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Discrete-Time Multirate Stabilization of Chained Form Systems: Convergence, Robustness, and Performance

♮ F. Conticelli ♯ L. Palopoli

♮ ABB Corporate Research
Milan, Italy
E-mail fabio.conticelli@it.abb.com

♯ Scuola Superiore Sant'Anna
Pisa, Italy
E-mail palopoli@sssup.it

Abstract

This paper presents a discrete-time multirate technique for the set-point stabilization of nonholonomic systems in chained form. Global practical exponential stabilization of the trivial equilibrium in the continuous-time system dynamics is achieved, by using multirate piecewise inputs and an inter-sampling analysis. As a case study, we consider the car-like robot stabilization problem. The control system quality is assessed through a quadratic cost function associated to the multi-sampling period. Robustness with respect to measurement errors and switching delays is also investigated.

Keywords: Chained form systems, Discrete-time representation, Practical exponential stability, Car-like robot

1 Introduction

Planning and control of nonholonomic mechanical systems [19] have been widely investigated in the literature, due to their interesting underlying geometry, and the consequent nontrivial stabilization problem, first pointed out by Brockett [3]. Nonstandard approaches have been then developed ranging from open-loop period control to time-varying, and discontinuous feedback, see for example [11, 4, 5, 18, 1, 20, 8, 13] and references therein.

Despite the rather large existing literature, robustness issues [2, 7], which are instrumental in practical control applications of nonholonomic mechanical systems [6], have not yet been fully treated. In [12] the control design of chained systems affected by smooth perturbations is considered. Recently Jiang [10] proposed a state-scaling and backstepping design to take into account nonlinear time-varying disturbances in chained systems, and a switching strategy to avoid singularities.

In this paper, we present a discrete-time multirate control law for the set-point stabilization of nonholonomic systems in $(2, n)$ chained form, namely

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_i &= x_{i-1}\, u_1 \quad i = 3, \ldots, n \quad , \end{aligned} \tag{1}$$

where $\mathbf{x} = [x_1, \ldots, x_n]^T \in \Re^n$ is the state vector, and $\mathbf{u} = [u_1,\ u_2]^T \in \Re^2$ is the control input vector. We use both tools from discrete-time nonlinear control theory [17, 16] and hybrid systems [14, 15] to synthesize a multirate digital controller, which ensures global practical exponential stability of the trivial equilibrium in the continuous-time system dynamics. The main idea in the control design is to transform the original system differential equation in a discrete-time representation, first derived by Monaco and D. Normand-Cyrot [17], which is equivalent to the original continuous dynamics at the sampling times. Then a two stage design is proposed. In particular the control input u_1 is sampled at a lower rate with respect to u_2, thus producing a net motion which is *time-invariant* during the sampling period. A multi-switching control law is designed for the input u_2, which globally exponentially stabilizes the state variables $(x_2, \ldots, x_n)$. The time-invariant nonzero net motion is guaranteed through a simple discrete-time gain adjustment strategy and state saturation. Inter-sampling analysis is performed using an approach presented in [14]. Moreover, control robustness analysis is presented in case of small bounded state perturbations. As a case study, we consider the car-like robot stabilization problem.

The paper is organized as follows. In Sect. 2, the discrete-time representation is recalled. Sect. 3 presents the multirate digital control law, stability results, and the robustness analysis. In Sect. 4 the proposed framework is applied to the car-like stabilization problem. Robustness with respect to switching delays and performance effects arising from the use of different multi-sampling periods are investigated. In Sect. 5 the major contribution of the paper is summarized and future investigations are outlined.

2 Discrete-Time Representation of Chained Form Systems

Consider the $(2,n)$ chained system in Eq. (1). Chosen a sampling period $T > 0$, denote with $\delta = \frac{T}{n-1}$ the sub-sampling period, where n indicates the dimension of the state space. We associate to the continuous-time axis $t \geq 0,\ t \in \Re$, a multi-switching discrete-time sequence $\{t_{k,l}\}, k \in \mathbf{N},\ l \in (0,1,\ldots,n-2)$, defined as $t_{k,l} = kT + \delta l$. We apply to the $(2,n)$ chained system in Eq. (1), multirate piecewise constant inputs of the following form:

$$u_1(t) = u_1^*(t_{k,0}) \quad t \in [kT,\ (k+1)T) \tag{2}$$

$$u_2(t) = \begin{cases} u_2^*(t_{k,0}) & t \in [kT,\ kT+\delta) \\ \cdots & \\ u_2^*(t_{k,l}) & t \in [kT+l\,\delta,\ kT+(l+1)\,\delta) \\ \cdots & \\ u_2^*(t_{k,n-2}) & t \in [kT+(n-2)\delta,\ kT+(n-1)\,\delta) \end{cases}$$

