Multiplex measures for higher-order networks

Quintino Francesco Lotito[1*], Alberto Montresor[1] and Federico Battiston[2*]

*Correspondence:
quintino.lotito@unitn.it;
battistonf@ceu.edu

[1] Department of Information
Engineering and Computer
Science, University of Trento, via
Sommarive 9, 38123 Trento, Italy
[2] Department of Network
and Data Science, Central
European University,
Quellenstraße 51, 1100 Vienna,
Austria

Abstract

A wide variety of complex systems are characterized by interactions of different types involving varying numbers of units. Multiplex hypergraphs serve as a tool to describe such structures, capturing distinct types of higher-order interactions among a collection of units. In this work, we introduce a comprehensive set of measures to describe structural connectivity patterns in multiplex hypergraphs, considering scales from node and hyperedge levels to the system's mesoscale. We validate our measures with three real-world datasets: scientific co-authorship in physics, movie collaborations, and high school interactions. This validation reveals new collaboration patterns, identifies trends within and across movie subfields, and provides insights into daily interaction dynamics. Our framework aims to offer a more nuanced characterization of real-world systems marked by both multiplex and higher-order interactions.

Keywords: Higher-order networks, Multiplex networks, Complex networks, Hypergraph algorithms

Introduction

From biological organisms to social groups, both natural and artificial systems demand sophisticated modeling tools to accurately capture their fundamental properties. Understanding how to represent interactions in such complex systems is crucial for unraveling their intricate architecture and emergent functionality. Networks have long offered a common language for studying these systems, representing units as nodes and interactions as dyadic links (Boccaletti et al. 2006; Cimini 2019). However, this approach overlooks group interactions involving three or more nodes, which are essential in systems where higher-order interactions are prevalent (Battiston 2020, 2021; Torres et al. 2021; Battiston and Petri 2022). Examples include collaboration networks (Patania et al. 2017), human face-to-face interactions (Cencetti et al. 2021, folksonomies (Ghoshal et al. 2009), species interactions within complex ecosystems (Grilli et al. 2017), brain networks (Petri 2014), and cognitive associations (Citraro 2023).

Hypergraphs (Berge 1973), able to explicitly encode group interactions as hyperedges, have emerged as a popular framework to represent higher-order networks (Battiston 2020, 2021). Recently, significant effort has been dedicated to the characterization of hypergraphs,

from centrality (Benson 2019; Tudisco and Higham 2021) and clustering (Benson et al. 2018) measures, to backboning (Musciotto et al. 2021, 2022) and reconstruction (Young et al. 2021). Investigations into higher-order networks cover both micro- (Lotito et al. 2022, 2023; Lee et al. 2020) and mesoscale levels (Wolf et al. 2016; Vazquez 2009; Carletti et al. 2021; Eriksson et al. 2021; Chodrow et al. 2021; Contisciani et al. 2022; Chodrow et al. 2023; Ruggeri et al. 2023; Lotito et al. 2024), unveiling structural principles essential to understanding group interactions in real-world systems. These higher-order interactions are known to impact the dynamic and collective phenomena within networked systems (Battiston 2021), affecting processes such as synchronization (Skardal and Arenas 2020; Millán et al. 2020; Lucas et al. 2020; Gambuzza 2021; Zhang et al. 2023), diffusion (Schaub et al. 2020; Carletti et al. 2020), spreading (Iacopini et al. 2019; Chowdhary et al. 2021; Neuhäuser et al. 2020) and evolution (Alvarez-Rodriguez 2021).

However, not all interactions in complex systems are alike; they may differ in nature, type, and scope. This observation led researchers to introduce the concept of multilayer and multiplex networks (Boccaletti 2014; Kivelä 2014), where links are encoded into different interaction layers, each representing a distinct type of relationship (De Domenico 2013; Battiston et al. 2014). Multilayer and multiplex networks can successfully describe systems such as trade networks (Barigozzi et al. 2010), transportations networks (Aleta et al. 2017), collaboration networks (Battiston et al. 2016), and the brain (De Domenico 2017). Multiplex hypergraphs, where layers encoding hyperedges of different type, could offer a robust tool for describing complex systems that involve group interactions of varying types. Despite a few exceptions (Vasilyeva 2021) and a significant potential, however, multiplex hypergraphs remain relatively unexplored, and a general set of tools for their analysis is still missing.

In this paper, we introduce some basic measures to characterize multiplex networks with higher-order interactions, spanning from the node/hyperedge level to the system's mesoscale. We propose measures for the activity of nodes in different layers and orders, as well as for node degree correlation. Moreover, we partition nodes in generalists or specialists based on how their higher-order degree is spread across layers. We characterize hyperedges by examining hyperedge order distributions and overlap in multiple layers. Additionally, we introduce measures to quantify the layer-dependent ability to connect either generalist or specialist nodes. Finally, we study the correlation of community and core-periphery structures across layers.

We apply our proposals to three different real-world datasets: scientific co-authorship in physics, collaborations in movies and face-to-face interactions in a high school. Our measures are able to highlight patterns of actors' collaborations across various film genres, discern co-authorship trends within and across physics subfields, and offer insights into the daily interaction dynamics of students.

Multiplex hypergraphs

Multiplex hypergraphs model systems where interactions among units (i) may belong to multiple types and (ii) are not necessarily dyadic, i.e. they may involve more than two units. A *multiplex hypergraph* **H** is defined as:

$$\mathbf{H} = \{H_1(V, E_1), \ldots, H_M(V, E_M)\}$$

where each *layer* α is a hypergraph $H_\alpha(V, E_\alpha)$. Each hypergraph $H_\alpha(V, E_\alpha)$ share the same set of entities V. $E_\alpha \subseteq \mathcal{P}(V)$ is the set of all interactions of a specific type α. Moreover, we require $|e| \geq 2$ for all $e \in E_\alpha$ for any α. In other words, each layer in our framework shares the same set of nodes and represents a distinct set of interactions of the same nature. In Fig. 1, we show a simple multiplex hypergraph with 7 nodes and hyperedges spread across 3 layers.

