigorous design process for heterogeneous systems.

1 Introduction

Modern computing systems are increasingly being built by composing components which are developed concurrently by different design teams. In such a paradigm, the distinction between what is constrained on environments, and what must be guaranteed by a system given the constraint satisfaction, reflects the different roles and responsibilities in the system design procedure. Such distinction can be captured by a component model called *contract* [1]. Formally, a contract is a pair of *assumptions* and *guarantees*, which intuitively are properties that must be satisfied by all inputs and outputs of a design, respectively. The separation between assumptions and guarantees supports the distributed development of complex systems and allows subsystems to synchronize by relying on associated contracts.

In the particular context of embedded systems, *heterogeneity* is a typical characteristic since these systems are usually composed from parts developed using different methods, time models and interaction mechanisms. To deal with heterogeneity, several modeling frameworks have been proposed oriented towards the representation and simulation of heterogeneous systems, such as the Ptolemy framework [2], or towards the unification of their interaction paradigms, such as those based on *tagged events* [3]. The latter can capture different notions of time, e.g., physical time, logical time, and relate them by mapping tagged events over a common tag structure [4]. However, due to the significant inherent complexity of heterogeneity, there have been only very few attempts at addressing heterogeneity in the context of contract-based models. For instance, the

HRC model from the SPEEDS project[3] was designed to deal with different viewpoints (functional, time, safety, etc.) of a single component [5,6]. However, the notion of heterogeneity in general is much broader than that between multiple viewpoints, and must take into account diverse interaction paradigms. Meanwhile, heterogeneous modeling frameworks have not been related to contract-based design flows. This has motivated us to study a methodology which allows heterogeneous systems to be modeled and interconnected in a contract-based fashion.

Our long term objective is to develop a modeling and analysis framework for the specification and verification of both heterogeneous components and contracts. In order to support formal correctness proofs, the framework must employ an underlying (or intermediate) semantically sound model that can be used to represent different computation and interaction paradigms uniformly. Because simulation is an essential design activity, the model must also be executable. At the same time, the semantic model must be able to retain the individual features of each paradigm to avoid losing their specific properties. In particular, the framework must interact with the user through a front end that exposes familiar models that feel native and natural. In this paper we focus on the intermediate semantic model and defer the discussion on how specific front ends may be constructed to our future work. To this end, we advocate the use of heterogeneous Tag Machines (TMs) as a suitable semantic model for system specification. The expressive power of TMs has been demonstrated though various concurrency models such as asynchronous, synchronous reactive, causality [7] as well as in job-shop modeling and specification [8]. In our previous work we have proposed and studied the compositional properties of heterogeneous Tag Machines (TMs) for *component* specification [9]. Here, we instead discuss their extension to a *contract* model, and define a full set of operations and relations such as contract satisfaction, contract refinement, contract dominance and contract compatibility. To do this, we rely on a generic meta-framework [10] that we extend with tags and mapping between tags to define model interactions. In this paper, we shall discuss extensively the technical difficulties in making such an extension.

The rest of the paper is organized as follows. In Sect. 3, we recall basic notions of tag behaviors and tag machines. Section 4 presents our tag contract methodology for heterogeneous systems built on top of TM operations such as composition, quotient, conjunction and refinement. In the same section, we discuss an application of our methodology to a simplified water control problem and model it using incrementing TMs. Finally we conclude in Sect. 5.

2 Related Work

The notion of contract was first introduced by Bertrand Meyer in his design-by-contract method [1], based on ideas by Dijkstra [11], Lamport [12], and others, where systems are viewed as abstract boxes achieving their common goal by verifying specified contracts. De Alfaro and Henzinger subsequently introduced interface automata [13] for documenting components and established a more general notion of contract, where pre-conditions and post-conditions, which originally appeared in the form of predicates, are

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generalized to behavioral interfaces. The differentiation between assumptions and guarantees, which is implicit in interface automata, is made explicit in the trace-based contract framework of the SPEEDS HRC model [5,14]. The relationship between specifications of component behaviors and contracts is further studied by Bauer et al. [10] where a contract framework can be built on top of any *specification theory* equipped with a composition operator and a refinement relation which satisfy certain properties. The mentioned trace-based contract theories [5,14] are also demonstrated to be instances of such framework. We take advantage of this formalization in this work to construct our tag contract theory. Assume-guarantee reasoning has also been applied extensively in declarative compositional reasoning [15] to help prove properties by decomposing the process into simpler and more manageable steps. Our objective is conceptually different: assumptions specify a set of legal environments and are used to prove (or disprove) contract compatibility and satisfaction. In contrast, classical assume-guarantee reasoning uses assumptions as hypotheses to establish whether a generic property holds. Naturally, this technique can be used in contract models, as well.

Heterogeneity theory has been evolving in parallel with contract theory, to assist designers in dealing with heterogeneous composition of components with various Models of Computation and Communication (MoCC). The idea behind these theories and frameworks is to be able to combine well-established specification formalisms to enable analysis and simulation across heterogeneous boundaries. This is usually accomplished by providing some sort of common mechanism in the form of an underlying rich semantic model or coordination protocol. In this paper we are mostly concerned with these lower level aspects. One such approach is the pioneering framework of Ptolemy II [2], where models, called *domains*, are combined hierarchically: each level of the hierarchy is homogeneous, while different interaction mechanisms are specified at different levels in the hierarchy. In the underlying model, intended for simulation, each domain is composed of a scheduler (the *director*) which exposes the same abstract interface to a global scheduler which coordinates the execution. This approach, which has clear advantages for simulation, has two limitations in our context. First, it does not provide access to the components themselves but only to their schedulers, limiting our ability to establish relations to only the models of computation, and not to the heterogeneous contracts of the components. Secondly, the heterogeneous interaction occurs implicitly as a consequence of the coordination mechanism, and can not be controlled by the user. The metroII framework [16] relaxes this limitation, and allows designers to build direct model adapters. However, metroII treats components mostly as black boxes using a wrapping mechanism to guarantee flexibility in the system integration, making the development of an underlying theory complex. These and other similar frameworks are mainly focused on handling heterogeneity at the level of simulation.

Another body of work is instead oriented towards the formal representation, verification and analysis of these system. The BIP framework uses the notion of connector, on top of a state based model, to implement both synchronous and asynchronous interaction patterns [17]. Their relationship, however, can not be easily altered, and the framework lacks a native notion of time. Benveniste et al. [4] propose a heterogeneous denotational semantics inspired by the Lee and Sangiovanni-Vincentelli formalism of tag signal models [3], which has been long advocated as a unified modeling framework capable of capturing heterogeneous MoCC. In both models, tags play an important role in capturing various notions of time, where each tag system has its own tag structure expressing an MoCC. Composing such system is thus done by applying mappings between different tag structures. TMs [7] are subsequently introduced as finite representations of homogeneous tag systems. We have chosen to use this formalism for our work, as it provides an operational representation based on rigorous and proven semantics, and extended their definition to encompass heterogeneous components [9]. TMs are quite expressive, and ways to map traditional interaction paradigms have been reported in the literature [7]. TMs have also been applied to model a job-shop specification [8] where any trace of the composite tag machine from the start to the final state results in a valid job-shop schedule. Alternatively, tag systems can be represented by functional actors forming a Kleene algebra [18]. The approach is similar to that of Ptolemy II in that both use actors to represent basic components.

