Combining logic with probability

Motivation

- First-order logic is a powerful language to represent complex relational information
- Probability is the standard way to represent uncertainty in knowledge
- Combining the two would allow to model complex probabilistic relationships in the domain of interest

Combining logic with probability

Problem of uncertainty

- In most real world scenarios, logic formulas are *typically* but not *always* true
- For instance:
  - "Every bird flies" : what about an ostrich (or Charlie Parker) ?
  - "Every predator of a bird is a bird": what about lions with ostriches (or heroin with Parker) ?
  - "Every prey has a predator": predators can be extinct
- A world failing to satisfy even a single formula would not be possible
- there could be no possible world satisfying all formulas

Combining logic with probability

Handling uncertainty

- We can relax the hard constraint assumption on satisfying all formulas
- A possible world not satisfying a certain formula will simply be less likely
- The more formula a possible world satisfies, the more likely it is
- Each formula can have a weight indicating how strong a constraint it should be for possible worlds
- Higher weight indicates higher probability of a world satisfying the formula wrt one not satisfying it

Example: probabilistic relational robotics

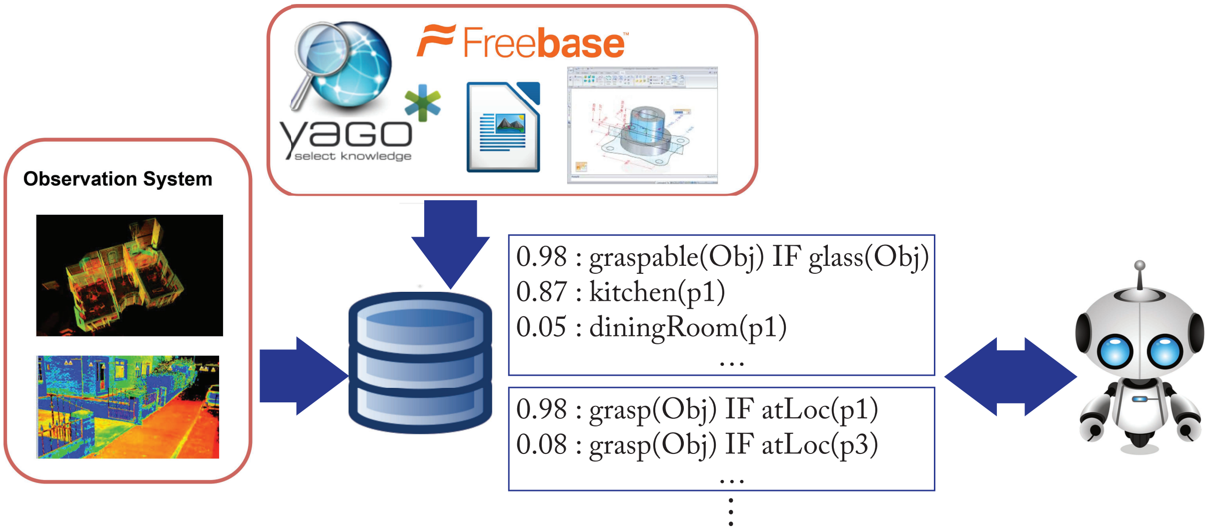


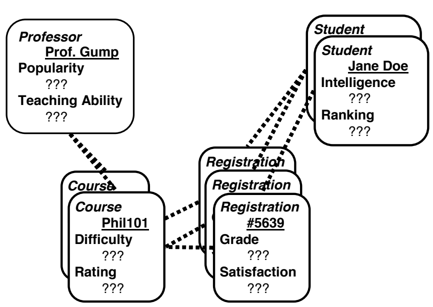
Image from De Raedt et al., 2016

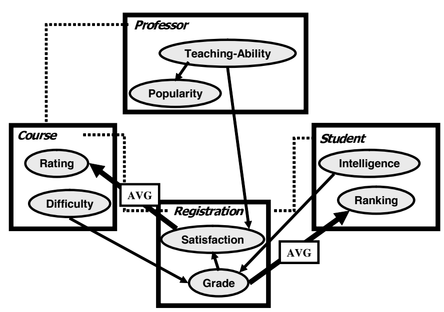
Combining logic with probability

logic graphical models

- Graphical models are a mean to represent joint probabilities highlighting the relational structure among variables
- A compressed representation of such models can be obtained using templates, cliques in the graphs sharing common parameters (e.g. as in HMM for BN or CRF for MN)
- Logic can be seen as a language to build templates for graphical models
- Logic based versions of HMM, BN and MN have been defined

Relational Probabilistic Models (RPM)





- Extend probabilistic models to include relations
- Exploit *exchangeability*: all individuals with the same properties are treated the same

- Template-based models: nodes contain random variables, model can be grounded over instantiation of variables (individuals)
- Alphabetic soup of relational probabilistic models

Image from Getoor et al., 2007

Representative Relational Probabilistic Models (RPM)

Markov Logic

- First-order logic version of Markov Networks
- Undirected RPM
- Extend probabilistic graphical model with logic