We remark that our proposed framework is different from a multiplex representation of higher-order interactions where layers are defined by interactions of different order (Lucas et al. 2020; Sun and Bianconi 2021).

In this work, we introduce a set of general tools to investigate multiplexity across different system scales in networks with higher-order interactions. We validate our measures and discuss relevant findings for three distinct real-world datasets:

- APS (Co-authorship network), where nodes are authors, and hyperedges represent groups of authors who have co-authored a paper. Each layer collects papers from the same subfield of physics, identified by a PACS code (Physics and Astronomy Classification Scheme) (Dataset 2021).
- IMDB (Co-starring network), where nodes represent actors, and hyperedges represent the cast of a specific movie. Each layer corresponds to a movie genre.
- HIGH SCHOOL (Social network), where nodes are students, and hyperedges represent groups of students interacting face-to-face, with each layer grouping interactions from the same day of the week (Mastrandrea et al. 2015).
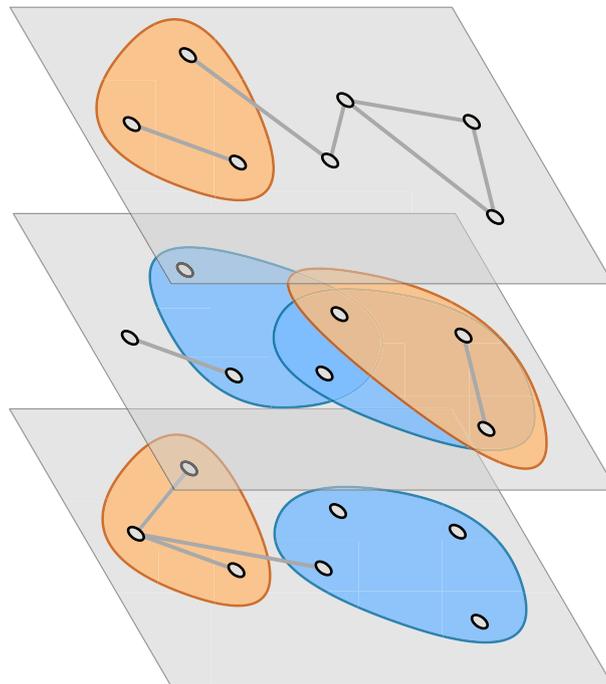

Fig. 1 Multiplex hypergraphs represent systems of units that display interactions of different orders and different types. Each type of interaction is encoded into a single layer of the hypergraph. All the layers share the same set of nodes

Lotito *et al. Applied Network Science*   (2024) 9:55

Page 4 of 14

Table 1 Statistics about three real-world multiplex hypergraphs

APS				IMDB				HIGH SCHOOL			
Layer	N	E	$\bar{d}$	Layer	N	E	$\bar{d}$	Layer	N	E	$\bar{d}$
AMPhys	30375	12562	3.7	Anim	5545	864	9.5	Mo	312	2655	1.3
CM1	63919	27241	3.2	Comedy	69303	9111	13.6	Tu	310	3002	1.2
CM2	103075	58075	4.0	Doc	13357	2007	7.4	We	303	2543	1.2
EMag	57056	30908	2.8	Drama	103163	15384	12.6	Th	295	2529	1.2
EPart	62997	26703	62.2	Family	12968	1274	12.6	Fr	299	2339	1.2
GAA	41535	10670	9.6	Fantasy	14793	1136	15.0				
GasPhy	16182	5120	5.3	Horror	28254	2964	11.6				
Gen	69074	35940	3.2	Thriller	44188	4739	13.8				
IntPhy	48136	17382	3.0								
NPhy	50142	20672	16.7								
H	315421	219769	12.2	**H**	195377	37465	12.6	**H**	327	7818	1.3

For each layer, we report the number of active nodes *N*, the number of hyperedges *E* and their average order $\bar{d}$. In the case of multiple PACS codes or genres associated with a paper or movie, only one code or genre is randomly selected. **H** is the layer-aggregated hypergraph
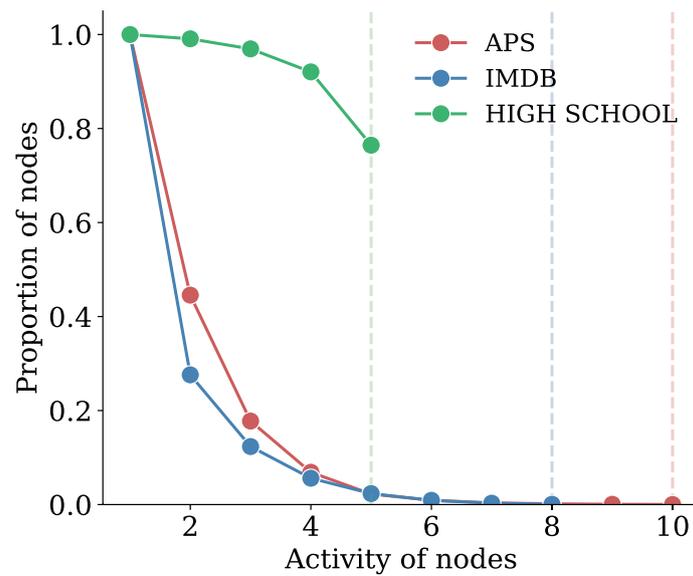

Fig. 2 Proportion of nodes active in at least *x* layers across three different datasets. Colored dashed lines indicate the number of layers in each respective dataset

Detailed statistics about datasets are provided in Table 1.