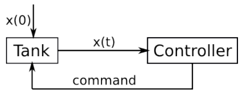
3 Background

We consider a component to be a set of behaviors in terms of sets of events that take place at its interface, intended as a collection of visible ports. Tags, which are associated to every event, characterize the temporal evolution of the behaviors. By changing the structure of tags, one can choose among different notions of time. Formally, a *tag structure* $\mathcal{T}$ is a pair $(T, \leq)$ where T is a set of *tags* and $\leq$ is a partial order on the tags. The tag ordering is used to resolve the ordering among events at the system interface.

3.1 Tag Behaviors

Events occur at the interface of a component. A component exposes a set V of *variables* (or *ports*) which can take values from a set D. An *event* is a snapshot of a variable state, capturing the variable value at some point in time. Formally, an *event* e on a variable $v \in V$ is a pair (τ, d) of a tag $\tau \in T$ and a value $d \in D$. The simplest way of characterizing a behavior is as a collection of events for each variable. To construct behaviors incrementally, the events of a variable are indexed into a sequence, with the understanding that events later in the sequence have larger tags [4]. A behavior for a variable v is thus a function $\mathbb{N} \mapsto (T \times D)$. A behavior σ for a component assigns a sequence of events to every variable in V, i.e. $\sigma \in V \mapsto (\mathbb{N} \mapsto (T \times D))$. Each event of behavior σ is identified by a tuple (v, n, τ, d), capturing the n-th occurrence of variable v as a pair of a tag τ and a value d. In the following, we denote with $\Sigma(V, \mathcal{T})$ the universe of all behaviors over a set of variables V and tag structure $\mathcal{T}$.

Combining behaviors σ_1 and σ_2 on the same tag structure, or *homogeneous* behaviors, amounts to computing their intersection provided that they are consistent, or *unifiable*, written $\sigma_1 \bowtie \sigma_2$, with each other on the shared variables, i.e. $\sigma_1|_{V_1 \cap V_2} = \sigma_2|_{V_1 \cap V_2}$, where $\sigma|_W$ denotes the restriction of behavior σ to the variables in set W. We may then construct a unified behavior $\sigma = \sigma_1 \sqcup \sigma_2$ on the set of variables $V_1 \cup V_2$ where $\sigma(v) = \sigma_1(v)$ for $v \in V_1$ and $\sigma(v) = \sigma_2(v)$ for $v \in V_2$. When behaviors are defined on different tag structures, before unifying them, the set of tags must be equalized by mapping them onto a third tag structure that functions as a common domain. The mappings are called *tag morphisms* and must preserve the order.



(a) System diagram

σ_1 :	m :		0.5;$\mathbf{p}$		1.5;$\mathbf{l}$	...
	x :	0;0	0.5;0	1;0.5	1.5;1	...
σ_2 :	m		1;$\mathbf{p}$		3;$\mathbf{l}$	...
	x :	0;0	1;0	2;0.5	3;1	...

(b) A tank (σ_1) and controller (σ_2) behavior

Fig. 1. Water controlling system

Definition 1 ([4]). *Let $\mathcal{T}$ and $\mathcal{T}'$ be two tag structures. A* tag morphism *from $\mathcal{T}$ to $\mathcal{T}'$ is a total map $\rho : \mathcal{T} \mapsto \mathcal{T}'$ such that $\forall \tau_1, \tau_2 \in \mathcal{T} : \tau_1 \leq \tau_2 \Rightarrow \rho(\tau_1) \leq \rho(\tau_2)$.*

Here, the tag orders must be taken on the respective domains. Using tag morphisms, we can turn a T-behavior $\sigma \in V \mapsto (\mathbb{N} \mapsto (T \times D))$ into a T'-behavior $\sigma \circ \rho \in V \mapsto (\mathbb{N} \mapsto (T' \times D))$ by simply replacing all tags τ in σ with the image $\rho(\tau)$. Unification of heterogeneous behaviors can be done on the common tag structure. Let $\rho_1 : \mathcal{T}_1 \mapsto \mathcal{T}$ and $\rho_2 : \mathcal{T}_2 \mapsto \mathcal{T}$ be two tag morphisms into a tag structure $\mathcal{T}$. Two behaviors σ_1 and σ_2 defined on $\mathcal{T}_1$ and $\mathcal{T}_2$ respectively are *unifiable in the heterogeneous sense*, written $\sigma_1 \ {}_{\rho_1}\!\bowtie_{\rho_2} \sigma_2$, if and only if $(\sigma_1 \circ \rho_1) \bowtie (\sigma_2 \circ \rho_2)$. The unified behavior σ over $\mathcal{T}$ is then $\sigma = (\sigma_1 \circ \rho_1) \sqcup (\sigma_2 \circ \rho_2)$. It is convenient, however, to retain some information of the original tag structures in the composition, since they are often referred to in the heterogeneous composition, as we will see in the sequel. To do so, we construct the behavior composition over the fibered product [4] $\mathcal{T}_1 \ {}_{\rho_1}\!\times_{\rho_2} \mathcal{T}_2 = (T_1 \ {}_{\rho_1}\!\times_{\rho_2} T_2, \leq)$ of the original tag structures, extending the order component-wise: $(\tau_1, \tau_2) \leq (\tau_1', \tau_2') \iff \tau_1 \leq \tau_1' \wedge \tau_2 \leq \tau_2'$, where $T_1 \ {}_{\rho_1}\!\times_{\rho_2} T_2 = \{(\tau_1, \tau_2) \in T_1 \times T_2 : \rho_1(\tau_1) = \rho_2(\tau_2)\}$.

Example 1. We consider a simplified version of the water controlling system proposed by Benvenuti et al. [14]. It consists of two components: a water tank and a water level controller, connected in a closed-loop fashion, c.f. Fig. 1. We assume that the water level $x(t)$ is changed linearly as follows:

$$x(t) \overset{\text{def}}{=} \begin{cases} \Delta_t * (\mathbf{f_i} - \mathbf{f_o}) & \text{when command is Open} \\ \mathbf{h} - \Delta_t * \mathbf{f_o} & \text{when command is Close} \end{cases} \tag{1}$$

where $\mathbf{f_i}$ and $\mathbf{f_o}$ denote the constant inlet and outlet flow respectively, $\mathbf{h}$ denotes the height when the tank is full of water and Δ_t denotes the time elapsed since t_0 at which the tank reaches the maximum/minimum water level $\mathbf{H}$, i.e., $\Delta_t = t - t_0$. Let $\epsilon_1 = \epsilon_2 = -\infty$, the tank behaviors are naturally defined on tag structure $\mathcal{T}_1 = (\mathbb{R}_+ \cup \{\epsilon_1\}, \leq)$ and the controller behaviors on $\mathcal{T}_2 = (\mathbb{N} \cup \{\epsilon_2\}, \leq)$ representing continuous and discrete time respectively. In addition, both components contain behaviors for two system variables, namely the command variable m and the water level x, thus $V_1 = V_2 = \{m, x\}$. The command values can be Open $(\mathbf{p})$ or Close $(\mathbf{l})$ and the water level is of positive real type and between 0 and $\mathbf{h}$, i.e., $D_m = \{\mathbf{p}, \mathbf{l}\}$ and $D_x = [0, \mathbf{h}]$.