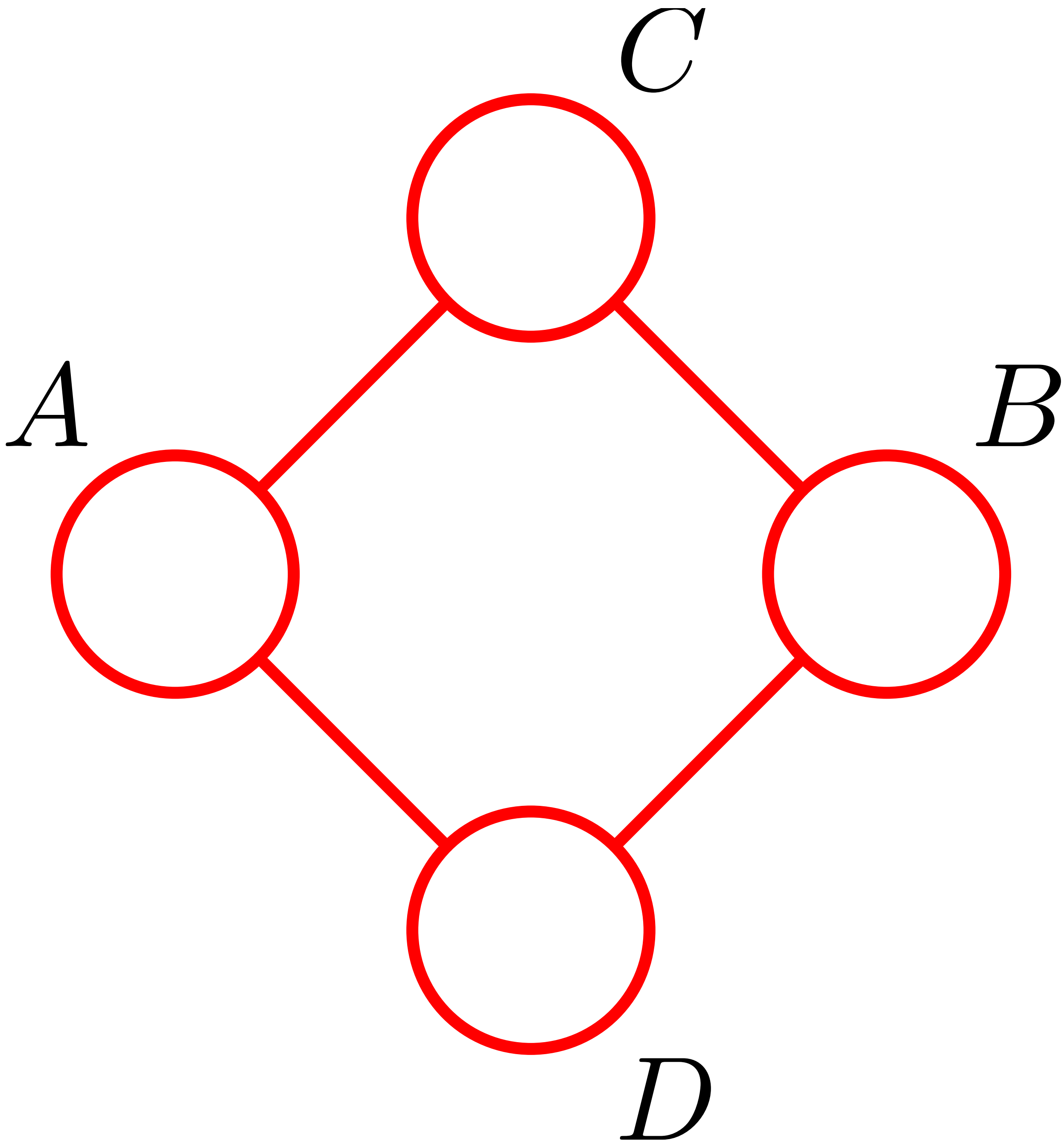
ProbLog

- Probabilistic version of the Prolog language
- Directed RPM
- Extend logic programming language with probabilities

Markov Networks (MN) or Markov random fields

Undirected graphical models

- Graphically model the joint probability of a set of variables encoding independency assumptions (as BN)
- Do not assign a directionality to pairwise interactions



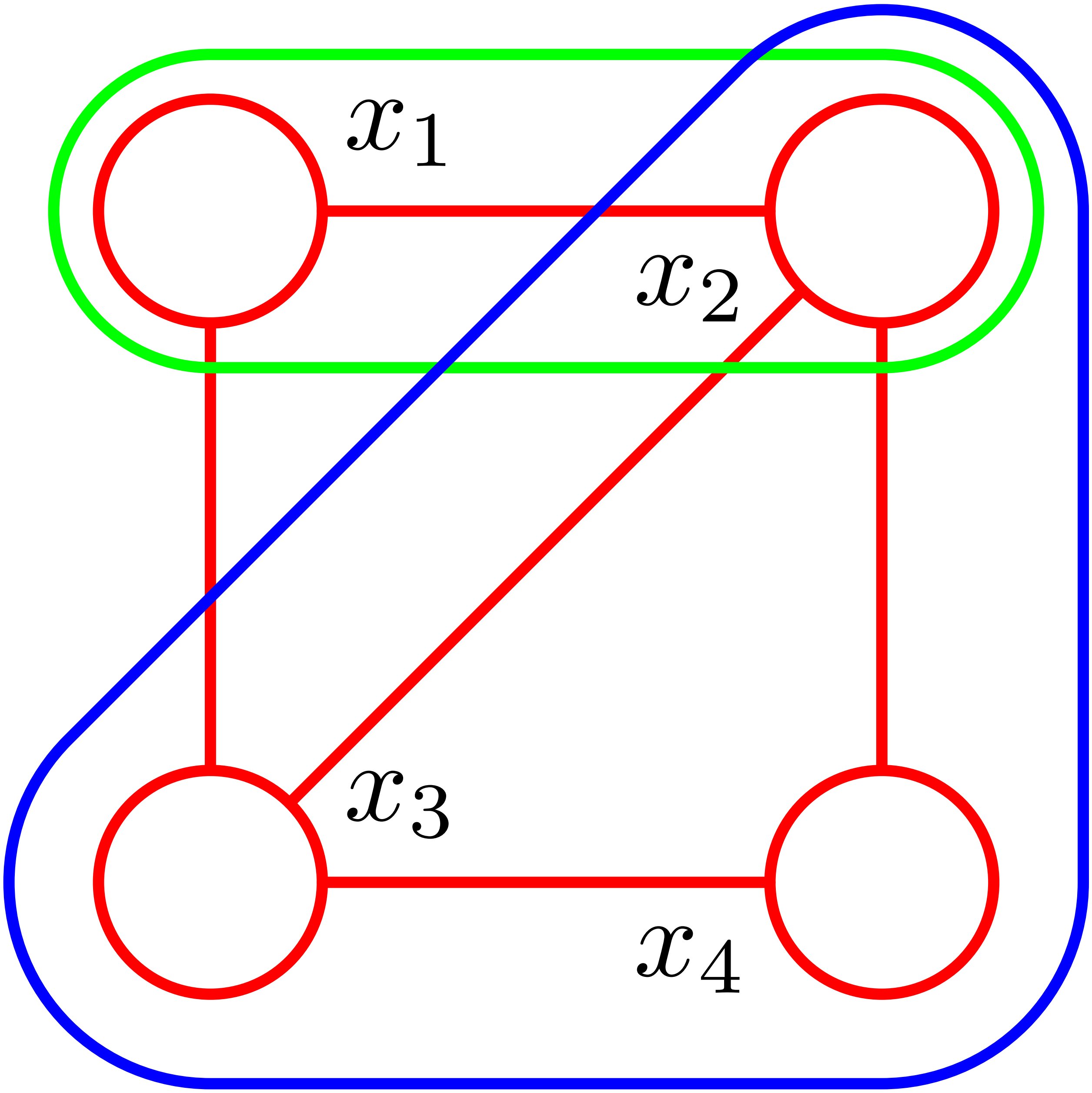
- Do not assume that local interactions are proper conditional probabilities (more freedom in parametrizing them)

Markov Networks (MN)

Factorization properties

- We need to decompose the joint probability in *factors* (like $P(x_i|pa_i)$ in BN) in a way consistent with the independency assumptions.
- Two nodes x_i and x_j not connected by a link are independent given the rest of the network (any path between them contains at least another node) $\Rightarrow$ they should stay in separate factors

- Factors are associated with *cliques* in the network (i.e. fully connected subnetworks)
- A possible option is to associate factors only with *maximal* cliques



Markov Networks (MN)

Joint probability (1)

$$p(\boldsymbol{x}) = \frac{1}{Z} \prod_C \Psi_C(\boldsymbol{x}_C)$$

- $\Psi_C(\boldsymbol{x}_C) : Val(\boldsymbol{x}_c) \rightarrow \mathbb{R}^+$ is a positive function of the value of the variables in clique $\boldsymbol{x}_C$ (called clique *potential*)

- The joint probability is the (normalized) product of clique potentials for all (maximal) cliques in the network
- The *partition function* Z is a normalization constant assuring that the result is a probability (i.e. $\sum_{\boldsymbol{x}} p(\boldsymbol{x}) = 1$):

$$Z = \sum_{\boldsymbol{x}} \prod_{C} \Psi_C(\boldsymbol{x}_C)$$

Markov Networks (MN)

Joint probability (2)

- In order to guarantee that potential functions are positive, they are typically represented as exponentials:

$$\Psi_C(\boldsymbol{x}_C) = \exp\left(-E(\boldsymbol{x}_c)\right)$$

- $E(\boldsymbol{x}_c)$ is called *energy* function: low energy means highly probable configuration
- The joint probability becames a sum of exponentials:

$$p(\boldsymbol{x}) = \frac{1}{Z}\exp\left(-\sum_{C} E(\boldsymbol{x}_c)\right)$$

Markov Networks (MN)

Comparison to BN

- Advantage:
  - More freedom in defining the clique potentials as they don't need to represent a probability distribution
- Problem:
  - The partition function Z has to be computed summing over all possible values of the variables
- Solutions:
  - Intractable in general
  - Efficient algorithms exists for certain types of models (e.g. linear chains)
  - Otherwise Z is approximated