where the constant values $u_1^*(t_{k,0})$, and $u_2^*(t_{k,l}), k \in \mathbf{N},\ l \in (0,1,\ldots,n-2)$ are control parameters to be chosen later. We briefly recall below the derivation of the state representation of system (1) subject to the control inputs as in (2). More results on discrete-time representation of general nonlinear systems are reported, for example, in [16] (see also references therein), where such representation is derived by using Lie exponential series, or by integrating the Volterra kernels which specify the input-state behavior of Eq. (1).

Since $u_1(t)$ is constant during the sampling period T, the sub-system given by the dynamics of the state variables $\mathbf{x}_r = (x_2,\ldots,\ x_n) \in \Re^{(n-1)}$ yields the following linear time-invariant system (LTI) in the time interval $[kT,\ (k+1)T)$ (for simplicity, the dependence on $t_{k,l}$ is dropped):

$$\dot{\mathbf{x}}_r = A_r(u_1^*)\,\mathbf{x}_r + b_r\,u_2(t) \quad , \tag{3}$$

where $A_r(u_1^*) \in \Re^{(n-1)\times(n-1)}$ has the following expression

$$A_r(u_1^*) = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 & \ldots & 0 & 0 \\ u_1^* & 0 & \ldots & 0 & 0 & \ldots & 0 & 0 \\ \ldots & & & & & & & \\ 0 & 0 & \ldots & u_1^* & 0 & \ldots & 0 & 0 \\ \ldots & & & & & & & \\ 0 & 0 & \ldots & 0 & 0 & \ldots & u_1^* & 0 \end{bmatrix} \quad , \tag{4}$$

and $b_r = [1,\ 0,\ldots,0]^T \in \Re^{(n-1)}$. The above LTI continuous-time system is controllable if u_1^* is nonzero. Notice that the matrix $A_r(u_1^*)$ has all the $n-1$ eigenvalues equal to 0, and since the matrix columns are linear independent if u_1^* is nonzero, the matrix kernel has dimension 1, i.e. $dim\,\mathrm{Ker}A_r(u_1^*) = 1,\ u_1^* \neq 0$. Integrating the differential equation (1) over the sub-interval $[t_{k,l},\ t_{k,l+1})$, the following discrete-time representation is obtained:

$$\begin{aligned} \mathbf{x}_r(t_{k,l+1}) &= A_\delta \mathbf{x}_r(t_{k,l}) + b_\delta\, u_2(t_{k,l}) \\ k &\in \mathbf{N},\ \ l \in (0,1,\ldots,n-2) \quad , \end{aligned} \tag{5}$$

where

$$A_\delta = \exp(A_r(u_1^*)\,\delta) = \begin{bmatrix} 1 & 0 & \ldots & 0 & 0 \\ u_1^*\delta & 1 & \ldots & 0 & 0 \\ \ldots & & & & \\ \frac{u_1^*\delta^{n-2}}{(n-2)!} & \frac{u_1^*\delta^{n-3}}{(n-3)!} & \ldots & u_1^*\delta & 1 \end{bmatrix}$$

and

$$\begin{aligned} b_\delta &= \int_{t_{k,l}}^{t_{k,l+1}} \exp(A_r(u_1^*)\,(t_{k,l+1}-\tau))\,b_r\,d\tau \\ &= \begin{bmatrix} 1 \\ u_1^*\delta^2 \\ \ldots \\ u_1^{*\,(n-2)}\,\frac{\delta^{(n-1)}}{(n-1)!} \end{bmatrix} \quad . \end{aligned} \tag{6}$$

Notice that A_δ, b_δ depend only on the parameters u_1^* and δ. The discrete-time system in Eq. (5), together with the x_1 dynamics described by: $x_1((k+1)T) = x_1(kT) + T\,u_1^*(kT)$, are equivalent (at the time instants $\{t_{k,l}\}$) to the continuous time system (1) controlled by the inputs (2). If $u_1^* \neq 0$, the discrete-time representation in Eq. (5) is controllable up to particular values of δ, see [17]. The discrete-time representation can also be viewed as a *family* of LTI discrete-time systems $\{\mathcal{S}_k\},\ k \in \mathbf{N}$, each one defined on the time interval $[kT,\ (k+1)T)$ and parametrized by u_1^* and δ.