Consider the tank behavior σ_1 and the controller behavior σ_2 described in Fig. 1(b), where $\sigma(v, n)$ is described when the parameter setting is $\mathbf{f_i} = 2, \mathbf{f_o} = 1, \mathbf{h} = 1$. These are different behaviors whose composition is only possible under the presence of morphisms such as $\rho_i : \mathcal{T}_i \mapsto \mathcal{T}_1$ given by $\rho_1(\tau_1) = \tau_1, \rho_2(\tau_2) = 0.5 * \tau_2$.

Our interest in this system is to prove the compatibility between the contracts of these components which will be provided later in this paper. Specifically, the tank contract guarantees a linear evolution of the water level $x(t)$ upon the reception of in-time commands. Meanwhile, the controller contract only assumes the initial emptiness of the tank and guarantees to send proper commands upon detecting its emptiness or fullness.

3.2 Operational Tag Machines

TMs were first introduced to represent sets of homogeneous behaviors [7] and have been recently extended to encompass the heterogeneous context [9]. To construct behaviors, the TM transitions must be able to *increment* time, i.e., to update the tags of the events. An operation of *tag concatenation* on a tag structure is used to accomplish this.

Definition 2 ([7]). *An* algebraic tag structure *is a tag structure* $\mathcal{T} = (T, \leq, \cdot)$ *where* $\cdot$ *is a binary operator on* T *called* concatenation*, such that:*

1. $(T, \cdot)$ *is a monoid with identity element* $\hat{\imath}_{\mathcal{T}}$
2. $\forall \tau_1, \tau_1', \tau_2, \tau_2' \in T : \tau_1 \leq \tau_1' \wedge \tau_2 \leq \tau_2' \Rightarrow \tau_1 \cdot \tau_2 \leq \tau_1' \cdot \tau_2'$
3. $\exists \epsilon_T \in T : \forall \tau \in T : \epsilon_T \leq \tau \wedge \epsilon_T \cdot \tau = \tau \cdot \epsilon_T = \epsilon_T$

Tags can be organized as *tag vectors* $\boldsymbol{\tau} = (\tau^{v_1}, \ldots, \tau^{v_n})$, where n is the number of variables in V. During transition, tag vectors evolve according to a matrix $\mu : V \times V \mapsto T$ called a *tag piece* [7]. The new tag vector is $\boldsymbol{\tau}_\mu \stackrel{\text{def}}{=} \boldsymbol{\tau} \cdot \mu$ where $\tau_\mu^{v_i} \stackrel{\text{def}}{=} \max(\tau^u \cdot \mu(u, v_i))^{u \in V}$ and the maximum is taken with respect to the tag ordering. As the order is partial, the maximum may not exist, in which case the operation is not defined.

Intuitively, a tag piece μ represents increments in all variable tags over a transition and provides a way to operationally renew them. To represent also changes in variable values, μ can be labeled with a partial assignment $\nu : V \to D$, which assigns new values to the variables. A *labeled* tag piece μ thus specifies events for all variables for which ν is defined. In the following, we denote by $\mathfrak{Dom}(\nu)$ the domain of ν and by $L(V, \mathcal{T})$ the universe of all labeled tag pieces, or simply labels, over a variable set V and tag structure $\mathcal{T}$. By abuse of notation, we assume that every tag piece μ has an associated assignment ν.

Example 2. The algebraic tag structure $(\mathbb{N} \cup \{-\infty\}, \leq, +)$, where $+$ is the concatenation operator, can be used to capture logical time by structuring tag pieces μ to represent an integer increment of 1. For instance, $\left[\begin{smallmatrix} 1 & 3 \end{smallmatrix}\right] \cdot \left[\begin{smallmatrix} 0 & 1 \\ -\infty & 1 \end{smallmatrix}\right] = \left[\begin{smallmatrix} 1 & 4 \end{smallmatrix}\right]$. The tag of the second variable is increased by 1 while that of the first variable remains the same since the least element $-\infty = \epsilon$ is used to cancel the contribution of an entry in the tag vector.

A tag machine M is a finite automaton where transitions are marked by labels.

Definition 3 ([9]). *A* tag machine *is a tuple* $(V, \mathcal{T}, S, s_0, F, E)$ *where:*

- V *is a set of variables,*
- $\mathcal{T}$ *is an algebraic tag structure,*
- S *is a finite set of states and* $s_0 \in S$ *is the initial state,*
- $F \subseteq S$ *is a set of accepting states,*

– $E \subseteq S \times L(V, \mathcal{T}) \times S$ *is the transition relation.*

A TM run r is a sequence of states and transitions $r : s_0 \overset{\mu_0}{\to} s_1 \overset{\mu_1}{\to} s_2 \dots s_{m-1} \overset{\mu_{m-1}}{\to} s_m$ such that $s_m \in F$ and for all i, $1 \leq i \leq m$, $(s_{i-1}, \mu_{i-1}, s_i) \in E$. Intuitively, a TM is used to construct a behavior (as defined in Sect. 3.1) by following its labeled transitions over a run, and concatenating the tag pieces sequentially to the initial tag vector $\boldsymbol{\tau} = (\hat{\imath}_{\mathcal{T}}, \dots, \hat{\imath}_{\mathcal{T}})$. A new event is added to the behavior whenever a new value is assigned by the label function ν_i. Run r is *valid* if the concatenation is always defined along the run and $s_m \in F$. The language $\mathcal{L}(M)$ of tag machine M is given by the behaviors of all its valid runs.

3.3 Tag Machine Composition

As TMs are used to represent sets of behaviors, combining TMs amounts to considering only behaviors which are consistent with every TM. In particular, over every transition, the TMs involved in the composition must agree on the tag increment and the value of the shared variables, i.e., their labels are *unifiable*. While TMs defined on the same tag structure, or *homogeneous* TMs, can always be composed, TMs on different tag structures, or *heterogeneous* TMs, can be composed if there exists a pair of *algebraic* tag morphisms mapping the tag structures $\mathcal{T}_1, \mathcal{T}_2$ to a common tag structure $\mathcal{T}$ and preserving the concatenation operator. The homogeneous composition can thus be regarded as a special case of the heterogeneous one when tag morphisms are identity functions mapping a tag to itself.