Node properties

We begin by investigating multiplex properties at the node level. The first basic measure we consider is *node activity* (Nicosia and Latora 2015). A node *i* is *active* at layer α if *i* participates in at least one interaction at layer α. Figure 2 shows statistics on nodes' simultaneous activity across multiple layers. Specifically, the *y*-axis plots the proportion of nodes (from the total node count) active in at least *x* layers. By definition, these curves exhibit a decreasing trend, with variations in the negative slopes reflecting the datasets'

diversity. While it is uncommon for scientists and actors to be active across more than 2 or 3 layers, students tend to be active in all layers. In fact, the inactivity of a student in a specific layer implies their absence from school on that day.

So far, we have grouped all interactions together, regardless of their order. To obtain more detailed insights about higher-order interactions, we can examine node activity for each specific interaction order d. To this scope, we introduce a list $\mathbf{A}$ of *node activity matrices*, one for each node i:

$$\mathbf{A}_i = \{a_{\alpha d}\} = \begin{cases} 1 & \text{if } i \text{ is active at order } d \text{ in } \alpha \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, one can define activity from a layer perspective and consider a list $\mathbf{B}$ of *layer activity matrices*, one for each layer α:

$$\mathbf{B}_\alpha = \{b_{id}\} = \begin{cases} 1 & \text{if } i \text{ is active at order } d \text{ in } \alpha \\ 0 & \text{otherwise.} \end{cases}$$

It can be useful to aggregate information about nodes and define an *aggregated layer activity matrix L* as:

$$L = \{l_{\alpha d}\} = |\{\text{nodes active at order } d \text{ in } \alpha\}|$$

In Fig. 3a, each dataset is represented as a graph where vertices are the layers of the multiplex hypergraph and links measure the similarity in activity patterns of two layers α and β, quantified as the cosine similarity of their node activity matrices $\mathbf{B}_\alpha$ and $\mathbf{B}_\beta$. The thicker the link, the higher the similarity. This figure emphasizes layers that not only share common active nodes, but also exhibit similar patterns of participation across different hyperedge orders. Particularly, a consistent higher-order similarity is observed across school days, reflecting recurring interaction patterns throughout the week. Other
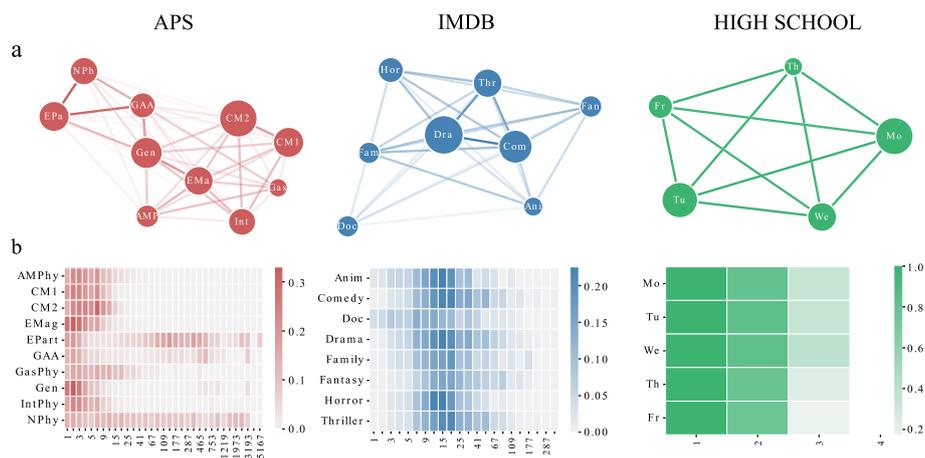

Fig. 3 **a** Each dataset is a graph in which vertices represent the layers of the multiplex hypergraphs and the thickness of an edge (α, β) quantifies the pairwise cosine similarity of layer activity matrices $\mathbf{B}_\alpha, \mathbf{B}_\beta$ associated with layers α and β. Vertex size is proportional to the number of nodes active in that layer. **b** Matrix *L* associated with each dataset. Rows are normalized by the number of nodes active in each layer. Interaction orders are binned exponentially

datasets show more heterogeneous behaviour, with documentary casts differing significantly from other layers, while drama and comedy casts exhibit similar patterns.

Figure 3b shows the aggregated layer activity matrices L for the three datasets. To account for variations in layer size, we normalize each row by the total number of active nodes in the respective layer. Distinct collaboration patterns emerge across the subfields of physics and movie genres. For instance, scientists in General or Electromagnetic Physics usually contribute to papers with a smaller number of co-authors, whereas co-author groups in fields like Elementary Particles and Nuclear Physics exhibit more variation in size. In movie collaborations, actor activity is concentrated in medium-size groups, typically between ten and twenty members. However, documentaries often feature smaller casts, while family and comedy movies tend to have larger ones. The figure once again highlights how students at school maintain a consistent group size in their interactions throughout the week.

Similar to node activity, node degree (defined as the number of interactions in which a node participates) is another property that can be used to measure the activity across the different layers and interaction orders. We define a list **K** of *node degree matrices*, one for each node i:

$$\mathbf{K}_i = \begin{matrix} \{k_{\alpha d}\} = |\{\text{hyperedges of order } d \\ \text{involving } i \text{ at } \alpha\}| \end{matrix}$$

We use $k_{i\alpha}$ to denote the *total number of interactions* involving i in layer α, irrespective of their order:

$$k_{i\alpha} = \sum_{d=1}^{D} \mathbf{K}_{i\alpha d},$$

where D is the order of the largest interaction in the dataset.