Learning in Markov Networks

Maximum likelihood estimation

- For general MN, the likelihood function cannot be decomposed into independent pieces for each potential (as happens for BN) because of the global normalization (Z)

$$\mathcal{L}(E, \mathcal{D}) = \prod_{i=1}^{N} \frac{1}{Z}\exp\left(-\sum_{C} E_c(\boldsymbol{x}_c(i))\right)$$

- However the likelihood is concave in E_C, the energy functionals whose parameters have to be estimated
- The problem is an unconstrained concave maximization problem solved by gradient ascent (or second order methods)

Learning in Markov Networks

Maximum likelihood estimation

- For each configuration $\mathbf{u}_c \in Val(\mathbf{x}_c)$ we have a parameter $E_{c,\mathbf{u}_c} \in \mathbb{R}$ (ignoring parameter tying)
- The partial derivative of the log-likelihood wrt $E_{c,\mathbf{u}_c}$ is:

$$\begin{aligned}\frac{\partial \log \mathcal{L}(E,\mathcal{D})}{\partial E_{c,\mathbf{u}_c}} &= \sum_{i=1}^{N} [\delta(\mathbf{x}_c(i),\mathbf{u}_c) - p(\boldsymbol{X}_c = \mathbf{u}_c|E)] \\ &= N_{\mathbf{u}_c} - N\, p(\boldsymbol{X}_c = \mathbf{u}_c|E)\end{aligned}$$

- The derivative is zero when the counts of the data correspond to the expected counts predicted by the model
- In order to compute $p(\boldsymbol{X}_c = \mathbf{u}_c|E)$, inference has to be performed on the Markov network, making learning quite expensive

Markov Logic networks

Definition

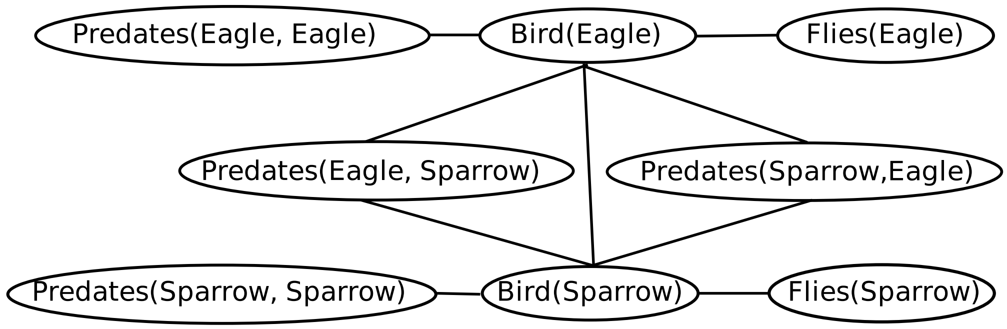
- A Markov Logic Network (MLN) L is a set of pairs (F_i, w_i) where:
  - F_i is a formula in first-order logic
  - w_i is a real number (the weight of the formula)
- Applied to a finite set of constants $C = \{c_1, \dots, c_{|C|}\}$ it defines a Markov network $M_{L,C}$:
  - $M_{L,C}$ has one binary node for each possible grounding of each atom in L. The value of the node is 1 if the ground atom is true, 0 otherwise.
  - $M_{L,C}$ has one feature for each possible grounding of each formula F_i in L. The value of the feature is 1 if the ground formula is true, 0 otherwise. The weight of the feature is the weight w_i of the corresponding formula

Markov Logic networks

Intuition

- A MLN is a *template* for Markov Networks, based on logical descriptions
- Single atoms in the template will generate nodes in the network
- Formulas in the template will be generate cliques in the network
- There is an edge between two nodes iff the corresponding ground atoms appear together in at least one grounding of a formula in L

Markov Logic networks: example



Ground network

- A MLN with two (weighted) formulas:

$$w_1 \quad \forall X \, (\texttt{bird(X)} \Rightarrow \texttt{flies(X)})$$
$$w_2 \quad \forall X, Y \, (\texttt{predates(X,Y)} \wedge \texttt{bird(Y)} \Rightarrow \texttt{bird(X)})$$

- applied to a set of two constants $\{\texttt{sparrow}, \texttt{eagle}\}$
- generates the Markov Network shown in figure

Markov Logic networks

Joint probability

- A ground MLN specifies a joint probability distribution over possible worlds (i.e. truth value assignments to all ground atoms)
- The probability of a possible world x is:

$$p(x) = \frac{1}{Z} \exp \left(\sum_{i=1}^{F} w_i n_i(x) \right)$$

where:

  - the sum ranges over formulas in the MLN (i.e. clique templates in the Markov Network)
  - $n_i(x)$ is the number of true groundings of formula F_i in x
  - The partition function Z sums over all possible worlds (i.e. all possible combination of truth assignments to ground atoms)

Markov Logic networks

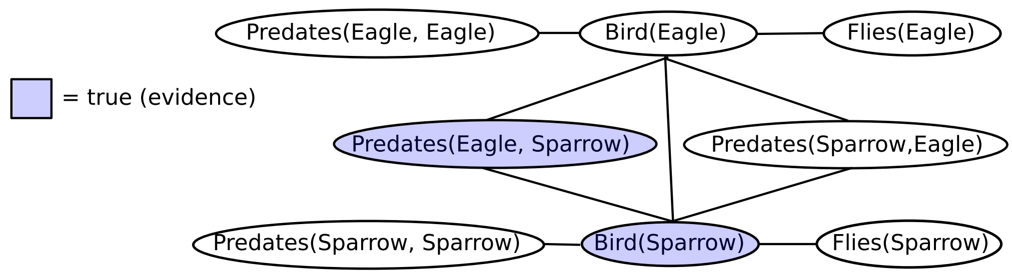
Adding evidence

- Evidence is usually available for a subset of the ground atoms (as their truth value assignment)
- The MLN can be used to compute the conditional probability of a possible world x (consistent with the evidence) given evidence e:

$$p(x|e) = \frac{1}{Z(e)} \exp\left(\sum_{i=1}^{F} w_i n_i(x)\right)$$

- where the partition function $Z(e)$ sums over all possible worlds consistent with the evidence.

Example: evidence

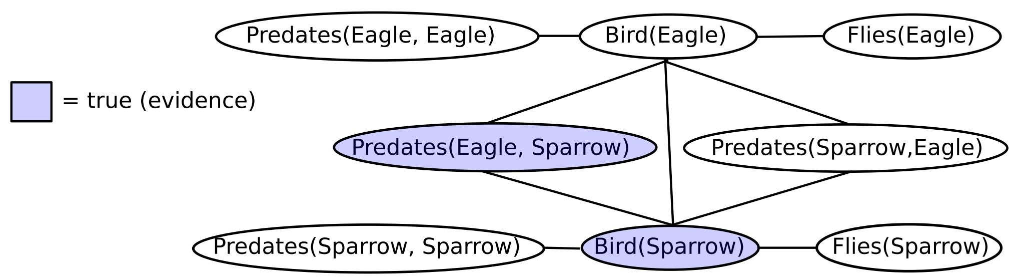


Including evidence

- Suppose that we have (true) evidence e given by these two facts:

```
bird(sparrow)
predates(eagle,sparrow)
```