3 Discrete-Time Multirate Stabilization

Consider an element of the system family $\{\mathcal{S}_k\},\ k \in \mathbf{N}$, since the the system is linear and time-invariant, the discrete-time flow is easily computed as

$$\mathbf{x}_r(t_{k,l+1}) = A_\delta^{(l+1)}\,\mathbf{x}_r(t_{k,0}) + \sum_{j=0}^{l} A_\delta^{l-j}\,b_\delta\,u_2^*(t_{k,j}) \quad . \tag{7}$$

Then, at time $(k+1)T$, the state variable x_r has the following expression

$$\begin{aligned} \mathbf{x}_r((k+1)T) &= A_\delta^{(n-1)}\,\mathbf{x}_r(kT) \\ &+ \ < A_\delta^{<(n-1)}, b_\delta > \mathbf{u}_2^*(kT) \end{aligned} \tag{8}$$

where we have introduced the controllability matrix $< A_\delta^{<(n-1)}, b_\delta >$ given by

$$< A_\delta^{<(n-1)}, b_\delta > = [\, A_\delta^{n-2}\, b_\delta \quad A_\delta^{n-3}\, b_\delta \quad \dots \quad b_\delta\,] \quad , \tag{9}$$

and the control input vector is given by $\mathbf{u}_2^*(kT) = [u_2^*(t_{k,0}), \dots, u_2^*(t_{k,n-2})]^T \in \Re^{(n-1)}$, i.e. the collection of the values of the piecewise signal $u_2(t)$ of Eq. (2). The family of LTI discrete-time systems $\{\mathcal{S}_k\}$, $k \in \mathbf{N}$ given in Eq. (8) is decoupled by the following control law:

$$\begin{aligned} \mathbf{u}_2^*(kT) &= < A_\delta^{<(n-1)}, b_\delta >^{-1} (\nu(kT) \\ &- A_\delta^{(n-1)}\, \mathbf{x}_r(kT)) \quad , \end{aligned} \tag{10}$$

where $\nu(kT) \in \Re^{n-1}$ denotes the re-defined control input, and the controllability matrix is nonsingular if $u_1^* \neq 0$, up to particular values of δ, i.e. the values corresponding to the loss of controllability of the system in Eq. (5) (see [17]).

Consider the LTI discrete-time system comprising the x_1-dynamics and the system (8) decoupled through (10):

$$\begin{aligned} x_1((k+1)T) &= x_1(kT) + T\, u_1^*(kT) \\ \mathbf{x}_r((k+1)T) &= \nu(kT) \quad , \end{aligned} \tag{11}$$

the following control law is proposed

$$\begin{aligned} u_1^*(kT) &= \frac{1}{T}\,((\lambda_1 - 1)\, x_1(kT) \\ &- \gamma^*(kT)\, \mathrm{sat}(\|\mathbf{x}_r\|, \overline{\phi}, \underline{\phi})) \\ \nu(kT) &= \Lambda\, \mathbf{x}_r(kT) \quad , \end{aligned} \tag{12}$$

where $\lambda_1 \in (0,1)$, the matrix $\Lambda \in \Re^{(n-1)\times(n-1)}$ is convergent, i.e. $\|\Lambda\| < 1$, the saturation thresholds satisfy $\overline{\phi} > \underline{\phi} > 0$, and the saturation function is defined as

$$\mathrm{sat}(\|\mathbf{x}_r(kT)\|, \overline{\phi}, \underline{\phi}) = \begin{cases} \overline{\phi} & \text{if } \|\mathbf{x}_r(kT)\| \geq \overline{\phi} \\ \|\mathbf{x}_r(kT)\| & \text{if } \|\mathbf{x}_r(kT)\| \in (\overline{\phi},\ \underline{\phi}] \\ \underline{\phi} & \text{otherwise} \end{cases} .$$