In Fig. 4a, we analyze the correlation of node degree across layers, exploring the extent to which a node with a high or low degree in one layer similarly exhibits a high or low degree in another layer. The correlation matrix for physics collaborations uncovers a hierarchical structure, with strong correlations among specific subfields sharing commonalities and notable scientists, such as in Nuclear and Elementary Particles Physics. In contrast, the degree correlations among actors are generally weak, though certain genres, like thriller and horror, show similarities. A significant correlation in node degrees across consecutive days in HIGH SCHOOL suggests stable and structured daily interaction patterns, implying that individuals with numerous interactions on one day tend to maintain similar levels of interactions on subsequent days, and vice versa.

We now define the *overlapping degree* o_i for a node i as the total number of interactions involving i, irrespective of both layers or orders:

$$o_i = \sum_{\alpha=1}^{M} k_{i\alpha}$$

It can be interesting to measure (i) how the overlapping degree of a node i is spread across the layers, i.e., if the degree is concentrated in certain layers or if it is uniformly distributed; (ii) how interactions involving node i are spread across orders. We measure
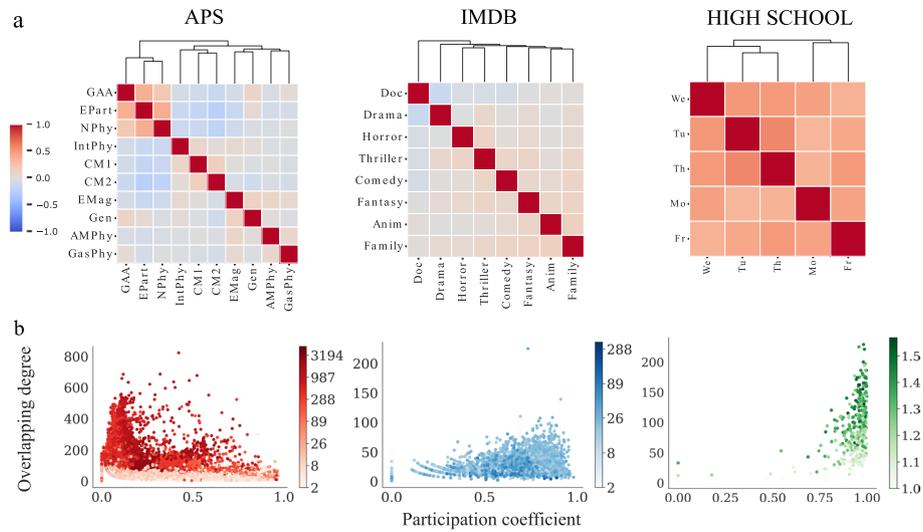
Fig. 4 **a** The heatmap shows the pairwise correlation between the degrees of nodes across different layers. The color scale indicates the strength of the correlation, with blue representing low correlation and red representing high correlation. **b** A system unit i is represented as a point on a Cartesian plane, with the overlapping degree o_i on the y-axis, the participation coefficient P_i on the x-axis, and the average order of the interactions in which the unit is involved indicated by color intensity

(i) by defining the *participation coefficient* P_i of a node i of the degree with respect to the layers:

$$P_i = \frac{M}{M-1}\left[1 - \sum_{\alpha=1}^{M}\left(\frac{k_{i\alpha}}{o_i}\right)^2\right]$$

where $k_{i\alpha}$ is the degree of node i at layer α, o_i is the overlapping degree of node i and M is the total number of layers. We measure (ii) by considering the average order of the interactions node i participates in.

In Fig. 4b, we represent each unit i of the different systems on a Cartesian plane, characterizing them across three distinct dimensions: their overlapping degree o_i (on the y-axis), their participation coefficient P_i (on the x-axis), and the average order of the interactions in which they are involved (indicated by color intensity). In general, such three dimensions provide different information about connectivity patterns and are only weakly correlated, or even uncorrelated. In APS, scientists are spread across the plane in terms of degrees and average interaction order, displaying an overall tendency towards specialization in a selected number of physics subfields, yet the behavior remains heterogeneous. IMDB displays isolated outliers with a very high degree, low dispersion around the average interaction order, and a general tendency towards uniform participation across multiple genres. HIGH SCHOOL shows students covering the entire spectrum of node degrees and average group orders, with an expected tendency to interact uniformly across school days.

Hyperedge properties

We now turn our attention to the properties of the hyperedges encoding interactions in the same three real-world systems.

We begin by considering the simplest measure for characterizing higher-order interactions, namely, the order of the groups. In Fig. 5, we plot the hyperedge order distribution disaggregated by layers. APS and IMDB reveal heterogeneity across layers, suggesting that different physics subfields and movie genres exhibit distinct patterns of collaboration in terms of the number of people involved in a paper or a movie cast. For example, genres such as documentaries and animated movies typically feature fewer actors compared to other genres. Conversely, papers in Elementary Particles and Nuclear Physics often include a larger number of authors compared to those in other areas of physics. The distributions in HIGH SCHOOL are stable across layers, indicating that patterns of face-to-face interactions tend not to change over the days, with a general preference for smaller groups over larger ones.

Another property frequently studied in the context of multiplex networks is edge overlap, which measures the extent to which interactions among the same nodes tend to repeat across multiple layers. We define *hyperedge overlap* as the maximum number of layers in which an interaction repeats exactly. In Fig. 6, we present the distribution of hyperedge overlap, including information about the order of the interactions. As expected, APS displays a high degree of hyperedge overlap, indicating that the same set of scientific authors consistently interact across multiple areas of physics. Conversely, for actors, hyperedge overlap decays very rapidly. Small interactions typically exhibit a higher degree of overlap than large interactions. Historically, edge overlap in multiplex networks with higher-order interactions has often been investigated by projecting hyperedges at different layers into cliques, frequently resulting in extremely high values of edge overlap. Our analysis suggests that patterns of hyperedge overlap are more complex and that projections of hyperedges can account for the high amount of overlap previously observed (Battiston et al. 2014).