Definition 4 ([9]). *A tag morphism $\rho : \mathcal{T} \mapsto \mathcal{T}'$ is* algebraic *if $\rho(\hat{\imath}_{\mathcal{T}}) = \hat{\imath}_{\mathcal{T}'}$ and $\rho(\epsilon_{\mathcal{T}}) = \epsilon_{\mathcal{T}'}$ and $\rho(\tau_1 \cdot \tau_2) = \rho(\tau_1) \cdot \rho(\tau_2)$ for all $\tau_1, \tau_2 \in \mathcal{T}$.*

The newly-composed TM will be defined on a unified tag structure and a unified label set. Referring to the previous notation, two labels μ_1 and μ_2 are *unifiable* under morphisms ρ_1 and ρ_2, written $\mu_1 \; {}_{\rho_1}\!\bowtie_{\rho_2} \; \mu_2$, whenever *a*) $\rho_1(\mu_1(w, v)) = \rho_2(\mu_2(w, v))$, and *b*) $\nu_1(v) = \nu_2(v)$, for all pairs $(w, v) \in W \times W$ where $W = V_1 \cap V_2$. Their unification $\mu = \mu_1 \; {}_{\rho_1}\!\sqcup_{\rho_2} \; \mu_2$ is defined over $\mathcal{T}_1 \; {}_{\rho_1}\!\times_{\rho_2} \mathcal{T}_2$ and is any of the members of the unification set of pieces given by

$$\mu(w, v) = \begin{cases} (\mu_1(w, v), \mu_2(w, v)) & \text{if } (w, v) \in W \times W \\ (\mu_1(w, v), \tau_2) & \text{if } w \in V_1, v \in V_1 \setminus V_2 \\ (\mu_1(w, v), \tau_2) & \text{if } w \in V_1 \setminus V_2, v \in V_1 \\ (\tau_1, \mu_2(w, v)) & \text{if } w \in V_2 \setminus V_1, v \in V_2 \\ (\tau_1, \mu_2(w, v)) & \text{if } w \in V_2, v \in V_2 \setminus V_1 \\ (\epsilon_{\mathcal{T}_1}, \epsilon_{\mathcal{T}_2}) & \text{otherwise} \end{cases}$$

where $\tau_2 \in T_2$ is such that $\rho_2(\tau_2) = \rho_1(\mu_1(w, v))$, and similarly $\tau_1 \in T_1$ is such that $\rho_1(\tau_1) = \rho_2(\mu_2(w, v))$. The unified labeling function agrees with individual functions on the shared variables:

$$\nu(v) = \begin{cases} \nu_1(v) \text{ if } v \in V_1 \\ \nu_2(v) \text{ if } v \in V_2 \end{cases}$$

The composition $M = M_1 \; {}_{\rho_1}\!\|_{\rho_2} \; M_2$ of heterogeneous TMs can then be defined over the unification of heterogeneous tag structures and labels.

Definition 5 ([9]). *The composition of M_1 and M_2 under algebraic tag morphisms ρ_1 and ρ_2 is the tag machine $M = M_1 \ {}_{\rho_1}\|_{\rho_2}\ M_2 = (V, \mathcal{T}_1\ {}_{\rho_1}\!\times_{\rho_2} \mathcal{T}_2, S, s_0, F, E)$ such that*

- $V = V_1 \cup V_2$,
- $S = S_1 \times S_2$, $s_0 = (s_{01}, s_{02})$, $F = F_1 \times F_2$,
- $E = \{((s_1, s_2), \mu_1\ {}_{\rho_1}\!\sqcup_{\rho_2} \mu_2, (s_1', s_2')) : \mu_1\ {}_{\rho_1}\!\bowtie_{\rho_2} \mu_2 \wedge (s_i, \mu_i, s_i') \in E_i, i = 1, 2\}$ *where $\mu_1\ {}_{\rho_1}\!\sqcup_{\rho_2} \mu_2$ extends to all the members of the unification set.*

As homogeneous composition is a special case of the heterogeneous one with identity morphisms, we shall omit the morphisms in the homogeneous notations in the sequel.

4 A Contract Framework for Heterogeneous Systems

Our goal is to use TMs as an operational means for modeling heterogeneous systems in contract-based design flows. To this end, we equip TMs with essential binary operators such as composition to combine two TMs [9] and refinement, quotient and conjunction to relate their sets of behaviors (Sect. 4.1). Moreover, we limit TMs to their *deterministic* form where labeled tag pieces annotated on transitions going out of a state are all different. On top of these TM operators, we propose a heterogeneous contract theory for TM-based specifications with universal contract operators such as composition, refinement and compatibility (Sect. 4.2).

4.1 Tag Machine Operators

Two TMs can be related in a refinement relation when the behavior set of one machine is included in that of the other under the morphisms. In the operational point of view, the refined TM can always take a transition unifiable with that taken by the refining TM. Let $M_i = (V_i, \mathcal{T}_i, S_i, s_{0i}, F_i, E_i)$ be TMs and $\rho_i : \mathcal{T}_i \mapsto \mathcal{T}$ be algebraic tag morphisms, where $i \in \{1, 2\}$. The TM refinement is defined as follows.

Definition 6. *M_1 refines M_2, written $M_1\ {}_{\rho_1}\!\preceq_{\rho_2} M_2$, if there exists a binary relation $R \subseteq S_1 \times S_2$ such that $(s_{01}, s_{02}) \in R$ and for all $(s_1, s_2) \in R$ and $(s_1, \mu_1, s_1') \in E_1$:*

$$\exists (s_2, \mu_2, s_2') \in E_2 : \mu_1\ {}_{\rho_1}\!\bowtie_{\rho_2} \mu_2 \wedge (s_1', s_2') \in R \wedge (s_1' \in F_1 \Rightarrow s_2' \in F_2)$$

The following theorem shows that our TM theory supports (homogenous) *independent implementability*: refinement is preserved when composing components.

Theorem 1. *Let M_i' be TMs defined on $\mathcal{T}_i$ and V_i :*

$$(M_1 \preceq M_1') \wedge (M_2 \preceq M_2') \Rightarrow (M_1\ {}_{\rho_1}\|_{\rho_2} M_2) \preceq (M_1'\ {}_{\rho_1}\|_{\rho_2} M_2').$$

We remark that Theorem 1 only holds for *homogenous* TM refinement, and note that heterogeneous refinement in general is *not* preserved even by homogeneous composition. The reason is that the morphisms involved in the former are generally many-to-one functions and can map two different tags into the same tag.