Example: assignment 1



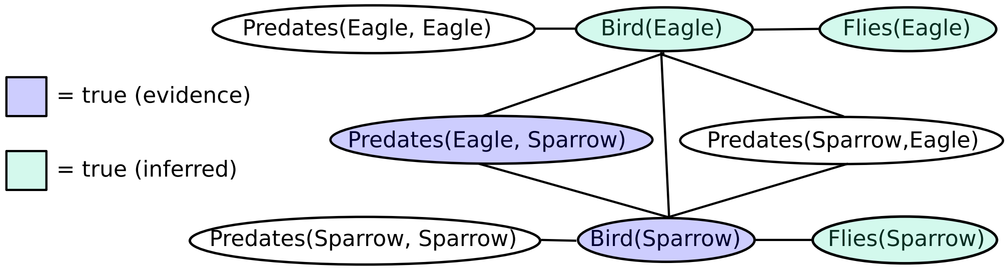
Computing probability

$$p(x) = \frac{1}{Z} \exp(w_1 + 3w_2)$$

- The probability of a world with only evidence atoms set as true violates two ground formulas:

```
bird(sparrow) ⇒ flies(sparrow)
predates(eagle,sparrow) ∧ bird(sparrow) ⇒ bird(eagle)
```

Example: assignment 2

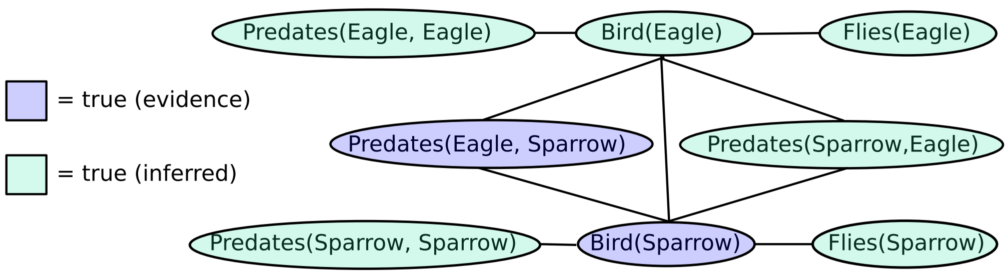


Computing probability

$$p(x) = \frac{1}{Z} \exp(2w_1 + 4w_2)$$

- This possible world is the most likely among all possible worlds as it satisfies all constraints.

Example: assignment 3



Computing probability

$$p(x) = \frac{1}{Z} \exp(2w_1 + 4w_2)$$

- This possible world has also highest probability.
- The problem is that we did not encode constraints saying that:
  - A bird is not likely to be predator of itself
  - A prey is not likely to be predator of its predator

Hard constraints

Impossible worlds

- It is always possible to make certain worlds impossible by adding constraints with infinite weight
- Infinite weight constraints behave like pure logic formulas: any possible world has to satisfy them, otherwise it receives zero probability

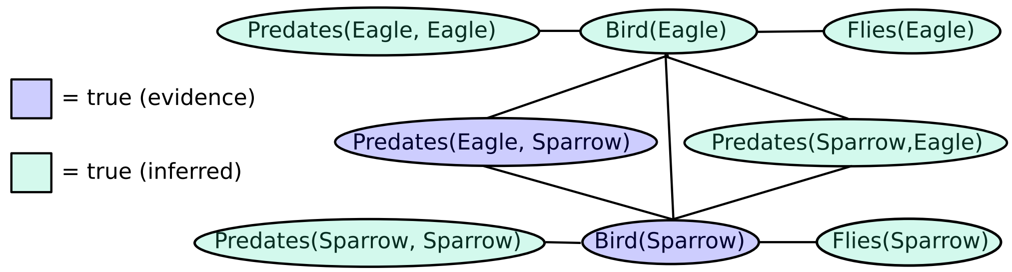
Example

- Let's add the infinite weight constraint:

"Nobody can be a self-predator"
w_3 $\forall X$ $\neg$`predates(X,X)`

to the previous example

Hard constraint: assignment 3

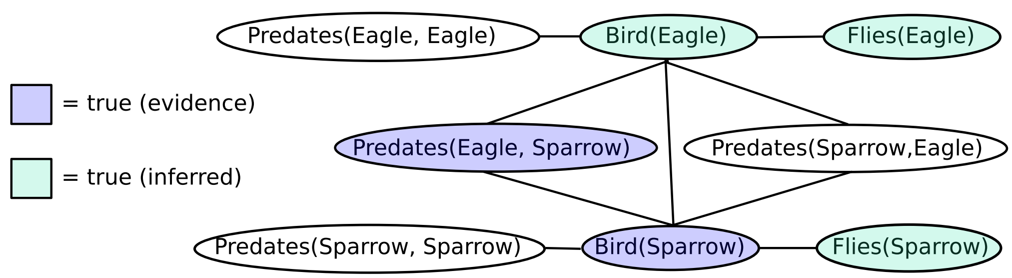


Computing joint probability

$$p(x) = \frac{1}{Z} \exp(2w_1 + 4w_2) = 0$$

- The numerator does not contain w_3, as the no-self-predator constraint is never satified
- However the partition function Z sums over all possible worlds, including those in which the constraint is satisfied.
- As $w_3 = \infty$, the partition function takes infinite value and the possible worlds gets zero probability.

Hard constraint: assignment 2



Computing joint probability

$$p(x) = \frac{1}{Z} \exp(2w_1 + 4w_2 + 2w_3) \neq 0$$

- The only non-zero probability possible worlds are those always satisying hard constraints
- Infinite weight features cancel out between numerator and possible worlds at denominator which also satisfy the constraints, while those which do not become zero

Inference

Assumptions

- For simplicity of presentation, we will consider MLN in form:
  - function-free (only predicates)
  - clausal
- However the methods can be applied to other forms as well
- We will use general first-order logic form when describing applications

Inference

MPE inference

- One of the basic tasks consists of predicting the most probable state of the world given some evidence (the *most probable explanation*)
- The problem is a special case of MAP inference (*maximum a posteriori* inference), in which we are interested in the state of a subset of variables which do not necessarily include all those without evidence.

Inference

MPE inference in MLN

- MPE inference in MLN reduces to finding the truth assignment for variables (i.e. nodes) without evidence maximizing the weighted sum of satisfied clauses (i.e. features)
- The problem can be addressed with any weighted satisfiability solver
- MaxWalkSAT has been successfully used for MPE inference in MLN.

MaxWalkSAT

Description

- Weighted version of WalkSAT
- Stochastic local search algorithm:
  1. Pick an unsatisfied clause at random
  2. Flip the truth value of an atom in the clause
- The atom to flip is chosen in one of two possible ways with a certain probability:
  - randomly
  - in order to maximize the weighted sum of the clauses satisfied with the flip
- The stochastic behaviour (hopefully) allows to escape local minima

MaxWalkSAT pseudocode

```
procedure MaxWalkSAT(weighted_clauses, max_flips, max_tries, target, p)
    vars ← variables in weighted_clauses
    for i ← 1 to max_tries do
        soln ← a random truth assignment to vars
        cost ← sum of weights of unsatisfied clauses in soln
        for j ← 1 to max_flips do
            if cost ≤ target then
                return "Success, solution is", soln
            end if
            c ← a randomly chosen unsatisfied clause
            if Uniform(0,1) < p then
                v_f ← a randomly chosen variable from c
            else
                for all variable v in c do
                    compute DeltaCost(v)
                end for
                v_f ← v with lowest DeltaCost(v)
            end if
            soln ← soln with v_f flipped
            cost ← cost+ DeltaCost(v_f)
        end for
    end for
    return "Failure, best assignment is", best soln found
end procedure
```

MaxWalkSAT

Ingredients

- $target$ is the maximum cost considered acceptable for a solution
- max_tries is the number of walk restarts
- max_flips is the number of flips in a single walk
- p is the probability of flipping a random variable
- Uniform(0,1) picks a number uniformly at random from [0,1]
- DeltaCost(v) computes the change in cost obtained by flipping variable v in the current solution