Finally, we assign a score P_e to each hyperedge e, defined in terms of the participation coefficient of the nodes involved in the interaction:
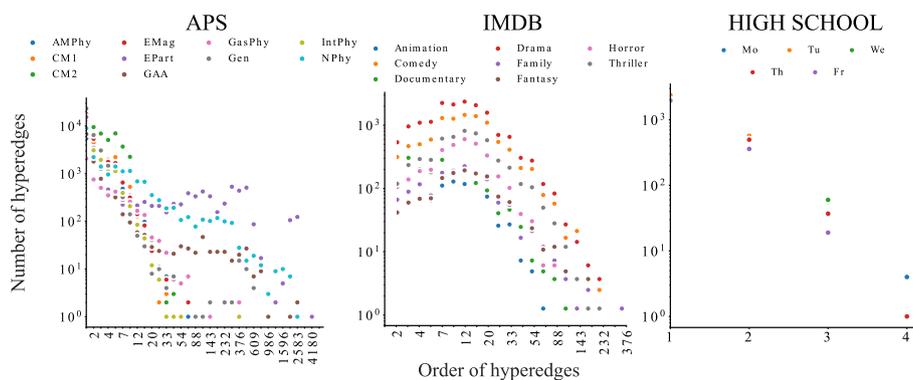

Fig. 5 Distribution of hyperedge orders disaggregated by layers in each dataset. Colors distinguish between different layers, with interaction orders binned exponentially
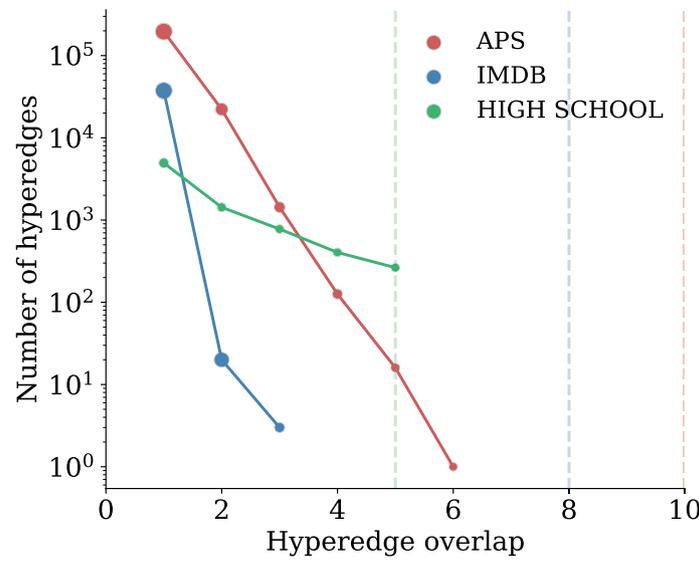
Fig. 6 Number of hyperedges as a function of their overlap, i.e., the maximal number of layers in which an interaction repeats. Markers are scaled proportionally to the average order of hyperedges. Colored dashed lines indicate the corresponding number of layers in each dataset
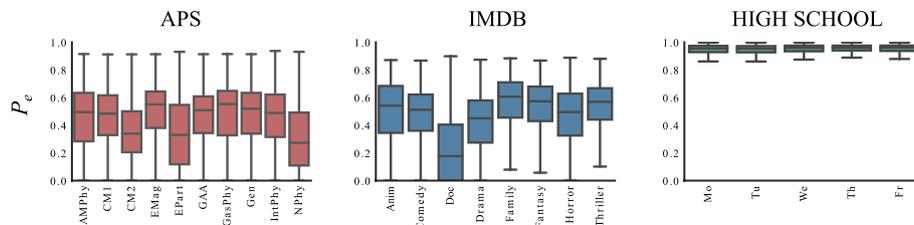

Fig. 7 Boxplots showing the distribution of hyperedge participation coefficient P_e across layers in each dataset

$$P_e = \frac{1}{|e|} \sum_{i \in e} P_i$$

where $|e|$ represents the number of nodes participating in hyperedge e and P_i is the participation coefficient of node i, as defined in the previous section.

This measure captures the tendency of hyperedges to connect nodes that either specialize in a few layers or act as generalists across multiple layers. In Fig. 7, we use boxplots to show distributions of P for hyperedges in various layers. It is noteworthy that layers can display heterogeneous mean values for the participation coefficient of their hyperedges. For example, casts in documentaries and co-authors in Nuclear and Elementary Particles Physics tend to include specialists. On the other hand, family and thriller movies are more likely to feature generalist actors. In HIGH SCHOOL, layers exhibit a consistent maximum mean value for the participation coefficient of hyperedges, attributed to students' regular attendance at school each day.

Mesoscale properties

We finally shift our focus towards mesoscale structures, examining the emergence of communities and core-periphery structures within different layers of real-world hypergraphs.

Communities are groups of nodes that display a higher degree of connectivity among themselves than with the rest of the nodes in the system. In hypergraphs, a *community* is defined as a subset of nodes that tend to form cohesive units by participating in common hyperedges. When analyzing multiplex systems, it is typical to examine the similarities in community structures observed across various layers. In this direction, we employ a method for hard clustering, applied independently to each layer, which is an extension of the well-established Infomap algorithm to the case of hypergraphs (Eriksson et al. 2021). In general, Infomap minimizes the map equation, which quantifies the description length required to represent the random walker's movements on the network (Rosvall et al. 2009). This optimization effectively partitions the network into communities that best capture the inherent modular structure. The method is publicly available (Eriksson et al. 2021) and we have used default parameters.