Example 3. We consider an example where:

- $\mathcal{T}_1 = \{\tau_1\}, \mathcal{T}_2 = \{\tau_2, \tau_2'\}$
- $V_1 = V_2 = \{z\}, D_z = \{\top\}$
- $\rho_1(\tau_1) = \rho_2(\tau_2) = \rho_2(\tau_2') = \tau$

Let M_i, M_i' be defined on $\mathcal{T}_i$ and V_i where $i \in \{1, 2\}$. For the sake of simplicity, assume all TMs have a single state which is both initial and accepting state. In addition, there is only one self-loop at this state annotated with μ_i for machine M_i and μ_i' for machine M_i' such that $\mu_1 = \mu_1' = [\tau_1], \mu_2 = [\tau_2], \mu_2' = [\tau_2'], \nu_1(z) = \nu_1'(z) = \nu_2(z) = \nu_2'(z) = \top$. It is easy to see that $M_1 \;{}_{\rho_1}\!\preceq_{\rho_2}\; M_2$ since $\mu_1 \;{}_{\rho_1}\!\bowtie_{\rho_2}\; \mu_2$ and $M_1' \;{}_{\rho_1}\!\preceq_{\rho_2}\; M_2'$ since $\mu_1' \;{}_{\rho_1}\!\bowtie_{\rho_2}\; \mu_2'$. However, $(M_1 \parallel M_1') \;{}_{\rho_1}\!\not\preceq_{\rho_2}\; (M_2 \parallel M_2')$ since the right composition is empty while the left is not.

While the refinement operator enables us to compare two TMs in terms of sets of behaviors, the composition and quotient operators allow us to synthesize specifications. The TM composition computes the most general specification that retains all unifiable behaviors of two TMs. The dual operator to TM composition is TM quotient which computes the maximal specification as follows.

Definition 7. *The quotient $M_1 \,{}_{\rho_1}\!/_{\rho_2} M_2$ is a machine $M = (V, \mathcal{T}_{12}, S, s_0, F, E)$, where*

- $V = V_1 \cup V_2, \mathcal{T}_{12} \stackrel{\text{def}}{=} \mathcal{T}_1 \;{}_{\rho_1}\!\times_{\rho_2}\; \mathcal{T}_2, s_0 = (s_{01}, s_{02})$,
- $S = (S_1 \times S_2) \cup \{\mathfrak{u}\}$, *where* $\mathfrak{u}$ *is a new* universal *state,*
- $F = ((S_1 \times S_2) \setminus ((S_1 \setminus F_1) \times F_2)) \cup \{\mathfrak{u}\} = (F_1 \times F_2) \cup (S_1 \times (S_2 \setminus F_2)) \cup \{\mathfrak{u}\}$,

$$
\begin{aligned}
E = \{&((s_1, s_2), \mu_1 \,{}_{\rho_1}\!\sqcup_{\rho_2} \mu_2, (s_1', s_2')) \mid \\
&\qquad (\mu_1 \;{}_{\rho_1}\!\bowtie_{\rho_2}\; \mu_2) \wedge ((s_1, \mu_1, s_1') \in E_1) \wedge ((s_2, \mu_2, s_2') \in E_2)\} \\
\cup\{&((s_1, s_2), \mu_1 \,{}_{\rho_1}\!\sqcup_{\rho_2} \mu_2, \mathfrak{u}) \mid \\
&\qquad (\forall s_2' \in S_2 : (s_2, \mu_2, s_2') \notin E_2) \wedge (\exists \mu_1 \in L(V_1, \mathcal{T}_1) : \mu_1 \,{}_{\rho_1}\!\bowtie_{\rho_2} \mu_2)\} \\
\cup\{&(\mathfrak{u}, \mu, \mathfrak{u}) \mid \mu \in L(V, \mathcal{T}_{12})\}.
\end{aligned}
$$

We give an example of a quotient construction in Fig. 4. The dual relation between composition and quotient is presented in the next theorem.

Theorem 2. *The quotient M satisfies refinement $(M_2 \;{}_{\mathsf{id}_2}\!\|_{\mathsf{proj}_2}\; M) \;{}_{\mathsf{proj}_1'}\!\preceq_{\mathsf{id}_1}\; M_1$ where:*

$$
\begin{aligned}
\forall i \in \{1, 2\}, \forall \tau_i \in \mathcal{T}_i : \mathsf{id}_i(\tau_i) &= \tau_i \\
\forall i \in \{1, 2\}, \forall (\tau_1, \tau_2) \in \mathcal{T}_{12} : \mathsf{proj}_i((\tau_1, \tau_2)) &= \tau_i \\
\forall (\tau_2, \tau_{12}) \in \mathcal{T}_2 \;{}_{\mathsf{id}_2}\!\times_{\mathsf{proj}_2}\; \mathcal{T}_{12} : \mathsf{proj}_1'((\tau_2, \tau_{12})) &= \mathsf{proj}_1(\tau_{12}) \\
\forall (\tau_1, \tau_{12}) \in \mathcal{T}_1 \;{}_{\mathsf{id}_1}\!\times_{\mathsf{proj}_1}\; \mathcal{T}_{12} : \mathsf{proj}_2'((\tau_1, \tau_{12})) &= \mathsf{proj}_2(\tau_{12})
\end{aligned}
$$

Moreover, for M' defined on $\mathcal{T}_{12}$ and V: $(M_2 \;{}_{\mathsf{id}_2}\!\|_{\mathsf{proj}_2}\; M') \;{}_{\mathsf{proj}_1'}\!\preceq_{\mathsf{id}_1}\; M_1 \Rightarrow M' \preceq M$.

Thus, the quotient M is the *greatest*, in the (homogeneous) refinement preorder, of all TMs M' defined in Theorem 2. This universal property is generally expected of quotients [10], and it alone implies that the quotient is uniquely defined up to two-sided homogeneous refinement [19]. As an example, Fig. 3(c) shows a homogeneous quotient and Fig. 4(b) shows a heterogeneous quotient using the morphisms of Example 1.

Finally, the operator of *heterogeneous conjunction*, denoted ${}_{\rho_1}\!\curlywedge_{\rho_2}$, is defined as the greatest lower bound of the refinement order. Conjunction, thus, amounts to computing the intersection of the behavior sets, in order to find the largest common refinement. Thus, for tag machines, conjunction can be computed similarly to composition. The two operators, however, serve very different purposes, and must not therefore be confused.

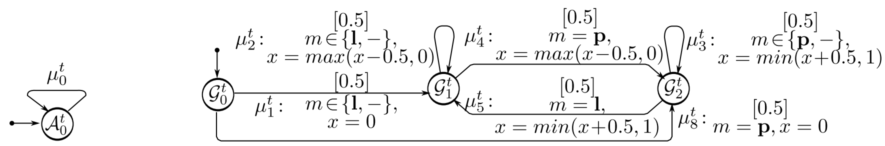
4.2 Tag Contracts

We use the term *tag contract* to mean that in our framework each contract is coupled with an algebraic tag structure, thereby allowing the contract assumption and guarantee to be represented as TMs.

Definition 8. *A tag contract is a homogeneous pair of TMs* $(M_{\mathcal{A}}, M_{\mathcal{G}})$ *where* $M_{\mathcal{A}}$ *- the assumption and* $M_{\mathcal{G}}$ *- the guarantee are TMs defined over the same tag structure* $\mathcal{T}$ *and variable set* V.

