Distributed Algorithms
Epidemic Dissemination

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Data dissemination

Problem

- Application-level broadcast/multicast is an important building block to create modern distributed applications
  - Video streaming
  - RSS feeds
- Want efficiency, robustness, speed when scaling
  - Flooding (reliable broadcast) is robust, but inefficient ($O(n^2)$)
  - Tree distribution is efficient ($O(n)$), but fragile
  - Gossip is both efficient ($O(n \log n)$) and robust, but has relative high latency
- Scalability:
  - Total number of messages sent by all nodes
  - Number of messages sent by each of the nodes
Data dissemination

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Introduction

Motivation

Data dissemination

- What if you want to deliver a message to millions of nodes?

- You have to trade robustness (the ability to withstand a large number of failures) versus scalability (the ability of delivering messages to millions of nodes).

- Evaluating scalability cannot be focused just on the total number of messages; even sending millions of messages from a single node is difficult.
Trade-offs

**Tree**

- **efficient**

**Flood**

- **fast**

**Gossip**

- **robust**

Figure: Courtesy: Robbert van Renesse
Introduction

Solution

- Reverting to a probabilistic approach based on epidemics / gossip
- Nodes infect each other through messages
- Total number of messages is less than $O(n^2)$
- No node is overloaded

But:

- No deterministic guarantee on reliability
- Only probabilistic ones
History of the Epidemic/Gossip Paradigm

- First defined by Alan Demers et al. (1987)
- Protocols based on epidemics predate Demers’ paper (e.g. NNTP)
- ’90s: gossip applied to the information dissemination problem
- ’00s: gossip beyond dissemination
- 2006: Workshop on the future of gossip (Leiden, the Netherlands)
Reality check (1987)

XEROX Clearinghouse Servers

- Database replicated at thousands of nodes
- Heterogeneous, unreliable network
- Independent updates to single elements of the DB are injected at multiple nodes
- Updates must propagate to all nodes or be supplanted by later updates of the same element
- Replicas become consistent after no more new updates
- Assuming a reasonable update rate, most information at any given replica is “current”
Reality check (today)

- Amazon uses a gossip protocol to quickly spread information throughout the S3 system
- Amazon’s Dynamo uses a gossip-based failure detection service
- The basic information exchange in BitTorrent is based on gossip
Basic assumptions

- System is asynchronous
  - No bounds on messages and process execution delays

- Processes fail by crashing
  - stop executing actions after the crash
  - We do not consider Byzantine failures

- Communication is subject to benign failures
  - Message omission
  - No message corruption, creation, duplication
Data model

- We consider a database that is replicated at a set of \( n \) nodes \( \Pi = \{p_1, \ldots, p_n\} \).

- The copy of the database at node \( p_i \) can be represented by a time-varying partial function:

\[
value : K \rightarrow V \cdot T
\]

where:
- \( K \) is the set of keys
- \( V \) is the set of values
- \( T \) is the set of timestamps

- The update operation is formalized as: \( value(k) \leftarrow (v, \text{now}()) \) where \( \text{now}() \) returns a globally unique timestamp.
Database Specification

The goal of the update distribution process is to drive the system towards consistency:

Definition (Eventual consistency)
If no new updates are injected after some time $t$, eventually all correct nodes will obtain the same copy of the database:

$$\forall p_i, p_j \in \Pi, \forall k \in K : value_i(k) = value_j(k)$$

where $value_i$ is the copy of the database at node $p_i$. 
Probabilistic Broadcast Specification

An alternative view

PB1 – Probabilistic validity
There is a given probability such that for any two correct nodes $p$ and $q$, every message PB-broadcast by $p$ is eventually PB-delivered by $q$

PB2 - Integrity
Every message is PB-delivered by a node at most once, and only if it was previously PB-broadcast
Best-Effort Broadcast vs Probabilistic Broadcast

Similarities:
- No agreement property

Differences:
- Probability in BEB is “hidden” in process life-cycle
- Probability in PB is explicit
Some simplifying assumptions

- Every node knows $\Pi$ (the communication network is a full graph)
- Communication costs between nodes are homogeneous
- We assume there is a single entry in the database
  - We simplify notation ($\text{value}(k) \rightarrow \text{value}$)
  - Managing multiple keys is more complex than adding $\text{foreach } k \in K$

To be relaxed later...
Models of epidemics

**Definition (Epidemiology)**

Epidemiology studies the spread of a disease or infection in terms of populations of infected/uninfected individuals and their rates of change.