Inference

Marginal and conditional probabilities

- Another basic inference task is that of computing the marginal probability that a formula holds, possibly given some evidence on the truth value of other formulas
- Exact inference in generic MLN is intractable (as it is for the generic MN obtained by the grounding)
- MCMC sampling techniques have been used as an approximate alternative

Inference

Constructing the ground MN

- In order to perform a specific inference task, it is not necessary in general to ground the whole network, as parts of it could have no influence on the computation of the desired probability
- Grounding only the needed part of the network can allow significant savings both in memory and in time to run the inference

Inference

Partial grounding: intuition

- A standard inference task is that of computing the probability that F_1 holds given that F_2 does.
- We will focus on the common simple case in which F_1, F_2 are conjunctions of ground literals:
  1. All atoms in F_1 are added to the network one after the other
  2. If an atom is also in F_2 (has evidence), nothing more is needed for it
  3. Otherwise, its Markov blanket is added, and each atom in the blanket is checked in the same way

Partial grounding: pseudocode

procedure CONSTRUCTNETWORK(F_1,F_2,L,C)
inputs:
F_1 a set of query ground atoms
F_2 a set of evidence ground atoms
L a Markov Logic Network
C a set of constants
output: M a ground Markov Network
calls: $MB(q)$ the Markov blanket of q in $M_{L,C}$
  $G \leftarrow F_1$
  while $F_1 \neq \emptyset$ **do**
    for all $q \in F_1$ **do**
      if $q \notin F_2$ **then**
        $F_1 \leftarrow F_1 \cup (MB(q) \setminus G)$
        $G \leftarrow G \cup MB(q)$
      end if
      $F_1 \leftarrow F_1 \setminus \{q\}$
    end for
  end while
  return M the ground MN composed of all nodes in G and all arcs between them in $M_{L,C}$, with features and weights of the corresponding cliques
end procedure

Inference

Gibbs sampling

- Inference in the partial ground network is done by Gibbs sampling.
- The basic step consists of sampling a ground atom given its Markov blanket
- The probability of X_l given that its Markov blanket has state $\boldsymbol{B}_l = \boldsymbol{b}_l$ is $p(X_l = x_l | \boldsymbol{B}_l = \boldsymbol{b}_l) =$

$$\frac{\exp \sum_{f_i \in F_l} w_i f_i(X_l = x_l, \boldsymbol{B}_l = \boldsymbol{b}_l)}{\exp \sum_{f_i \in F_l} w_i f_i(X_l = 0, \boldsymbol{B}_l = \boldsymbol{b}_l) + \exp \sum_{f_i \in F_l} w_i f_i(X_l = 1, \boldsymbol{B}_l = \boldsymbol{b}_l)}$$

  where:
  - F_l is the set of ground formulas containing X_l
  - $f_i(X_l = x_l, \boldsymbol{B}_l = \boldsymbol{b}_l)$ is the truth value of the ith formula when $X_l = x_l$ and $\boldsymbol{B}_l = \boldsymbol{b}_l$
- The probability of the conjuction of literals is the fraction of samples (at chain convergence) in which all literals are true

Inference

Multimodal distributions

- As the distribution is likely to have many modes, multiple independently initialized chains are run
- Efficiency in modeling the multimodal distribution can be obtained starting each chain from a mode reached using MaxWalkSAT

Inference

Handling hard constraints

- Hard constraints break the space of possible worlds into separate regions
- This violate the MCMC assumption of reachability
- Very strong constraints create areas of very low probability difficult to traverse
- The problem can be addressed by *slice sampling* MCMC, a technique aimed at sampling from slices of the distribution with a frequency proportional to the probability of the slice

Learning

Maximum likelihood parameter estimation

- Parameter estimation amounts at learning weights of formulas
- We can learn weights from training examples as possible worlds.
- Let's consider a single possible world as training example, made of:
  - a set of constants $\mathcal{C}$ defining a specific MN from the MLN
  - a truth value for each ground atom in the resulting MN
- We usually make a *closed world* assumption, where we only specify the true ground atoms, while all others are assumed to be false.
- As all groundings of the same formula will share the same weight, learning can be also done on a single possible world

Learning

Maximum likelihood parameter estimation

- Weights of formulas can be learned maximizing the likelihood of the possible world:

$$w^{\max} = \text{argmax}_w p_w(x) = \text{argmax}_w \frac{1}{Z} \exp\left(\sum_{i=1}^{F} w_i n_i(x)\right)$$

- As usual we will equivalenty maximize the log-likelihood:

$$\log(p_w(x)) = \sum_{i=1}^{F} w_i n_i(x) - \log(Z)$$

Priors

- In order to combat overfitting Gaussian priors can be added to the weights as usual (see CRF)

Learning

Maximum likelihood parameter estimation

- The gradient of the log-likelihood wrt weights becomes:

$$\frac{\partial}{\partial w_i} \log p_w(x) = n_i(x) - \sum_{x'} p_w(x') n_i(x')$$

  where the sum is over all possible worlds x', i.e. all possible truth assignments for ground atoms in the MN

- Note that $p_w(x')$ is computed using the current parameter values w
- The i-th component of the gradient is the difference between number of true grounding of the i-th formula, and its expectation according to the current model

Logic Programming

As disjunction	As implication	In Prolog
¬ bird(X) ∨ flies(X)	bird(X) ⇒ flies(X)	flies(X) :- bird(X).
¬ predates(X,Y) ∨ ¬ bird(Y) ∨ bird(X)	predates(X,Y) ∧ bird(Y) ⇒ bird(X)	bird(X) :- predates(X,Y), bird(Y).

Horn clauses

- Clauses (disjunctions of literals) with at most one positive literal
- Variables are implicitly universally quantified
- Can be written as implications (the head of the implication is a single atom)
- Amenable to efficient inference by SLD resolution (Prolog programming language)

ProbLog

```
bird(sparrow).
bird(eagle).
bird(ostrich).
predates(eagle,sparrow).
predates(cheetah, ostrich).

0.9 :: flies(X) :- bird(X).
0.8 :: bird(X) :- predates(X,Y), bird(Y).
```

Probabilistic Logic Programming

- **rules**: definite clauses h :- $b_1, \ldots, b_n$ where h is the head and $b_1, \ldots, b_n$ is the body of the rule.
- **facts**: atoms a representing deterministic outcomes.
- **probabilistic rules**: definite clauses p :: h :- $b_1, \ldots, b_n$ where $p \in [0, 1]$ is the probability of the rule.
- **probabilistic facts**: p :: a representing probabilistic outcomes.

ProbLog

```
bird(sparrow).
bird(eagle).
bird(ostrich).
predates(eagle,sparrow).
predates(cheetah, ostrich).

0.9 :: flies(X) :- bird(X).
0.8 :: bird(X) :- predates(X,Y), bird(Y).

query(flies(cheetah)).
```

Probabilistic Queries

P(`flies(cheetah)`) = 0.72

ProbLog: probabilistic inference

```
bird(sparrow).
bird(eagle).
bird(ostrich).
predates(eagle,sparrow).
predates(cheetah, ostrich).

0.9 :: bird_fly(X).
flies(X) :- bird(X), bird_fly(X).

0.8 :: bird_predator_is_bird(X,Y).
bird(X) :- predates(X,Y), bird(Y), bird_predator_is_bird(X,Y).
```

Take probabilities out of rules

probabilistic rules can be made deterministic by introducing auxhiliary probabilistic facts.