Tag contract $\mathcal{C}$ can also be associated with a *profile* $\pi = (V^{\mathfrak{i}}, V^{\mathfrak{o}})$ which is a partition of its variables into inputs and outputs, i.e. $V = V^{\mathfrak{i}} \cup V^{\mathfrak{o}}$ and $V^{\mathfrak{i}} \cap V^{\mathfrak{o}} = \emptyset$. When composing contracts $\mathcal{C}_i$ with profiles π_i, we enforce the property that each output port should be controlled by at most one contract, i.e., $V_1^{\mathfrak{o}} \cap V_2^{\mathfrak{o}} = \emptyset$. The composite contract profile is then $\pi = ((V_1^{\mathfrak{i}} \cup V_2^{\mathfrak{i}}) \setminus (V_1^{\mathfrak{o}} \cup V_2^{\mathfrak{o}}), V_1^{\mathfrak{o}} \cup V_2^{\mathfrak{o}})$. As we will see in the sequel, the notion of profile is closely connected to that of contract compatibility. Thus we will only mention it when elaborating contract compatibility for the sake of readability, .

Example 4. We consider the simplified water controlling system in Example 1 and present a contract for each component. To simplify the behavioral construction, we rely on a special clock $\mathfrak{inc}$ added to the variable set of both components. Tag pieces μ are then structured to represent an increment of δ by always assigning δ to $\mu(\mathfrak{inc}, \mathfrak{inc})$ and assigning δ to all entries $\mu(\mathfrak{inc}, v)$ where $v \in \mathfrak{Dom}(\mu)$, and the least element $-\infty$ to other entries. The tags of x and m are thus renewed to the tag of clock $\mathfrak{inc}$ over every transition. To keep the figures readable we represent tag pieces as $[\delta]$. In addition, the clock value is always equal to its tag and thus is omitted from the labeling function.

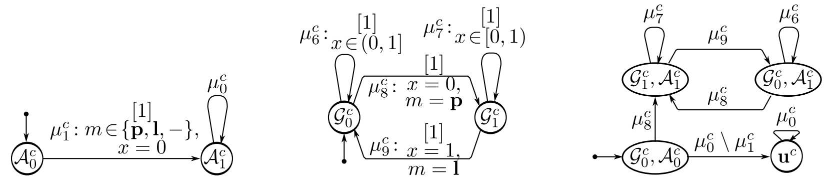


(a) $M_{\mathcal{A}_t}$ (b) $M_{\mathcal{G}_t}$

Fig. 2. The tank contract

Figure 2 depicts the tank contract $\mathcal{C}_t = (M_{\mathcal{A}_t}, M_{\mathcal{G}_t})$ which guarantees a linear evolution of the water level $x(t)$ (Fig. 2(b)) given the assumption satisfaction (Fig. 2(a)). That is, the water level will evolve linearly as specified in Example 1, provided that the controlling command is received at the right time (i.e., open when the tank is empty and close when it is full). For the sake of simplicity, the events described by the tank contract are timestamped periodically every 0.5 time unit.

The controller contract is shown Fig. 3, where it assumes the tank to be empty initially (Fig. 3(a)), i.e., $x = 0$ and places no requirement on its output which is the command signal. As long as such assumption is satisfied, the controller guarantees



(a) $M_{\mathcal{A}_c}$ (b) $M_{\mathcal{G}_c}$ (c) $M_{\mathcal{G}_c}/M_{\mathcal{A}_c}$

Fig. 3. The controller contract

(Fig. 3(b)) to send a proper command upon knowing of the tank emptiness or fullness. Intuitively, the controller behaviors ensure timely control over the water evolution while the tank behaviors accept untimely control and allow water spillages or shortages. While the tank system uses physical time to stamp its behaviors, the controller system instead timestamps its events logically, which can be described by the integer tag set $\mathbb{N}$. In both figures, the initial states are marked with short arrows arriving at them and all states are accepting states. For the sake of expressiveness, some of the labeled tag pieces can be represented symbolically. For example, to capture any event of variable x happening at a specific time point within an interval, we label with the tag piece expressions such as $x \in (0, 1)$, meaning that in such an event x can take any value between 0 and 1. Similarly, $m \in \{\mathbf{p}, \mathbf{l}, -\}$ means the command value can either be open, close or undefined. In addition, we use μ_0^t to denote the universe set of labels $L(V_1, \mathcal{T}_1)$ and μ_0^c the set of labels $L(V_2, \mathcal{T}_2)$.

Example 5. The tank and controller contracts in Example 4 are naturally associated respectively with profiles $\pi_1 = (\{x\}, \{\mathit{cmd}\})$ and $\pi_2 = (\{\mathit{cmd}\}, \{x\})$. The profile of their composition is then $\pi_1 = (\emptyset, \{x, \mathit{cmd}\})$.

The tag contract semantics is subsequently defined through the notions of contract environments and implementations. Let $M_{\mathcal{I}}$ and $M_{\mathcal{E}}$ be TMs defined over tag structure $\mathcal{T}$ and variable set V in Def. 8. We call $M_{\mathcal{E}}$ an environment of contract $\mathcal{C}$ when $M_{\mathcal{E}}$ refines $M_{\mathcal{A}}$. Let $[\![\mathcal{C}]\!]_{\mathsf{e}}$ be the set of all such environments, we call $M_{\mathcal{I}}$ an implementation of contract $\mathcal{C}$, if it holds that $\forall M_{\mathcal{E}} \in [\![\mathcal{C}]\!]_{\mathsf{e}} : M_{\mathcal{I}} \parallel M_{\mathcal{E}} \preceq M_{\mathcal{G}} \parallel M_{\mathcal{E}}$. The set of implementations is similarly denoted by $[\![\mathcal{C}]\!]_{\mathsf{p}}$. Hence, the implementation checking is done based on instantiating all possible environments of a contract. When the contract is *normalized*, such a check can be done independently of the assumption instantiation.

Definition 9. *A tag contract $\mathcal{C} = (M_{\mathcal{A}}, M_{\mathcal{G}})$ is in normalized form if and only if:*

$$\forall M_{\mathcal{I}} : M_{\mathcal{I}} \in [\![\mathcal{C}]\!]_{\mathsf{p}} \Leftrightarrow M_{\mathcal{I}} \preceq M_{\mathcal{G}}.$$

The following theorem states the preservation of tag contract semantics under the normalization operation: whenever a tag contract is in a normalized form, checking contract satisfaction is reduced to finding a refinement relation between two TMs.

Theorem 3. *Tag contract $(M_{\mathcal{A}}, M_{\mathcal{G}}/M_{\mathcal{A}})$ is in normalized form and has the same semantics as $\mathcal{C} = (M_{\mathcal{A}}, M_{\mathcal{G}})$ does.*

Example 6. We use the tag contracts in Example 4 and perform the quotient between the guarantees and assumptions in order to normalize them. Since the tank assumption is the universe of all possible behaviors, i.e., $\Sigma(V_1, \mathcal{T}_1)$, normalizing the tank guarantee adds no more behaviors to the guarantee, i.e., $M_{\mathcal{G}_t}/M_{\mathcal{A}_t} = M_{\mathcal{G}_t}$. Figure 3(c), on the other hand, shows the normalized controller guarantee having more behaviors than the un-normalized one. It is easy to see that the behavior σ_1 in Example 1 is included in $M_{\mathcal{G}_t}$ and σ_2 is in $M_{\mathcal{G}_c}/M_{\mathcal{A}_c}$.