Fixed the interval $I_{\gamma^*} = [\underline{\gamma^*},\ \overline{\gamma^*}]$, $\overline{\gamma^*}, \underline{\gamma^*} \in \Re$ sufficiently large, define the threshold $\overline{\epsilon} > 0$, and the incremental step $\delta_\gamma > 0$. Define the scalar quantity

$$\begin{aligned} C(\gamma^*, k) &= |\frac{1}{T}\,((\lambda_1 - 1)\, x_1(kT) \\ &- \gamma^*\, \mathrm{sat}(\|\mathbf{x}_r(kT)\|, \overline{\phi}, \underline{\phi}))| \in \Re^+ \quad ; \end{aligned} \tag{13}$$

the time-varying control parameter $\gamma^*(kT)$, $k \in \mathbf{N}$ is chosen according the following *predictive* updating strategy: $\gamma^*(0) = \gamma_0^* \in I_{\gamma^*}$ such that $C(\gamma_0^*, 0) \geq \overline{\epsilon}$, and

$$\gamma^*(kT) = \begin{cases} \gamma_{k-1}^* & \text{if } C(\gamma_{k-1}^*, k) \geq \overline{\epsilon} \\ \gamma_{k-1}^* \pm \delta_\gamma & C(\gamma_{k-1}^*, k) < \overline{\epsilon} \text{ and } \gamma_{k-1}^* \pm \delta_\gamma \in I_{\gamma^*} \\ \gamma_{k-1}^* - \delta_\gamma & C(\gamma_{k-1}^*, k) < \overline{\epsilon} \text{ and } \gamma_{k-1}^* + \delta_\gamma > \overline{\gamma^*} \\ \gamma_{k-1}^* + \delta_\gamma & C(\gamma_{k-1}^*, k) < \overline{\epsilon} \text{ and } \gamma_{k-1}^* - \delta_\gamma < \underline{\gamma^*} \end{cases} \tag{14}$$

We are now in the position to state the following result.

Lemma 1 *Consider the discrete-time system in Eq. (11) controlled by (12). If the control parameters T, δ_γ, $\overline{\epsilon}$, $\overline{\phi}$ and $\underline{\phi}$ satisfy the condition*

$$\delta_\gamma \in (\frac{2\,T\,\overline{\epsilon}}{\underline{\phi}},\ \frac{\overline{\gamma^*} - \underline{\gamma^*}}{2}) \quad , \tag{15}$$

then the origin $\mathbf{x} = 0$ of the closed-loop system (11), (12) is globally practically stable, i.e. the discrete-time x_1-dynamics is globally exponentially ultimate bounded, and the discrete-time x_r-dynamics is globally exponentially asymptotically stable in the sense of Lyapunov.

Proof:

As first step, we prove that the following condition holds

$$|u_1^*(kT)| \geq \overline{\epsilon},\ k \in N \quad . \tag{16}$$

Notice that $|u_1(kT)|$ is by definition equal to $C(\gamma^*(kT), k)$. For any k, if $C(\gamma_{k-1}^*, k) \geq \overline{\epsilon}$, then $\gamma^*(kT)$ remains unchanged and equal to γ_{k-1}^*. Therefore, in this case:

$$|u_1(kT)| = C(\gamma^*(kT), k) = C(\gamma_{k-1}^*, k) \geq \overline{\epsilon} \quad .$$

On the contrary, suppose $C(\gamma_{k-1}^*, k) < \overline{\epsilon}$; the value of $\gamma^*(kT)$ is changed in:

$$\gamma^*(kT) = \gamma_{k-1} \ \pm\ \delta_\gamma$$

where the sign of the δ_γ term is chosen according to the above rule.

By using this parameter update the following inequality can be written for $|u_1(kT)|$:

$$\begin{aligned} & |u_1(kT)| = C(\gamma^*(kT), k) \\ = \ & |\frac{1}{T}((\lambda_1 - 1)\, x_1(kT) \\ - \ & \gamma_{k-1}^*\, sat(\|\mathbf{x}_r\|,\ \overline{\phi},\ \underline{\phi})) \pm \frac{\delta_\gamma}{T}\, sat(\|\mathbf{x}_r\|,\ \overline{\phi},\ \underline{\phi})| \\ \geq \ & ||\frac{\delta_\gamma}{T}\ sat(\|\mathbf{x}_r\|,\ \overline{\phi},\ \underline{\phi})| - C(\gamma_{k-1}^*, k)| \quad . \end{aligned} \tag{17}$$

Condition (15), yields: $|\frac{1}{T}\ \delta_\gamma\ sat(\|\mathbf{x}_r\|,\ \overline{\phi},\ \underline{\phi}))| > 2\ \overline{\epsilon}$. Hence $|u_1(kT)| \geq |\frac{1}{T}\ \delta_\gamma\ sat(\|\mathbf{x}_r\|,\ \overline{\phi},\ \underline{\phi}))| - C(\gamma_{k-1}^*, k) > \overline{\epsilon}$.