To assess the similarity and consistency of community structure across different hypergraph layers, we use Normalized Mutual Information (NMI), taking into account the set of nodes active in both layers. NMI is defined as:

$$\text{NMI}(C_1, C_2) = \frac{I(C_1, C_2)}{\sqrt{H(C_1)H(C_2)}}$$

where $I(C_1, C_2)$ denotes the mutual information between partitions C_1 and C_2. Mutual information quantifies the amount of information shared between the two partitions, i.e., measures how much knowing the community structure in one partition informs about the structure in the other. $H(C_1)$ and $H(C_2)$ represent the entropies of the partitions C_1 and C_2, respectively. By normalizing the mutual information $I(C_1, C_2)$ with the geometric mean of these entropies, NMI adjusts for the variability in partition sizes and the number of communities, allowing for a fair comparison of community structures across different partitions. NMI ranges from 0 (indicating no mutual information) to 1 (indicating perfect agreement).

In Fig. 8a, we present the outcomes of this analysis through heatmaps, highlighting the strength of community structure similarities across layers. For example, community structures in animation and fantasy movie casts are closely related, as are those in comedy and family movie genres, whereas documentary casts show a completely uncorrelated structure. The community structure within HIGH SCHOOL interactions remains consistent across days. Physics collaborations reveal a significant degree of similarity across fields, though some layers exhibit more pronounced similarities than others. Overall, communities tend to be preserved within physics subfields and school days, while movie genres often demonstrate predominantly uncorrelated communities.

We then direct our attention to core-periphery structures. Core-periphery structures delineate the existence of a group of central and tightly connected nodes, the core, surrounded by less densely connected peripheral nodes, forming a distinctive organizational pattern often crucial for system functionality (Borgatti and Everett 2000).
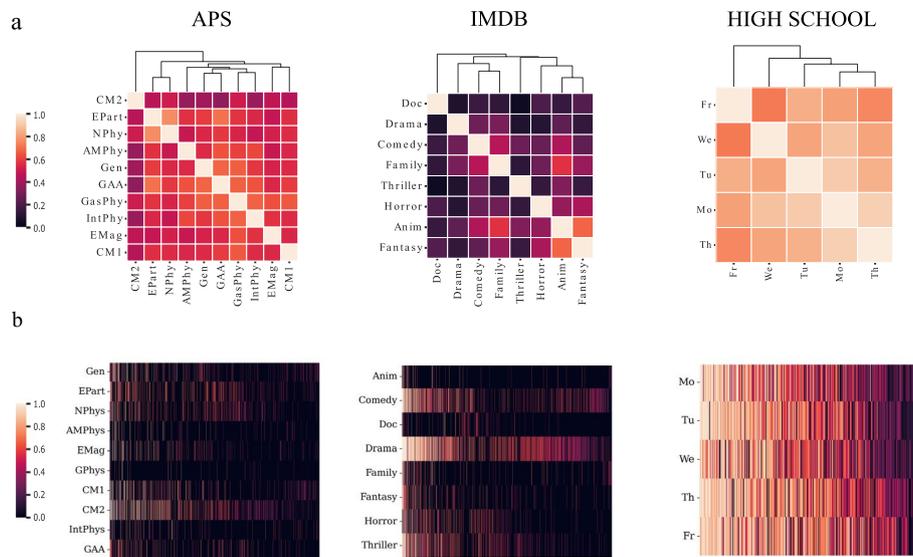
Fig. 8 a Heatmaps illustrating the similarity of community structures across layers, measured by Normalized Mutual Information (NMI), for the three datasets. In these heatmaps, high NMI values are represented by light colors, while low NMI values are represented by dark colors. **b** Core-periphery score c_i across layers for each node in the datasets, visualized as heatmaps. Rows are layers and columns represent nodes. Lighter colors are higher coreness values. Nodes are consistently sorted across layers based on their coreness in the aggregated hypergraphs, i.e., the hypergraph obtained by collapsing all layers into a single layer

We detect core-periphery structure for each layer independently using a method tailored for hypergraphs (Tudisco and Higham 2023). Such a method ranks nodes assigning to each node i a value c_i, where c_i is a real number within the range of 0 to 1. This value delineates the extent to which a node participates in the core (value closer to 1) or the periphery (value closer to 0) structure of the system. Vector **c** is selected for each layer α independently. Following the work by Tudisco and Higham (Tudisco and Higham 2023), for each layer α we select the vector **c** that optimizes the following function:

$$\max_{\mathbf{c}} \sum_{e \in E_\alpha} \frac{1}{|e|} \cdot \left(\sum_{i \in e} c_i \right) \quad \text{subject to} \quad \|\mathbf{c}\|_2 = 1 \text{ and } c_i \geq 0 \text{ for all } i$$

This continuous scale allows for a nuanced characterization of each node's role within the core-periphery framework. Additionally, by comparing the coreness score c_i for each node i across the different layers of a multiplex hypergraph, we can analyze the variation or consistency of a node centrality across the layers. This method is publicly available (Tudisco and Higham 2023). We use the implementation from Hypergraphx (Lotito 2023).

To provide a visually appealing way of highlighting correlations of core-periphery structures and node behaviour across layers, Fig. 8b shows heatmaps in which rows are layers and columns are nodes, and each entry is coloured depending on node coreness. To compare the coreness value of single nodes across layers and visualize to which extent it keeps its core value, we maintain a consistent sorting of the nodes on the x-axis. For each dataset, nodes are sorted according to their core values in the aggregated hypergraphs (i.e., the hypergraph obtained by dropping information about layers

and collapsing every hyperedge to a single layer). We observe that coreness values of nodes are maintained across layers exhibiting patterns similar to those seen in community structure correlations.