- probabilistic rule: $p :: h$:- $b_1, \ldots, b_n$
- deterministic version of the rule: h :- $b_1, \ldots, b_n, a$
- auxhiliary probabilistic fact: $p :: a$ (a must be parameterized by the logical variables in the rule)

ProbLog: probabilistic inference

$$P(\omega) = \prod_{\substack{p\,:\,a \in \mathcal{F},\\ a \in A(\omega)}} p \prod_{\substack{p\,:\,a \in \mathcal{F},\\ a \notin A(\omega)}} (1-p)$$

Probability of a possible world

- $\mathcal{F}$ is the set of ground instances of the probabilistic facts in the logic program.
- ω is a possible world, i.e., a truth assignments to the elements of $\mathcal{F}$.
- $A(\omega)$ is the set of ground instances in $\mathcal{F}$ that are true according to ω.

ProbLog: probabilistic inference

$$P(\omega) = \prod_{\substack{p\,:\,a \in \mathcal{F},\\ a \in A(\omega)}} p \prod_{\substack{p\,:\,a \in \mathcal{F},\\ a \notin A(\omega)}} (1-p)$$

```
bird(sparrow).
bird(eagle).
bird(ostrich).

0.9 :: bird_fly(X).
flies(X) :- bird(X), bird_fly(X).
```

Probability of a possible world: example

- $\mathcal{F} = \{$`bird_fly(sparrow), bird_fly(eagle), bird_fly(ostrich)`$\}$
- $A(\omega) = \{$`bird_fly(sparrow), bird_fly(eagle)`$\}$
- $P(\omega) = 0.9 \cdot 0.9 \cdot (1 - 0.9) = 0.081$

ProbLog: probabilistic inference

$$P(\phi) = \sum_{\omega \models \phi} P(\omega)$$

Probability of a formula (query)

The probability of a formula ϕ is the sum of the probabilities of the possible worlds where the formula holds.

ProbLog: probabilistic inference

```
bird(sparrow).
bird(eagle).
bird(ostrich).
predates(eagle,sparrow).
predates(cheetah, ostrich).

0.9 :: flies(X) :- bird(X).

0.8 :: bird(X) :- predates(X,Y), bird(Y).
```

Probability of a query: example

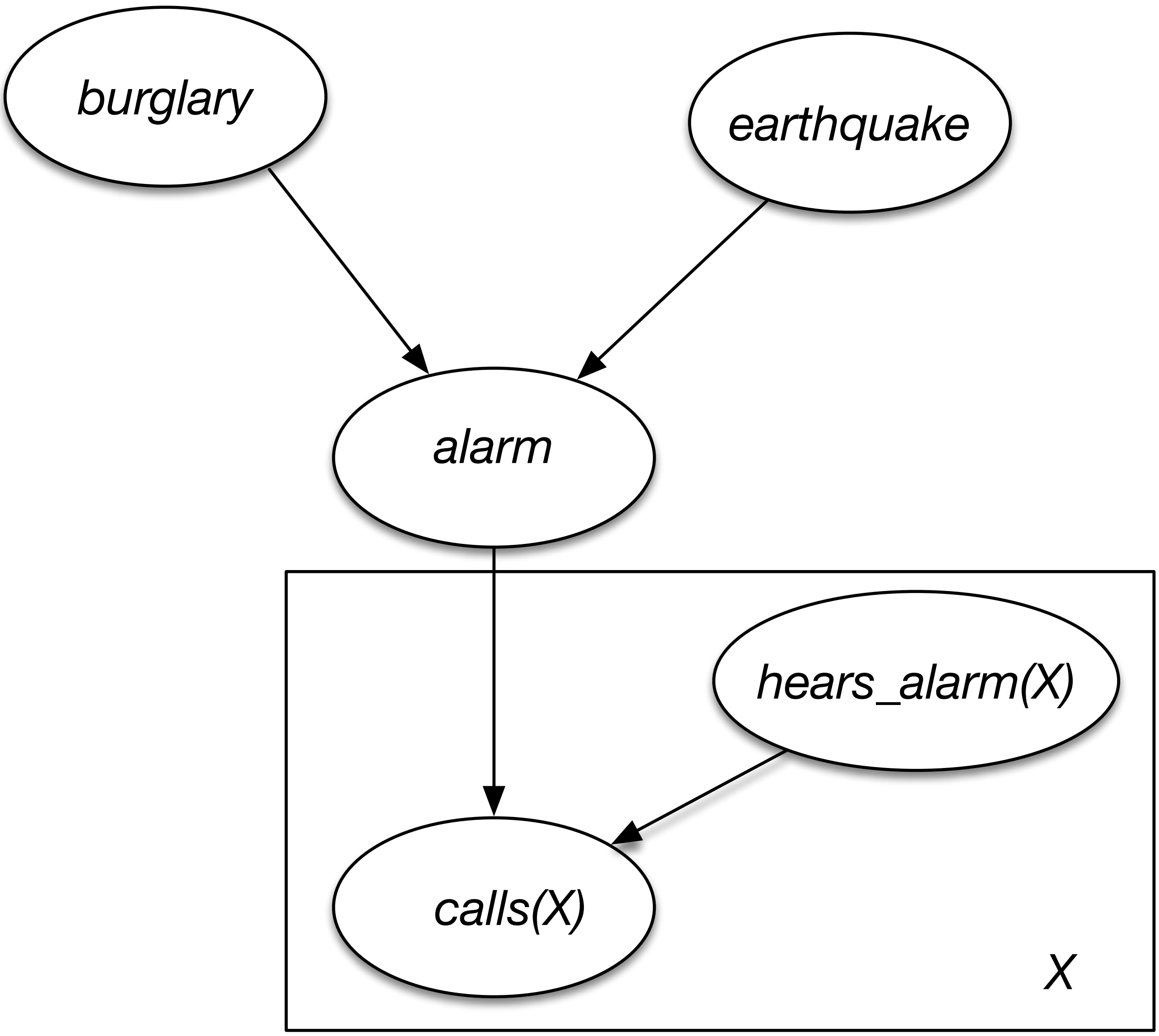
P(`flies(cheetah)`) = 0.9 * 0.8 = 0.72

- In all possible worlds where `flies(cheetah)` holds, `bird(ceetach)` also holds (it's the only way to prove `flies(cheetah)`).
- The probabilities associated to the other ground instances sum to one as all combinations are present in the possible worlds where `flies(cheetah)` holds.

Bayesian Logic Networks in ProbLog

```
0.1 :: burglary.
0.2 :: earthquake.
0.7 :: hears_alarm(john).
0.7 :: hears_alarm(mary).

alarm :- burglary.
alarm :- earthquake.
calls(X) :- alarm, hears_alarm(X).
```



ProbLog: efficient probabilistic inference

Probablistic inference as Weighted Model Counting (WMC)

$$P(\phi) = WMC(\phi) = \sum_{\omega \models \phi} \prod_{\omega \models \ell} w(\ell)$$

- The logic program + the query (and/or evidence) are grounded (instanting variables to constants) and converted into a format amenable to efficient computation (Clark's completion).
- Ground probabilistic facts (and their negation) are given as weights the probability of the fact (or 1 minus it if negated).
- All other literals have weight equal to 1.