**Why?**

- To understand if an epidemic will turn into a pandemic
- To adopt countermeasures to reduce the rate of infection
  - Inoculation
  - Isolation
SIR Model

Definition (SIR Model – Kermack and McKendrick, 1927)

An individual $p$ can be:

- **Susceptible**: if $p$ is not yet infected by the disease
- **Infective**: if $p$ is infected and capable to spread the disease
- **Removed**: if $p$ has been infected and has recovered from the disease
SIR Model

How does it work?

- Initially, a single individual is infective
- Individuals get in touch with each other, spreading the disease
- Susceptible individuals are turned into infective ones
- Eventually, infective individuals will become removed

SIR is not the only model...

Definition (SIRS Model)
The SIR model plus temporary immunity, so recovered nodes may become susceptible again.

Definition (SEIRS Model)
The SEIRS model takes into consideration the exposed or latent period of the disease.
From Epidemiology to Distributed Systems

The idea

- Disease spread quickly and robustly
- Our goal is to spread an update as fast and as reliable as possible
- Can we apply these ideas to distributed systems?

Definition (SIR Model for Database replication)

- **Susceptible**: if $p$ has not yet received an update
- **Infective**: if $p$ holds an update and is willing to share it
- **Removed**: if $p$ has the update but is no longer willing to share it

Note: Rumor spreading, or gossiping, is based on the same principles
Algorithm – Summary

- Best effort

- Anti-entropy (simple epidemics)
  - Push
  - Pull
  - Push-pull

- Rumor mongering (complex epidemics)
  - Push
  - Pull
  - Push-pull

- Probabilistic broadcast
### Best-effort

#### Best-effort (direct mail) algorithm

- Notify all other nodes of an update as soon as it occurs.
- When receiving an update, check if it is “news”

```
Direct mail protocol executed by process $p_i$:

**upon** $value \leftarrow (v, \text{now}())$ **do**
- **foreach** $p_j \in \Pi$ **do**
  - **send** $\langle \text{UPDATE}, value \rangle$ **to** $p_j$

**upon** receive $\langle \text{UPDATE}, (v, t) \rangle$ **do**
- **if** $value.time < t$ **then**
  - $value \leftarrow (v, t)$
```

Not randomized nor epidemic algorithm: just the simplest
Anti-entropy

Anti-entropy: Algorithm

- Every node regularly chooses another node at random and exchanges database contents, resolving differences.
- Nodes are either
  - susceptible – they know the update
  - infective – they know the update

Anti-entropy protocol executed by process $p_i$:

```
repeat every $\Delta$ time units
  PROCESS $p_j \leftarrow \text{random}(\Pi)$  % Select a random neighbor
  { exchange messages to resolve differences }
```
Anti-entropy: graphical representation

**Rounds**

During a *round* of length $\Delta$

- every node has the possibility of contacting one random node
- can be contacted by several nodes
Resolving differences – Summary

\[ p_i \xrightarrow{\text{PUSH, } v, t} p_j \]

\[ p_i \xrightarrow{\text{PULL, } p_i} p_j \]

\[ p_i \xrightarrow{\text{REPLY, } v, t} p_j \]

\[ p_i \xrightarrow{\text{PUSHPULL, } p_i, v, t} p_j \]

\[ p_i \xrightarrow{\text{REPLY, } v, t} p_j \]
Resolving differences – Push

Anti-entropy, Push protocol executed by process \( p_i \):