Conclusions

Networks have established themselves as a fundamental tool in a variety of disciplines to encode and study systems of interacting units. With the idea of capturing richer information about interactions, novel and more comprehensive network models have emerged: (i) multiplex networks, describing links of different types, and (ii) hypergraphs, encoding non-dyadic ties. Bridging these two notions, in this work we have introduced a general set of measures to characterize the structure of multiplex hypergraphs at multiple scales. We introduced a description of nodes in terms of higher-order activity patterns and degrees, to quantify the extent and magnitude of node participation in interactions of different orders across layers. Nodes have been also characterized in terms of how their degree is correlated and spread in the different layers, and by their preferred order of group interactions. For hyperedges, we have studied their order distribution disaggregated by layers, highlighting different patterns of group interactions depending on the hyperedge type. We have quantified the extent to which hyperedge tends to repeat exactly in multiple layers and we have analyzed the layer-dependant property of hyperedges of linking nodes with low or high participation coefficients. Finally, we have analyzed hypergraphs at their mesoscale, quantifying similarities of communities and core-periphery participation across layers. We have validated our proposed measures on three datasets from different domains, describing collaboration patterns across physics subfields, movie genres and daily interactions among students.

In summary, we believe that our measures can be useful in describing the structure of various empirical datasets characterized by both multiplex and higher-order interactions. We also hope that this initial characterization of multiplex hypergraphs will spark interest from a methodological perspective, such as proposing frameworks for extracting multiplex communities in hypergraphs. Further characterizations could be enhanced by considering the complex patterns of temporality in hyperedges, a common feature of higher-order systems (Cencetti et al. 2021; Ceria and Wang 2023; Gallo et al. 2023; Iacopini et al. 2023; Di Gaetano et al. 2024; Arregui-García et al. 2024; Mancastroppa et al. 2024; Chodrow 2020).

Supplementary Information

The online version contains supplementary material available at https://doi.org/10.1007/s41109-024-00665-9.

> Supplementary file 1.

Data Availability
Data is publicly available as part of HGX data repository (Lotito 2023).

Code Availability
Multiplex measures for higher-order networks, as well as basic functions for handling multiplex hypergraphs, are available as part of Hypergraphx (HGX) (Lotito 2023).

Declarations

References

https://journals.aps.org/datasets (2021)

Aleta A, Meloni S, Moreno Y (2017) A multilayer perspective for the analysis of urban transportation systems. Sci Rep 7:44359

Alvarez-Rodriguez U et al (2021) Evolutionary dynamics of higher-order interactions in social networks. Nat Hum Behav 5:586–595

Arregui-García B, Longa A, Lotito QF, Meloni S, Cencetti G (2024) Patterns in temporal networks with higher-order ego-centric structures. arXiv:2402.03866

Barigozzi M, Fagiolo G, Garlaschelli D (2010) Multinetwork of international trade: a commodity-specific analysis. Phys Rev E-Stat Nonlinear Soft Matter Phys 81:046104

Battiston F et al (2020) Networks beyond pairwise interactions: structure and dynamics. Phys Rep 874:1–92

Battiston F et al (2021) The physics of higher-order interactions in complex systems. Nat Phys 17:1093–1098

Battiston F, Iacovacci J, Nicosia V, Bianconi G, Latora V (2016) Emergence of multiplex communities in collaboration networks. PLoS ONE 11:e0147451

Battiston F, Nicosia V, Latora V (2014) Structural measures for multiplex networks. Phys Rev E 89:032804

Battiston F, Petri G (2022) Higher-order systems. Springer

Benson AR (2019) Three hypergraph eigenvector centralities. SIAM J Math Data Sci 1:293–312

Benson AR, Abebe R, Schaub MT, Jadbabaie A, Kleinberg J (2018) Simplicial closure and higher-order link prediction. Proc Natl Acad Sci 115:E11221–E11230

Berge C (1973) Graphs and hypergraphs. North-Holland Pub. Co

Boccaletti S et al (2014) The structure and dynamics of multilayer networks. Phys Rep 544:1–122

Boccaletti S, Latora V, Moreno Y, Chavez M, Hwang D-U (2006) Complex networks: structure and dynamics. Phys Rep 424:175–308

Borgatti SP, Everett MG (2000) Models of core/periphery structures. Soc Netw 21:375–395

Carletti T, Battiston F, Cencetti G, Fanelli D (2020) Random walks on hypergraphs. Phys Rev E 101:022308

Carletti T, Fanelli D, Lambiotte R (2021) Random walks and community detection in hypergraphs. J Phys Complex 2:015011

Cencetti G, Battiston F, Lepri B, Karsai M (2021) Temporal properties of higher-order interactions in social networks. Sci Rep 11:1–10

Ceria A, Wang H (2023) Temporal-topological properties of higher-order evolving networks. Sci Rep 13:5885

Chodrow PS (2020) Configuration models of random hypergraphs. J Complex Netw 8:cnaa018

Chodrow P, Eikmeier N, Haddock J (2023) Nonbacktracking spectral clustering of nonuniform hypergraphs. SIAM J Math Data Sci 5:251–279

Chodrow PS, Veldt N, Benson AR (2021) Generative hypergraph clustering: from blockmodels to modularity. Sci Adv 7:eabh1303

Chowdhary S, Kumar A, Cencetti G, Iacopini I, Battiston F (2021) Simplicial contagion in temporal higher-order networks. J Phys Complex 2:035019

Cimini G et al (2019) The statistical physics of real-world networks. Nat Rev Phys 1:58–71

Citraro S et al (2023) Hypergraph models of the mental lexicon capture greater information than pairwise networks for predicting language learning. New Ideas Psychol 71:101034

Contisciani M, Battiston F, De Bacco C (2022) Inference of hyperedges and overlapping communities in hypergraphs. Nat Commun 13:1–10