ProbLog: efficient probabilistic inference

WMC by knowledge compilation (d-DNNF)

The ground weighted program (+ query/evidence) is compiled into a compact graphical representation, like a d-DNNF:

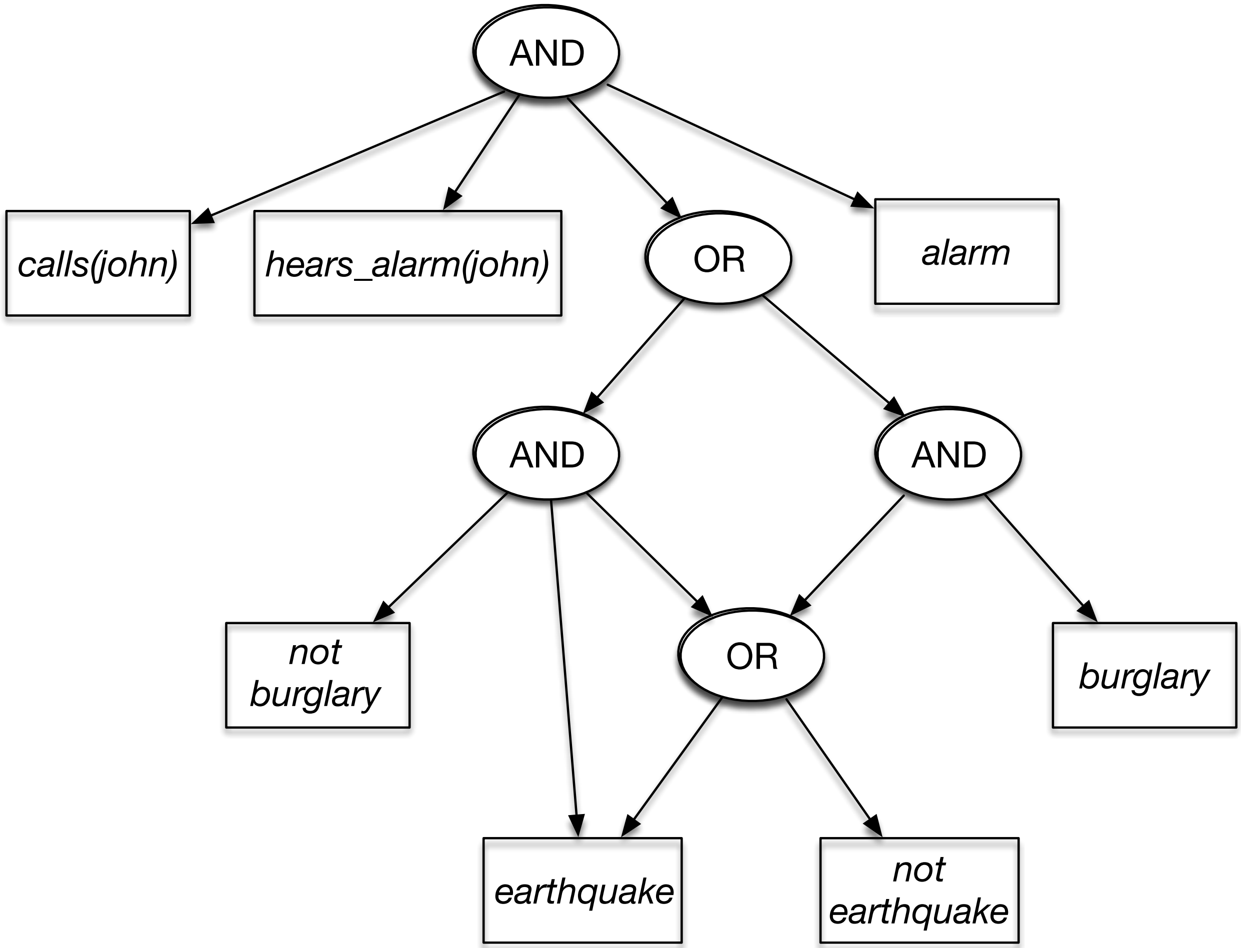
- **NNF**: each leaf is a literal, each internal node is AND or OR
- **DNNF**: decomposable NNF, no two children of an AND node share any atom (can multiply)
- **d-DNNF**: deterministic DNNF, for any OR node, each pair of children should represent logically inconsistent alternatives (can sum)
- **smooth d-DNNF**: all children of an OR node should use exactly the same set of atoms.

Knowledge compilation example: d-DNNF

```
0.1 :: burglary.
0.2 :: earthquake.
0.7 :: hears_alarm(john).
0.7 :: hears_alarm(mary).

alarm :- burglary.
alarm :- earthquake.
calls(X) :- alarm,
            hears_alarm(X).

query(calls(john)).
```



ProbLog: efficient probabilistic inference

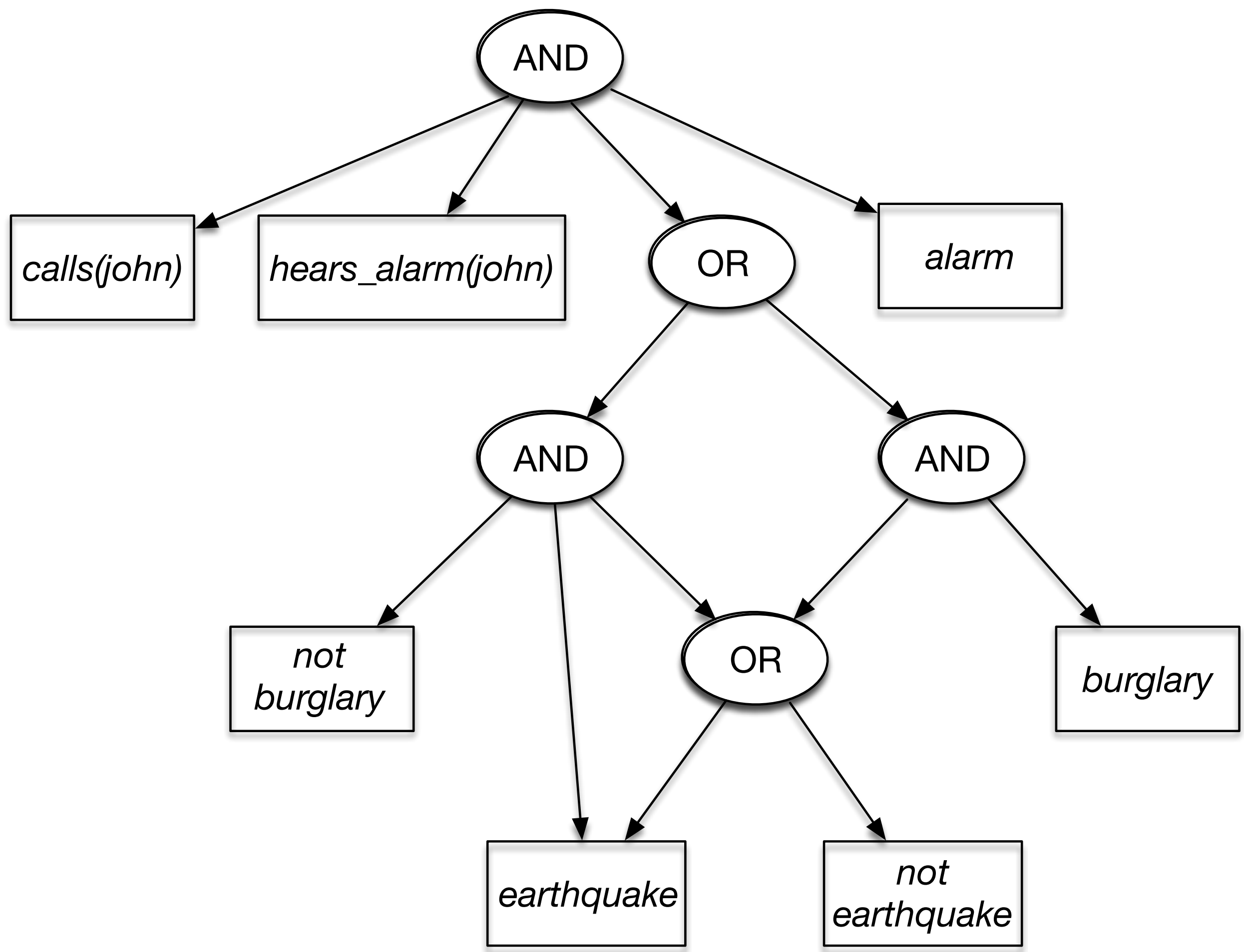
WMC by knowledge compilation (d-DNNF)