\[
\text{repeat every } \Delta \text{ time units}
\]

\[
\text{PROCESS } p_j \leftarrow \text{random}(\Pi) \quad \% \text{ Select a random neighbor}
\]

\[
\text{send } \langle \text{PUSH, value} \rangle \text{ to } p_j
\]

\[
\text{upon receive } \langle \text{PUSH, (v, t)} \rangle \text{ do}
\]

\[
\text{if } \text{value.time} < t \text{ then}
\]

\[
\text{value } \leftarrow (v, t)
\]
Resolving differences – Pull

Anti-entropy, Pull protocol executed by process $p_i$:

\[
\text{repeat every } \Delta \text{ time units} \\
\quad \text{PROCESS } p_j \leftarrow \text{random}(\Pi) \quad \% \text{ Select a random neighbor} \\
\quad \text{send } \langle \text{PULL}, p_i, \text{value}.time \rangle \text{ to } p_j \\
\text{upon receive } \langle \text{PULL}, p_j, t \rangle \text{ do} \\
\quad \text{if } \text{value}.time > t \text{ then} \\
\quad \quad \text{send } \langle \text{REPLY}, \text{value} \rangle \text{ to } p_j \\
\text{upon receive } \langle \text{REPLY}, (v, t) \rangle \text{ do} \\
\quad \text{if } \text{value}.time < t \text{ then} \\
\quad \quad \text{value } \leftarrow (v, t)
\]
# Resolving differences – Push-Pull

Anti-entropy, Push-Pull protocol executed by process $p_i$:

```
repeat every $\Delta$ time units
    PROCESS $p_j \leftarrow \text{random}(\Pi)$ \hspace{1cm} \% Select a random neighbor
    send $\langle \text{PUSHPULL}, p_i, \text{value} \rangle$ to $p_j$

upon receive $\langle \text{PUSHPULL}, p_j, (v, t) \rangle$ do
    if $\text{value}.\text{time} < t$ then
        $\text{value} \leftarrow (v, t)$
    else if $\text{value}.\text{time} > t$ then
        send $\langle \text{REPLY}, \text{value} \rangle$ to $p_j$

upon receive $\langle \text{REPLY}, (v, t) \rangle$ do
    if $\text{value}.\text{time} < t$ then
        $\text{value} \leftarrow (v, t)$
```

Analytical results

Definition (Compartmental model analysis)

We want to evaluate convergence of the protocol based on the size of the populations of susceptible and infected nodes (compartments)

- \( s_t \): the probability of a node being susceptible after \( t \) anti-entropy rounds
- \( i_t = 1 - s_t \): the probability of a node being infective after \( t \) anti-entropy rounds

Probability at round \( k + 1 \)

- Initial condition: \( s_0 = \frac{n-1}{n} \)
- Pull: \( E[s_{t+1}] = s_t^2 \)
- Push: \( E[s_{t+1}] = s_t(1 - \frac{1}{n})(1-s_t)^n \approx s_t e^{-(1-s_t)} \)
• Probability at round 0: 1 node is infective, all the others $n - 1$ are susceptible

• In case of pull, a node will remain susceptible at round $t + 1$ if it was susceptible at round $t$ (prob.: $s_t$) and it contacts a susceptible node (prob.: $s_t$).

• In case of push, a node will remain susceptible at round $t + 1$ if it was susceptible at round $t$ (prob.: $s_t$) and it is not selected randomly (prob: $1 - \frac{1}{n}$) by all nodes that are not susceptible, which are $(1 - s_t)n$. 

Definition (Compartmental model analysis)

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- $s_{i+1} = 1 - s_t$: the probability of a node being infective after $t$ anti-entropy rounds.

Probability at round $k + 1$

- Initial condition: $s_0 = \frac{1}{n}$.
- Pull: $E[s_{t+1}] = s_t^2$.
- Push: $E[s_{i+1}] = s_t(1 - \frac{1}{n})^{(1-s_t)n} \approx s_t e^{-(1-s_t)}$. 