As we will see later, working with normalized tag contracts can simplify the formalization of contract operators (e.g. contract refinement and dominance) as well as provide a unique representation for equivalent contracts, thus we will often assume contracts to be in normalized form hereafter.

Tag Contract Refinement. The refinement relation between two tag contracts is subject to the tag morphisms and is determined by that between their sets of implementations and environments as follows. Let $\mathcal{C}_i = (M_{\mathcal{A}_i}, M_{\mathcal{G}_i})$ be tag contracts defined on $\mathcal{T}_i$ and V_i and $\rho_i : \mathcal{T}_i \mapsto \mathcal{T}$ be algebraic tag morphisms where $i \in \{1, 2\}$

Definition 10. *Contract $\mathcal{C}_1$ refines contract $\mathcal{C}_2$ under morphisms ρ_1 and ρ_2, written $\mathcal{C}_1 \;{}_{\rho_1}\!\preceq_{\rho_2} \mathcal{C}_2$, if the following two conditions hold:*

1. $\forall M_{\mathcal{E}_2} \in [\![\mathcal{C}_2]\!]_{\mathsf{e}} : \exists M_{\mathcal{E}_1} \in [\![\mathcal{C}_1]\!]_{\mathsf{e}} : M_{\mathcal{E}_2} \;{}_{\rho_2}\!\preceq_{\rho_1} M_{\mathcal{E}_1}$
2. $\forall M_{\mathcal{I}_1} \in [\![\mathcal{C}_1]\!]_{\mathsf{p}} : \exists M_{\mathcal{I}_2} \in [\![\mathcal{C}_2]\!]_{\mathsf{p}} : M_{\mathcal{I}_1} \;{}_{\rho_1}\!\preceq_{\rho_2} M_{\mathcal{I}_2}$

The following theorem shows that for two normalized tag contracts, checking refinement can be done at the *syntactic* level, i.e., by finding a TM refinement relation between their assumptions and guarantees.

Theorem 4. $\mathcal{C}_1 \;{}_{\rho_1}\!\preceq_{\rho_2} \mathcal{C}_2 \Leftrightarrow (M_{\mathcal{A}_2} \;{}_{\rho_2}\!\preceq_{\rho_1} M_{\mathcal{A}_1}) \wedge (M_{\mathcal{G}_1} \;{}_{\rho_1}\!\preceq_{\rho_2} M_{\mathcal{G}_2})$

Tag Contract Composition and Dominance. In composing two heterogeneous tag contracts, it is essential to guarantee that composing implementations of each contract results in a new implementation of the composite contract. In addition, every environment of the composite contract should be able to work with any implementation of an individual contract in a way that their composition does not violate the other contract assumption. In fact, there exists a class of contracts, including the composite contract, able to provide such desirable consequences. We refer to them as *dominating* contracts [10].

Definition 11. *A contract $\mathcal{C} = (M_{\mathcal{A}}, M_{\mathcal{G}})$ is said to* dominate *the tag contract pair $(\mathcal{C}_1, \mathcal{C}_2)$ under morphisms ρ_1 and ρ_2 if :*

1. *$\mathcal{C}$ is defined over tag structure $\mathcal{T}_{12} \stackrel{\text{def}}{=} \mathcal{T}_1 \;{}_{\rho_1}\!\times_{\rho_2} \mathcal{T}_2$ and variable set $V = V_1 \cup V_2$*
2. $\forall M_{\mathcal{I}_1} \in [\![\mathcal{C}_1]\!]_{\mathsf{p}}, \forall M_{\mathcal{I}_2} \in [\![\mathcal{C}_2]\!]_{\mathsf{p}} : M_{\mathcal{I}_1} \;{}_{\rho_1}\!\|_{\rho_2} M_{\mathcal{I}_2} \in [\![\mathcal{C}]\!]_{\mathsf{p}}$
3. $\forall M_{\mathcal{E}} \in [\![\mathcal{C}]\!]_{\mathsf{e}} : \begin{cases} \forall M_{\mathcal{I}_1} \in [\![\mathcal{C}_1]\!]_{\mathsf{p}} : (M_{\mathcal{I}_1} \;{}_{\mathsf{id}_1}\!\|_{\mathsf{proj}_1} M_{\mathcal{E}}) \;{}_{\mathsf{proj}'_2}\!\preceq_{\mathsf{id}_2} M_{\mathcal{A}_2} \wedge \\ \forall M_{\mathcal{I}_2} \in [\![\mathcal{C}_2]\!]_{\mathsf{p}} : (M_{\mathcal{I}_2} \;{}_{\mathsf{id}_2}\!\|_{\mathsf{proj}_2} M_{\mathcal{E}}) \;{}_{\mathsf{proj}'_1}\!\preceq_{\mathsf{id}_1} M_{\mathcal{A}_1} \end{cases}$

where the morphisms are defined as in Theorem 2.

The composition of heterogeneous tag contracts can then be defined as follows.

Definition 12. *The composition of tag contracts* $\mathcal{C}_1$ *and* $\mathcal{C}_2$*, written* $\mathcal{C}_1\ {}_{\rho_1}\|_{\rho_2}\ \mathcal{C}_2$*, is another tag contract* $((M_{\mathcal{A}_1\ \rho_1}/_{\rho_2} M_{\mathcal{G}_2}) \curlywedge (M_{\mathcal{A}_2\ \rho_2}/_{\rho_1} M_{\mathcal{G}_1})_{\mathsf{swap}}, M_{\mathcal{G}_1\ \rho_1}\|_{\rho_2}\ M_{\mathcal{G}_2})$ *where* $\mathsf{swap} : \mathcal{T}_2\ {}_{\rho_2}\!\times_{\rho_1} \mathcal{T}_1 \mapsto \mathcal{T}_1\ {}_{\rho_1}\!\times_{\rho_2} \mathcal{T}_2$ *is such that* $\mathsf{swap}((\tau_2, \tau_1)) = ((\tau_1, \tau_2))$ *and* M_{swap} *is* M *where all pieces* μ *are replaced with* $\mu \circ \mathsf{swap}$.

Let $\mathcal{C}'_i$ be normalized tag contracts defined on $\mathcal{T}_i$ and V_i such that $\mathcal{C}'_i \preceq \mathcal{C}_i$ where $i \in \{1, 2\}$. The following theorem states important results: the composition of two normalized contracts dominates the individual contracts and is the *least*, in the homogeneous refinement order, of all contracts dominating them under the same morphisms.

Theorem 5. *Let* $\mathcal{C} = \mathcal{C}_1\ {}_{\rho_1}\|_{\rho_2}\ \mathcal{C}_2$*, then:*

1. $\mathcal{C}$ *dominates the contract pair* $(\mathcal{C}_1, \mathcal{C}_2)$ *under morphisms* ρ_1 *and* ρ_2.
2. *If* $\mathcal{C}'$ *dominates* $(\mathcal{C}_1, \mathcal{C}_2)$ *under morphisms* ρ_1 *and* ρ_2 *then* $\mathcal{C} \preceq \mathcal{C}'$.