Since $|u_1^*(kT)| \geq \overline{\epsilon}$, $k \in N$, the system (8) is decoupled through (10) for every $k \in N$. Then the use of the static state feedback $\nu(kT) = \Lambda\, \mathbf{x}_r(kT)$, where Λ is convergent, ensures that the origin $\mathbf{x}_r = 0$ of the discrete-time x_r-dynamics is globally exponentially stable.

The closed-loop x_1-dynamics results:

$$x_1((k+1)T) = \lambda_1 x_1(kT) - \gamma^*(kT)\,\mathrm{sat}(\|\mathbf{x}_r\|, \overline{\phi}, \underline{\phi}) \quad , \tag{18}$$

this equation can be viewed as a asymptotically stable linear system with a bounded perturbation $\gamma^*(kT)\,\mathrm{sat}(\|\mathbf{x}_r\|, \overline{\phi}, \underline{\phi})$, in fact $\gamma^*(kT) \in I_{\gamma^*} = [\overline{\gamma^*},\ \underline{\gamma^*}]$, $k \in \mathbf{N}$. Hence, the x_1-dynamics is Bounded Input Bounded Output (BIBO). Moreover, the discrete-time x_1-dynamics is globally exponentially ultimate bounded, in fact the perturbation converges exponentially to $\gamma^*(kT)\,\underline{\phi}$ as $k \to \infty$. It follows that x_1 converges exponentially, at least, to the residual set $|x_1| \leq (1-\lambda_1)^{-1}\max(|\overline{\gamma^*}|, |\underline{\gamma^*}|)\,\underline{\phi}$.

□

To analyze the behavior of the continuous-time dynamics (1), subject to the piecewise control inputs (10), (12), we use hybrid systems theory [14, 15]. The main result of this paper is the following.

Proposition 1 *Consider the continuous-time* $(2, n)$ *chained system (1), subject to the piecewise control inputs (10), (12), then the origin* $\mathbf{x} = 0$ *of the closed-loop system is globally practically stable, i.e. the continuous-time* x_1*-dynamics is globally exponentially ultimate bounded, and the continuous-time* x_r*-dynamics is globally uniformly exponentially asymptotically stable in the sense of Lyapunov.*

Proof:

We consider the continuous-time x_r-dynamics in the time interval $[kT,\ (k+1)T)$, $k \in \mathbf{N}$. Fixed $t \in [kT,\ (k+1)T)$, there exists $\bar{l} \in (0, \ldots, n-2)$, such that $t \in [t_{k,\bar{l}},\ t_{k,\bar{l}+1})$, and the flow of $\mathbf{x}_r$ results:

$$\mathbf{x}_r(t) = \exp(A_r(u_1^*)\,(t-kT))\,\mathbf{x}_r(kT) + \sum_{s=0}^{\bar{l}-1}\int_{t_{k,s}}^{t_{k,s+1}} \exp(A_r(u_1^*)\,(t-\tau))\,b_r d\tau\, u_2^*(t_{k,s}) + \int_{t_{k,\bar{l}}}^{t} \exp(A_r(u_1^*)\,(t-\tau))\,b_r d\tau\, u_2^*(t_{k,\bar{l}}) \quad , \tag{19}$$

The control law $\mathbf{u}_2^*(kT)$, $k \in \mathbf{N}$ can be rewritten as: $\mathbf{u}_2^*(kT) = \mathcal{C}(u_1^*(kT))\mathbf{x}_r(kT)$, $\mathcal{C}(u_1^*(kT)) = [\mathcal{C}_0(u_1^*(kT)), \ldots, \mathcal{C}_{n-2}(u_1^*(kT))]^T \in \Re^{(n-1)\times(n-1)}$, where $\mathcal{C}_l^T(u_1^*(kT))^T \in \Re^{n-1}$, $l = 0, \ldots, n-2$ are the rows of the matrix $\mathcal{C}(u_1^*(kT))$, which has the following expression

$$\mathcal{C}(u_1^*(kT)) = \ < A_\delta^{<(n-1)}, b_\delta >^{-1} (\Lambda - A_\delta^{(n-1)}) \quad . \tag{20}$$