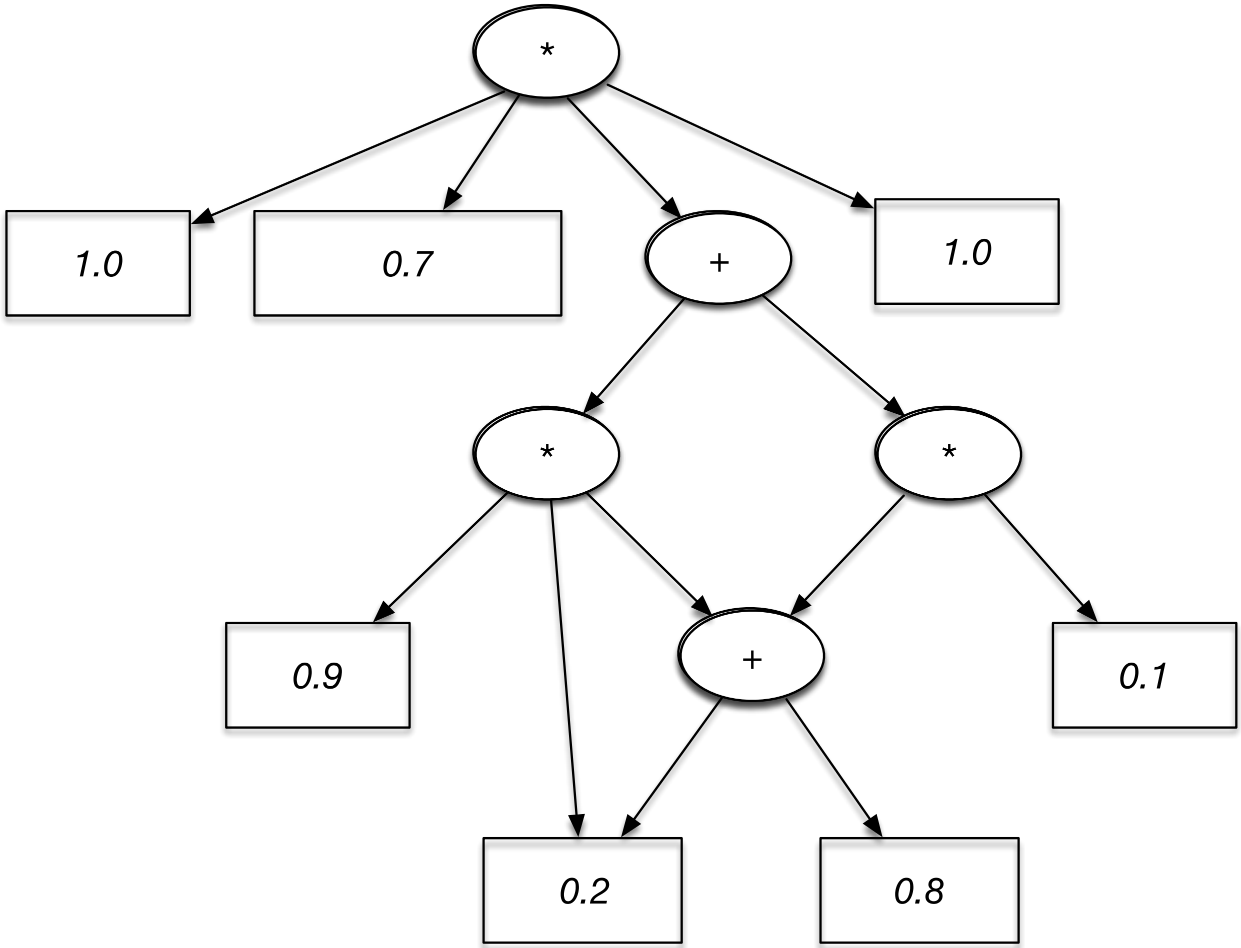
The d-DNNF is converted into an Algebraic Circuit (AC):

- AND are replaced by products
- OR are replaced by sums
- Literals are replaced by their weight

Knowledge compilation example: d-DNNF to AC

```
0.1 :: burglary.                 alarm :- burglary.
0.2 :: earthquake.               alarm :- earthquake.
0.7 :: hears_alarm(john).        calls(X) :- alarm, hears_alarm(X).
0.7 :: hears_alarm(mary).        query(calls(john)).
```


AND
calls(john)
hears_alarm(john)
OR
alarm
AND
AND
not burglary
OR
burglary
earthquake
not earthquake



ProbLog: efficient probabilistic inference

Further improvements

- **Lifted inference**: exploit symmetries (individuals behaving the same) to avoid full grounding (sets of individuals grouped together).
- **Approximate inference**: using e.g. sampling techniques (as in MLN), possibly combined with decomposition strategies (hashing functions).

ProbLog: parameter learning

```
w1 :: burglary.                alarm :- burglary.
w2 :: earthquake.              alarm :- earthquake.
w3 :: hears_alarm(X).          calls(X) :- alarm, hears_alarm(X).
```

Maximum likelihood parameter learning

$$\mathbf{w}^* = \operatorname*{argmax}_{\mathbf{w}} \prod_{\omega \in \mathcal{T}} P(\omega; \mathbf{w})$$

- Probabilities associated to probabilistic facts are unknown (parameters)
- There exists a training set $\mathcal{T}$ of (possibly partial) intepretations (i.e., possible worlds)
- Learning amounts at finding parameters maximizing the likelihood of $\mathcal{T}$

ProbLog: parameter learning

```
w1 :: burglary.                alarm :- burglary.
w2 :: earthquake.              alarm :- earthquake.
w3 :: hears_alarm(X).          calls(X) :- alarm, hears_alarm(X).
```

Complete interpretations: fractional counts

$$w_k^* = \frac{\sum_{\omega \in \mathcal{T}} n_k(A(\omega))}{\sum_{\omega \in \mathcal{T}} n_k(\mathcal{F}(\omega))}$$

- The parameter of a probabilistic fact is estimated as the fraction of its groundings that hold in the training set over the total number of its possibile groundings (same as for BN).
- $n_k(A(\omega))$ is the number of groundings of the k-th probabilistic fact that hold in possible world ω.
- $n_k(\mathcal{F}(\omega))$ is the total number of groundings of the k-th probabilistic fact for possible world ω (true and false).

ProbLog: parameter learning

```
w1 :: burglary.                alarm :- burglary.
w2 :: earthquake.              alarm :- earthquake.
w3 :: hears_alarm(X).          calls(X) :- alarm, hears_alarm(X).
```

Partial interpretations: Expectation-Maximization

$$w_k^{i+1} = \frac{\sum_{\omega \in \mathcal{T}} \sum_{f'_k \in \mathcal{F}_k(\omega)} P(f'_k | \mathbf{E}(\omega) = \mathbf{e}(\omega); w^i)}{\sum_{\omega \in \mathcal{T}} n_k(\mathcal{F}(\omega))}$$

- $\mathbf{E}(\omega)$ are the observed groundings in ω, and $\mathbf{e}(\omega)$ their values.
- $\mathcal{F}_k(\omega)$ is the subset of $\mathcal{F}(\omega)$ containing groundings of the k-th probabilistic fact.
- f'_k ranges over these groundings.
- $P(f'_k | \mathbf{E}(\omega) = \mathbf{e}(\omega); w^i)$ is the probability that f'_k holds in ω given the observed facts and the current estimate of the parameters (initialized randomly).

References

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