The next theorem is another of *independent implementability*: homogeneous tag contract refinement is preserved under the heterogeneous contract composition.

Theorem 6. *Let* $\mathcal{C} = \mathcal{C}_1\ {}_{\rho_1}\|_{\rho_2}\ \mathcal{C}_2$*, then:*

1. *If* $\mathcal{C}$ *dominates* $(\mathcal{C}_1, \mathcal{C}_2)$ *under morphisms* ρ_1 *and* ρ_2 *then it also dominates* $(\mathcal{C}'_1, \mathcal{C}'_2)$ *under the same morphisms.*
2. $(\mathcal{C}'_1\ {}_{\rho_1}\|_{\rho_2}\ \mathcal{C}'_2) \preceq (\mathcal{C}_1\ {}_{\rho_1}\|_{\rho_2}\ \mathcal{C}_2)$.

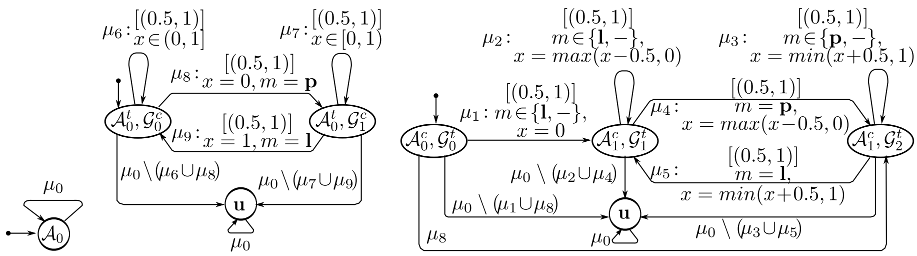
Tag Contract Compatibility. Of particular interest is the notion of *compatibility* between contracts. This notion depends critically on the contract profiles. Intuitively, a contract can only constrain its inputs provided by its environment and provide certain guarantees on its outputs. This is visualized by enforcing the contract assumption to be *output-enabled* and the contract guarantee to be *input-enabled*. Certain models are not input-enabled, e.g. interface automata, because they use input refusal to represent assumptions implicitly. We instead can afford this desirable property as assumptions are represented separately in our framework. A tag machine is said to be input(output)-enabled when it accepts all possible combinations of the input(output) values.

When composing different contracts, it is often desirable to ensure that there exists some environment which can discharge all assumptions made by the composition. The contract compatibility is therefore essential in caring for such a need. Two tag contracts $\mathcal{C}_1$ and $\mathcal{C}_2$ are said to be *compatible* if there exists a contract $\mathcal{C}_e$ defined over the composite tag structure $\mathcal{T}_{12} = \mathcal{T}_1\ {}_{\rho_1}\!\times_{\rho_2} \mathcal{T}_2$ and variable set $V = V_1 \cup V_2$ with profile $\pi_e = (V_1^{\mathsf{o}} \cup V_2^{\mathsf{o}}, (V_1^{\mathsf{i}} \cup V_2^{\mathsf{i}}) \setminus (V_1^{\mathsf{o}} \cup V_2^{\mathsf{o}}))$ such that:

- $M_{\mathcal{A}_e} \equiv M_{\mathfrak{u}}$, c.f. Fig. 4(a), meaning that $\mathcal{C}_e$ makes no assumptions on its inputs and accepts all possible behaviors defined on $L(V, \mathcal{T}_{12})$. In addition, the composition of $\mathcal{C}_1\ {}_{\rho_1}\|_{\rho_2}\ \mathcal{C}_2 = (M_{\mathcal{A}}, M_{\mathcal{G}}) = ((M_{\mathcal{A}_1\ \rho_1}/_{\rho_2} M_{\mathcal{G}_2}) \curlywedge (M_{\mathcal{A}_2\ \rho_2}/_{\rho_1} M_{\mathcal{G}_1})_{\mathsf{swap}}, M_{\mathcal{G}_1\ \rho_1}\|_{\rho_2}\ M_{\mathcal{G}_2})$ and $\mathcal{C}_e$ should also weaken the assumption made on its environment to the greatest extent. That is $(M_{\mathcal{A}_e}/M_{\mathcal{G}}) \curlywedge (M_{\mathcal{A}}/M_{\mathcal{G}_e}) \equiv M_{\mathfrak{u}}$ as well.

– $M_{\mathcal{G}_e}$ is input-enabled so as to make contract $\mathcal{C}_e$ consistent.

In looking for such a contract, it is important to notice that $M_{\mathcal{A}_e} \equiv M_{\mathrm{u}}$, therefore the condition of $(M_{\mathcal{A}_e}/M_{\mathcal{G}}) \curlywedge (M_{\mathcal{A}}/M_{\mathcal{G}_e}) \equiv M_{\mathrm{u}}$ holds when $M_{\mathcal{G}_e}$ is a refinement of $M_{\mathcal{A}}$. Therefore, the compatibility check is reduced to finding a refinement of $M_{\mathcal{A}}$ such that it is input-enabled.



(a) M_{u} (b) $M_{\mathcal{A}_t\ \rho_1}/_{\rho_2} M_{\mathcal{G}_c}$ (c) $(M_{\mathcal{A}_c\ \rho_2}/_{\rho_1} M_{\mathcal{G}_t})_{\mathsf{swap}}$

Fig. 4. Quotient components of the composite assumption of $\mathcal{C}_1\ {}_{\rho_1}\|_{\rho_2}\ \mathcal{C}_2$

Example 7. We consider again the water tank controlling problem in Example 4 and the two contracts on the tank and the controller. The composite assumption of these two contracts is the conjunction of the two quotients shown in Fig. 4(b) and 4(c). Since it is easy to verify that both quotients are equivalent to M_{u}, therefore an input-enabled refinement of the composite assumption exists and we can take for example $(M_{\mathcal{A}_t\ \rho_1}/_{\rho_2} M_{\mathcal{G}_c})$ or $(M_{\mathcal{A}_c\ \rho_2}/_{\rho_1} M_{\mathcal{G}_t})_{\mathsf{swap}}$. Hence the two contracts are compatible.

5 Conclusions

We have presented a modeling methodology based on contracts for designing heterogeneous distributed systems. Heterogeneous systems are usually characterized by their heterogeneity of components which can be of very different nature, e.g. real-time component or logical control component. Without a heterogeneous mechanism, modeling the interaction between components may not be feasible, thereby making it difficult to do verification and analysis based on the known properties of the components. This problem is further complicated for distributed systems where components are developed concurrently by different design teams and are synchronized by relying on their associated contracts. To deal with such problem, we adopt the TM formalism [7,9] for specifying components in terms of operational behaviors. We subsequently propose a contract methodology for synchronizing heterogeneous components based on a set of useful operations on TMs such as composition, quotient and refinement. Our next step is to demonstrate our methodology through a prototype tool and validate it through case studies. The development of such a tool is therefore included in our future work.

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