It follows that

$$\mathbf{x}_r(t) = \{\exp(A_r(u_1^*)\,(t-kT)) + \sum_{s=0}^{\bar{l}-1}\int_{t_{k,s}}^{t_{k,s+1}} \exp(A_r(u_1^*)\,(t-\tau))\,b_r d\tau\, \mathcal{C}_s^T(u_1^*(kT)) + \int_{t_{k,\bar{l}}}^{t} \exp(A_r(u_1^*)\,(t-\tau))\,b_r d\tau\, \mathcal{C}_{\bar{l}}^T(u_1^*(kT))\}x_r(kT) = \mathcal{D}(u_1^*, t)\,x_r(kT) \quad , \tag{21}$$

where we have indicated with $\mathcal{D}(u_1^*, t)$ the matrix:

$$\mathcal{D}(u_1^*, t) = \exp(A_r(u_1^*)\,(t-kT)) + \sum_{s=0}^{\bar{l}-1}\int_{t_{k,s}}^{t_{k,s+1}} \exp(A_r(u_1^*)\,(t-\tau))\,b_r d\tau\, \mathcal{C}_s^T(u_1^*(kT)) + \int_{t_{k,\bar{l}}}^{t} \exp(A_r(u_1^*)\,(t-\tau))\,b_r d\tau\, \mathcal{C}_{\bar{l}}^T(u_1^*(kT)) \quad . \tag{22}$$

Since $u_1^*(kT)$, $k \in \mathbf{N}$ is bounded, and $t \in [kT,\ (k+1)T)$ is also bounded, then the norm of the matrix $\mathcal{D}(u_1^*, t)$, $t \geq 0$ is bounded, and it does not depend on the initial time $t_0 = 0$. From the relation $\|\mathbf{x}_r(t)\| = \|\mathcal{D}(u_1^*, t)\,\mathbf{x}_r(kT)\| \leq \|\mathcal{D}(u_1^*, t)\|\,\|\mathbf{x}_r(kT)\|$, and Lemma 1, the origin of the continuous-time x_r-dynamics is globally uniformly exponentially asymptotically stable.

A similar argumentation shows that, since the continuous-time x_1-dynamics results:

$$|x_1(t)| = |x_1(kT) + u_1^*(kT)\,(t-kT)| \leq |x_1(kT)| + u_1^*\,T$$

and, being $x_1(kT)$, $u_1^*(kT)$ exponentially ultimate bounded from Lemma 1, also $x_1(t)$, $t \geq 0$ is exponentially ultimate bounded.

□

Remark 1 *Consider the* $(2, n)$ *chained form system in Eq. (1), it has been shown in [8] (see also references therein) that, by using a nonlinear dynamic feedback control law, the chained form can be transformed in a linear system provided that* $u_1(t) \neq 0$, $t \geq 0$. *In our case, the condition* $u_1(t) \neq 0$, $t \geq 0$ *is satisfied by imposing a piecewise form to this signal and by using the* predictive *strategy in Eq. (14).*

Remark 2 *The result given by the above proposition ensures that the equilibrium* $\mathbf{x}_r = 0$ *is uniformly asymptotically stable. From a known result from nonlinear systems theory, see for example [9], this kind of stability implies* stability under persistent disturbances *(i.e. robustness) acting as additive perturbation terms on the system vector field. Moreover, the scalar continuous-time* x_1*-dynamics remains exponentially ultimate bounded if a bounded disturbance acts as an additive term on the bounded perturbation* $\gamma^*(kT)\ sat(\|\mathbf{x}_r\|, \overline{\phi}, \underline{\phi})$.

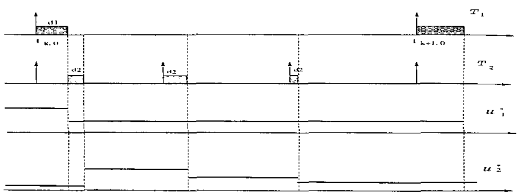

Figure 1: Computation delays on the command variables switchings.

4 A case study

The proposed technique has been applied to a case study consisting of a car-like robot. The kinematic model is nonholonomic and can be described by the following set of state variables: $\mathbf{q} = [x,\ y,\ \theta,\ \phi]^T \in \Re^2 \times S^2$, where S^2 denotes the unit sphere. x and y are the Cartesian coordinates of the rear axle, θ is the angle between the x axis and the robot axle and ϕ is the *steering angle.*

If the robot has *rear-wheel driving*, the kinematic model turns out to be:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi \\ \sin\theta\cos\phi \\ \frac{\sin\phi}{l} \\ 0 \end{bmatrix} v_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v_2, \qquad (23)$$

where v_1 is the driving velocity referred to the front wheel and v_2 is the steering velocity, and the control input vector is denoted as $\mathbf{v} = [v_1,\ v_2]^T \in \Re^2$.