Analytical results
Analytical results

Definition (Termination time)

Let $S_n$ be the first cycle where $s_t = 0$ in a population of $n$ individuals

- **Push**: Pittel (1987) shows that in probability,
  $$S_n = \log n + \ln n + O(1)$$

- **Push-pull**: Karp et al. (2000) shows that in probability,
  $$S_n = \log \log n$$

Relevant bibliography:


The proof is rather long and technical, but the intuitive explanation is rather simple.

In the initial cycles, most nodes are susceptible.

In this phase, the number of infected nodes will double in each cycle to a good approximation.

However, in the last cycles, where \( s_t \) is small, we can see that
\[
E[s_{t+1}] \approx s_t e^{-1}.
\]

This suggests that there is a first phase, lasting for approximately \( \log_2 n \) cycles

There is a last phase lasting for \( \ln N \) cycles.

The middle phase, between these two phases, can be shown to be very fast, lasting a constant number of cycles.
Analytical results

In summary:
All methods converge to 0, but pull and push-pull are much more rapid

Figure: \( n = 10000 \)
Comments

Benefits
- Simple epidemic eventually "infects" all the population
- It is extremely robust

Drawbacks
- Propagates updates much slower than direct mail (best effort)
- Requires examining contents of database even when most data agrees, so it cannot practically be used too often
- Normally used as support to best effort/rumor mongering, i.e. left running in the background
Working with multiple values

Examples of techniques to compare databases:

- Maintain checksum, compare databases if checksums unequal
- Maintain recent update lists for time $T$, exchange lists first
- Maintain inverted index of database by timestamp; exchange information in reverse timestamp order, incrementally re-compute checksums

Notes:

- Those ideas apply to databases
- Strongly application-dependent
- We will see how anti-entropy may be used beyond information dissemination
Rumor mongering in brief

- Nodes initially “susceptive”
- When a node receives a new update it becomes a “hot rumor” and the node “infective”
- A node that has a rumor periodically chooses randomly another node to spread the rumor
- Eventually, a node will “lose interest” in spreading the rumor and becomes “removed”
  - Spread too many times
  - Everybody knows the rumor
- A sender can hold (and transmit) a list of infective updates rather than just one
Rumor mongering: loss of interest

- **When**: Counter vs coin (random)
  - Coin (random): lose interest with probability $1/k$
  - Counter: lose interest after $k$ contacts

- **Why**: Feedback vs blind
  - Feedback: lose interest only if the recipient knows the rumor.
  - Blind: lose interest regardless of the recipient.
Analytical results

Question
How fast does the system converge to a state where all nodes are not infective? (inactive state)

Compartmental analysis again – Feedback, coin
Let $s$, $i$ and $r$ denote the fraction of susceptible, infective, and removed nodes, respectively. Then:

$$s + i + r = 1$$
$$\frac{ds}{dt} = -si$$
$$\frac{di}{dt} = +si - \frac{1}{k}(1-s)i$$

Solving the differential equations:

- $s = e^{-(k+1)(1-s)}$
- Increasing $k$ increases the probability that the nodes get the rumor
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Solving the differential equations:

- $s = e^{-(s+1)(1-s)}$
- Increasing $k$ increases the probability that the nodes get the rumor

- $si$ corresponds to the proportion of nodes that are susceptible and are contacted by infective nodes at any given round
- $\frac{ds}{dt} = -si$ and $\frac{di}{dt} = +si$ mean that those nodes will be not susceptible any more and will become infective
- $-(1/k)(1-s)i$ means that with probability $\frac{1}{k}$, an infective node $(i)$ will become removed if it contacts a non-susceptible node $(1-s)$.
Quality measures

**Definition (Residue)**
- The nodes that remain susceptible when the epidemic ends: value of \( s \) when \( i = 0 \).
- Residue must be as small as possible.