De Domenico M et al (2013) Mathematical formulation of multilayer networks. Phys Rev X 3:041022

De Domenico M (2017) Multilayer modeling and analysis of human brain networks. Giga Sci 6:gix004

Eriksson A, Edler D, Rojas A, de Domenico M, Rosvall M (2021) How choosing random-walk model and network representation matters for flow-based community detection in hypergraphs. Commun Phys 4:1–12

Di Gaetano L, Battiston F, Starnini M (2024) Percolation and topological properties of temporal higher-order networks. Phys Rev Lett 132:037401

Gallo L, Lacasa L, Latora V, Battiston F (2023) Higher-order correlations reveal complex memory in temporal hypergraphs. arXiv:2303.09316

Gambuzza LV et al (2021) Stability of synchronization in simplicial complexes. Nat Commun 12:1–13

Ghoshal G, Zlatić V, Caldarelli G (2009) Random hypergraphs and their applications. Phys Rev E 79:066118

Grilli J, Barabás G, Michalska-Smith MJ, Allesina S (2017) Higher-order interactions stabilize dynamics in competitive network models. Nature 548:210–213

Iacopini I, Petri G, Barrat A, Latora V (2019) Simplicial models of social contagion. Nat Commun 10:1–9

Iacopini I, Karsai M, Barrat A (2023) The temporal dynamics of group interactions in higher-order social networks. arXiv:2306.09967

Kivelä M et al (2014) Multilayer networks. J Complex Netw 2:203–271

Lee G, Ko J, Shin K (2020) Hypergraph motifs: concepts, algorithms, and discoveries. Proc VLDB Endow 13:2256–2269

Lotito QF et al (2023) Hypergraphx: a library for higher-order network analysis. J Complex Netw 11:cnad019

Lotito QF, Musciotto F, Montresor A, Battiston F (2022) Higher-order motif analysis in hypergraphs. Commun Phys 5:79

Lotito QF, Musciotto F, Montresor A, Battiston F (2024) Hyperlink communities in higher-order networks. J Complex Netw 12:cnae013

Lotito QF, Musciotto F, Battiston F, Montresor A (2023) Exact and sampling methods for mining higher-order motifs in large hypergraphs. Computing, pp 1–20

Lucas M, Cencetti G, Battiston F (2020) Multiorder Laplacian for synchronization in higher-order networks. Phys Rev Res 2:033410

Mancastroppa M, Iacopini I, Petri G, Barrat A (2024) The structural evolution of temporal hypergraphs through the lens of hyper-cores. arXiv:2402.06485

Mastrandrea R, Fournet J, Barrat A (2015) Contact patterns in a high school: a comparison between data collected using wearable sensors, contact diaries and friendship surveys. PLoS ONE 10:e0136497

Millán AP, Torres JJ, Bianconi G (2020) Explosive higher-order Kuramoto dynamics on simplicial complexes. Phys Rev Lett 124:218301

Musciotto F, Battiston F, Mantegna RN (2021) Detecting informative higher-order interactions in statistically validated hypergraphs. Commun Phys 4:1–9

Musciotto F, Battiston F, Mantegna RN (2022) Identifying maximal sets of significantly interacting nodes in higher-order networks. arXiv:2209.12712

Neuhäuser L, Mellor A, Lambiotte R (2020) Multibody interactions and nonlinear consensus dynamics on networked systems. Phys Rev E 101:032310

Nicosia V, Latora V (2015) Measuring and modeling correlations in multiplex networks. Phys Rev E 92:032805

Patania A, Petri G, Vaccarino F (2017) The shape of collaborations. EPJ Data Sci 6:1–16

Petri G et al (2014) Homological scaffolds of brain functional networks. J R Soc Interface 11:20140873

Rosvall M, Axelsson D, Bergstrom CT (2009) The map equation. Eur Phys J Special Top 178:13–23

Ruggeri N, Contisciani M, Battiston F, De Bacco C (2023) Community detection in large hypergraphs. Sci Adv 9:eadg9159

Schaub MT, Benson AR, Horn P, Lippner G, Jadbabaie A (2020) Random walks on simplicial complexes and the normalized hodge 1-laplacian. SIAM Rev 62:353–391

Skardal PS, Arenas A (2020) Higher order interactions in complex networks of phase oscillators promote abrupt synchronization switching. Commun Phys 3:1–6

Sun H, Bianconi G (2021) Higher-order percolation processes on multiplex hypergraphs. Phys Rev E 104:034306

Torres L, Blevins AS, Bassett D, Eliassi-Rad T (2021) The why, how, and when of representations for complex systems. SIAM Rev 63:435–485

Tudisco F, Higham DJ (2021) Node and edge nonlinear eigenvector centrality for hypergraphs. Commun Phys 4:1–10

Tudisco F, Higham DJ (2023) Core-periphery detection in hypergraphs. SIAM J Math Data Sci 5:1–21

Vasilyeva E et al (2021) Multilayer representation of collaboration networks with higher-order interactions. Sci Rep 11:5666

Vazquez A (2009) Finding hypergraph communities: a bayesian approach and variational solution. J Stat Mech Theory Exp 2009:P07006

Wolf MM, Klinvex AM, Dunlavy DM (2016) Advantages to modeling relational data using hypergraphs versus graphs. In: 2016 IEEE high performance extreme computing conference (HPEC), pp 1–7

Young J-G, Petri G, Peixoto TP (2021) Hypergraph reconstruction from network data. Commun Phys 4:1–11

Zhang Y, Lucas M, Battiston F (2023) Higher-order interactions shape collective dynamics differently in hypergraphs and simplicial complexes. Nat Commun 14:1605

Publisher's Note