The system can be put in $(2,4)$ chained form [8]. In the rest of this section, we will present simulation results for a robot having $l = 1m$. The considered point stabilization problem is defined as follows. The robot starts from an initial configuration $x(0) = 0,\ y(0) = -5,\ \theta(0) = 0,\ \phi(0) = 0,$ and the desired configuration is the origin of the state space. The controller parameters have been chosen as follows: $T = 7,\ \lambda_1 = 0.3,\ \Lambda = 0.6\,I_3,\ \underline{\phi} = \overline{\epsilon} = 0.005,\ \overline{\phi} = 1,\ \delta_\gamma = 14,\ \overline{\gamma}^* = 15,$ and $\underline{\gamma}^* = -15$. Such a choice for the parameters satisfies the condition (15).

The control robustness has been verified considering two types of disturbances (see also Remark 2). The first problem is the measurement noise on the sensor readings. To simulate its effects, we used four noise sources producing stationary and normal processes, having zero mean and 10^{-4} standard deviation. Such processes have been sampled with a Zero Order Hold (ZOH) at a frequency of $1Khz$. As far as the problem of the computation delays is concerned, we modeled the control system implementation assuming two distinct computation activities (threads): $\mathcal{T}_1$ and $\mathcal{T}_2$. The $\mathcal{T}_1$ thread is activated at the beginning of every period $t_{k,0}$; it does the following actions:compute $u_1^*(t_{k,0})$; compute $u_2^*(t_{k,0})$, $u_2^*(t_{k,1})$, $u_2^*(t_{k,2})$ and store them into intermediate memory buffers; store u_1^* into a memory location whence it will be taken for continuously computing v_1 and v_2, until the next sampling period; trigger the execution of three instances for $\mathcal{T}_2$ thread; the first of them is to be executed at the end of $\mathcal{T}_1$'s execution, the second and the third of them are to be executed at time $t_{k,1}$ and $t_{k,2}$.

The i-th instance of the $\mathcal{T}_2$ thread just reads the value $u_2^*(t_{k,i})$ and stores it into a memory location which is continuously read in order to compute v_2. The $\mathcal{T}_1$ and $\mathcal{T}_2$ threads have uniformly distributed random execution times, d_1 and d_2 respectively. The d_1 variable has been assumed to be in the $[0.02ms\ 5ms]$ range, while d_2's range has been assumed to be $[0.001ms\ 0.1ms]$. We remark that such ranges are quite realistic with respect to modern computer architectures. The temporal behavior of the two threads is summarized in Fig. 1. The simulation results considered in this section are reported in Figures 2, and 3.

5 Conclusions

In this paper, a novel approach for the set-point stabilization of nonholonomic systems in chained form has been presented. The multirate digital stabilizer uses a simple gain adjustment strategy to ensure a nonzero time-invariant net motion. Global practical exponential stabilization of the trivial equilibrium in the continuous-time system dynamics is achieved, by using piecewise inputs and an inter-sampling analysis. As future work, we are considering, both theoretically and numerically, the stability and performance effects of activation frequencies, and scheduling policy in nonlinear multirate digital control systems.

References

[1] M. Aicardi, G. Casalino, A. Balestrino and A. Bicchi. "Closed loop smooth steering of unicycle-like vehicles", IEEE Conference on Decision and Control, Lake Buena Vista, FL, December 1994.

[2] B. Allotta, F. Conticelli and C. Colombo: "Robust Decoupling Control of an Image-Based Visual System", 4th IFAC Nonlinear Control Systems Design Sympo-

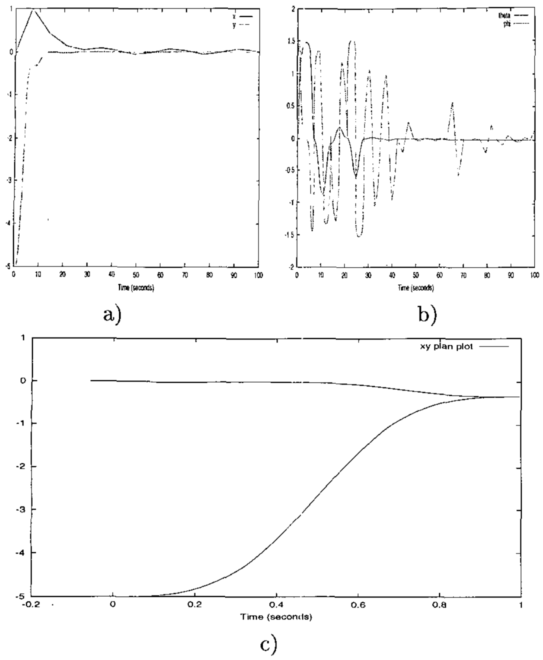

Figure 2: Simulation results with sensor noise and computation delays: initial point [0, -5, 0, 0]. a) x and y dynamics; b) θ and ϕ dynamics; c) Cartesian motion of the vehicle.

sium (NOLCOS'98), Enschede, the Netherlands, July 1998.