**Definition (Traffic)**
- The average number of database updates sent between nodes
- \( m = \text{total update traffic} / \# \text{ of nodes} \)

**Definition (Convergence)**
- \( t_{avg} \) : average time it takes for all nodes to get an update
- \( t_{last} \) : time it takes for the last node to get the update
# Simulation results

## Using feedback and counter

<table>
<thead>
<tr>
<th>Counter $k$</th>
<th>Residue $s$</th>
<th>Traffic $m$</th>
<th>Convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$t_{avg}$</td>
</tr>
<tr>
<td>1</td>
<td>0.176</td>
<td>1.74</td>
<td>11.0</td>
</tr>
<tr>
<td>2</td>
<td>0.037</td>
<td>3.30</td>
<td>12.1</td>
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<tr>
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<td>0.011</td>
<td>4.53</td>
<td>12.5</td>
</tr>
<tr>
<td>4</td>
<td>0.0036</td>
<td>5.64</td>
<td>12.7</td>
</tr>
<tr>
<td>5</td>
<td>0.0012</td>
<td>6.68</td>
<td>12.8</td>
</tr>
</tbody>
</table>

## Using blind and random

<table>
<thead>
<tr>
<th>Counter $k$</th>
<th>Residue $s$</th>
<th>Traffic $m$</th>
<th>Convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$t_{avg}$</td>
</tr>
<tr>
<td>1</td>
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<td>19</td>
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<td>2</td>
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<td>0.021</td>
<td>3.91</td>
<td>14.1</td>
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<tr>
<td>5</td>
<td>0.008</td>
<td>4.95</td>
<td>13.8</td>
</tr>
</tbody>
</table>
Push vs pull

- Push (what we have assumed so far)
  - If the database becomes quiescent, push stops trying to introduce updates

- Pull
  - If many independent updates, pull is more likely to find a source with a non-empty rumor list
  - But if the database is quiescent, several update requests are wasted
Rumor Mongering + Anti-Entropy

- Quick & Dirty Push Rumor Mongering
  - spreads updates fast with low traffic.
  - however, there is still a nonzero probability of nodes remaining susceptible after the epidemic

- Not-so-fast Push-Pull Anti-Entropy
  - can be run (infrequently) in the background to ensure all nodes eventually get the update with probability 1
Dealing with deletions

Deletion

- We cannot delete an entry just by removing it from a node - the absence of the entry is not propagated.
- If the entry has been updated recently, there may still be an update traversing the network!

Definition (Death certificate)

Replace the deleted item with a death certificate that has a timestamp and spreads like an ordinary update.
Dealing with deletions

Problem
We must, at some point, delete DCs or they may consume significant space

- **Strategy 1**: retain each DC until all nodes have received it
  - requires a protocol to determine which nodes have it and to handle node failures

- **Strategy 2**: hold DCs for some time (e.g. 30d) and discard them
  - pragmatic approach
  - we still have the “resurrection” problem
  - increasing the time requires more space
Observation:

- we can delete very old DCs but retain only a few “dormant” copies in some nodes
- if an obsolete update reaches a dormant DC, it is “awakened” and re-propagated

Analogy with epidemiology:

- the awakened DC is like an antibody triggered by an immune reaction
Epidemics vs probabilistic broadcast

**Anti-entropy:**
DB is a collection of message received

**Rumor mongering**
Updates $\equiv$ Message broadcast
What’s next?

Problems

- **Membership:**
  How do processes get to know each other, and how many do they need to know?

- **Network awareness:**
  How to make the connections between processes reflect the actual network topology such that the performance is acceptable?

- **Buffer management:**
  Which updates to drop when the storage buffer of a process is full?

Does not end here...

Epidemic/gossip approach has been successfully applied to other problems. We will provide a brief overview of them.
http://www.disi.unitn.it/~montreso/ds/papers/demers87.pdf

M. Jelasity. Gossip.