[3] R.W. Brockett, "Asymptotic stability and feedback stabilization", In Differential Geometric Control Theory, R.S. Millman R.W. Brockett and H.H. Sussmann, (eds), 1983.

[4] J.M. Coron, "Global Asymptotic Stabilization for controllable systems without drift", Mathematical Control Signals Systems, n. 5, pp. 295-312, 1992.

[5] C. Canudas de Wit and O.J. Sordalen, "Exponential stabilization of mobile robots with nonholonomic constraints", IEEE Trans. on Automatic Control, 1992, pp. 1791-1797.

[6] F. Conticelli, B. Allotta, P. K. Khosla: "Image-based visual servoing of nonholonomic mobile robots", 1999 IEEE Conference on Decision and Control, Phoenix, Arizona, USA, december 1999.

[7] F. Conticelli, B. Allotta: "Robust Stabilization of Second Order Image-Based Systems", Systems & Control Letters, , Vol. 39, No.4, pp. 245 - 253, April 2000.

[8] A. De Luca and G. Oriolo and C. Samson, "Feedback control of a nonholonomic car-like robot", In Robot Motion Planning and Control, J.P. Laumond, (eds), 1998.

[9] A. Isidori: "Nonlinear Control Systems", 3rd ed., Springer Verlag, 1995.

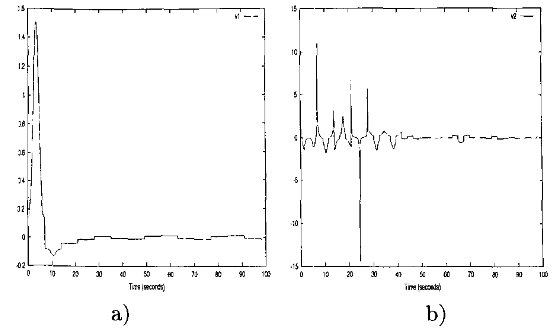

Figure 3: Simulation results with sensor noise and computation delays: initial point [0, -5, 0, 0]. a) v_1 command, b) v_2 command.

[10] Z.P. Jiang, "Robust Exponential Regulation of nonholonomic systems with uncertainties", Automatica, 36, pp. 189-209, 2000.

[11] G. Lafferriere and H.J. Sussmann, "Motion planning for controllable systems without drift", *IEEE Int. Conf. Robotics and Automation*, Sacramento, CA, 1991.

[12] M. Maini, P. Morin, J. B. Pomet, and C. Samson: "On the robust stabilization of chained systems by continouos feedback", 1999 IEEE Conference on Decision and Control, Phoenix, Arizona, USA, december 1999.

[13] A. Marigo, A. Bicchi, "Rolling Bodies with Regular Surface: Controllability Theory and Applications", to appear in *IEEE Trans. Automat. Contr., 2000.*

[14] Bo Hu and Anthony N. Michel, "Some Qualitative Properties of Multirate Digital Control Systems", *IEEE Trans. Automat. Contr.*, vol. 44, no. 4, pp. 765-770, April 1999.

[15] Anthony N. Michel and Bo Hu, "Towards a Stability Theory of General Hybrid Dynamical Systems", *Automatica*, 35, pp. 371-384, 1999.

[16] S. Monaco and D. Normand-Cyrot, *Nonlinear Systems*, vol. 3, chap. 5, Edit. A. J. Fossard and D. Normand-Cyrot, Chapman & Hall, 1996.

[17] S. Monaco and D. Normand-Cyrot, "An introduction to motion planning under multirate digital control", *31st IEEE Conference on Decision and Control*, Tucson, AZ December 1992.

[18] R.M. Murray and S.S. Sastry, "Nonholonomic motion planning: steering using sinusoids", IEEE Trans. on Automatic Control, 38(5), 1993, pp. 700-716.

[19] J.I. Neimark and N.A. Fufaev, *Dynamics of Nonholonomic Systems*, American Math. Society, Providence, Rhode Island, 1972.

[20] C. Samson, "Control of chained systems. application to path following and time-varying point stabilization", IEEE Trans. on Automatic Control, vol. 40, 1995, pp. 64-77.