

Algoritmi e Strutture Dati

Alberi

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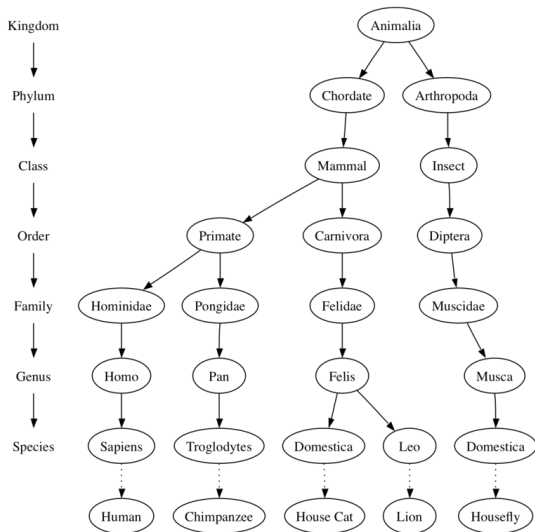
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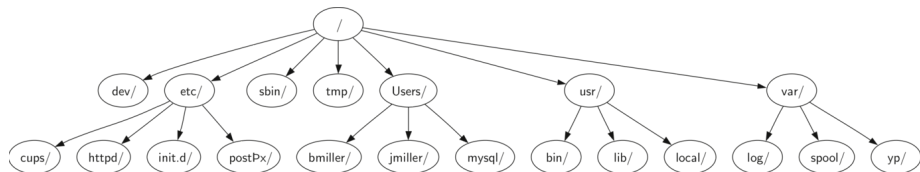
Sommario

- 1 Introduzione
 - Esempi
 - Definizioni
- 2 Alberi binari
 - Introduzione
 - Implementazione
 - Visite
- 3 Alberi generici
 - Visite
 - Implementazione

Esempio 1



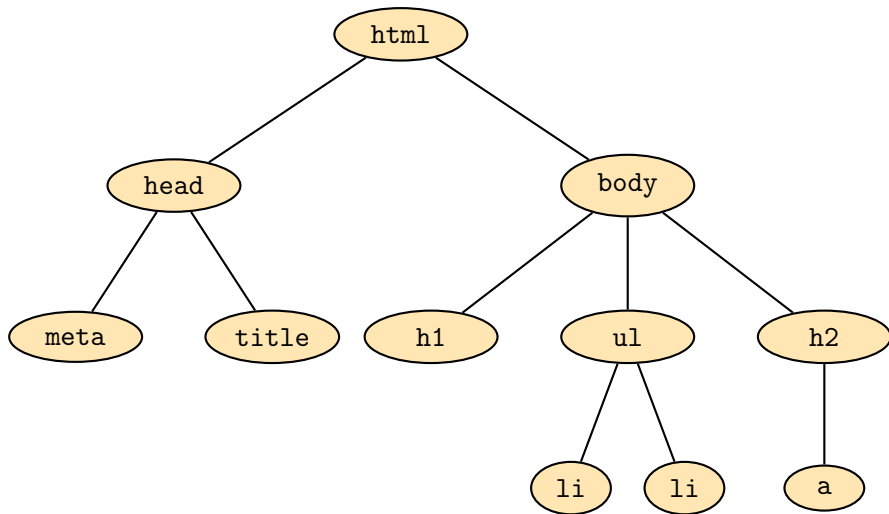
Esempio 2



Esempio 3

```
<html>
  <head>
    <meta http-equiv="Content-Type" content="text/html"/>
    <title>simple</title>
  </head>
  <body>
    <h1>A simple web page</h1>
    <ul>
      <li>List item one</li>
      <li>List item two</li>
    </ul>
    <h2>
      <a href="http://www.google.com">Google</a>
    </h2>
  </body>
</html>
```

Esempio 3



Albero radicato – Definizione 1

Albero radicato (Rooted tree)

Un albero consiste di un insieme di nodi e un insieme di archi orientati che connettono coppie di nodi, con le seguenti proprietà:

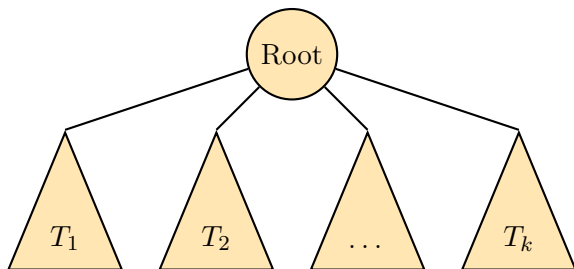
- Un nodo dell'albero è designato come nodo **radice**;
- Ogni nodo n , a parte la radice, ha esattamente un arco entrante;
- Esiste un cammino unico dalla radice ad ogni nodo;
- L'albero è connesso.

Albero radicato – Definizione 2 (Ricorsiva)

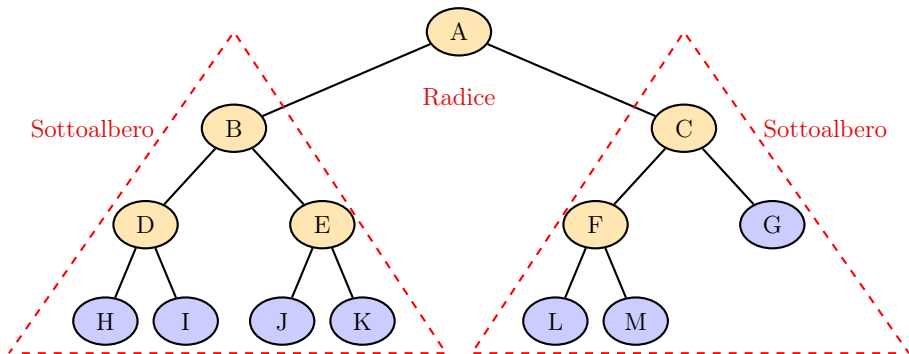
Albero radicato (Rooted tree)

Un albero è dato da:

- un insieme vuoto, oppure
- un nodo **radice** e zero o più **sottoalberi**, ognuno dei quali è un albero; la radice è connessa alla radice di ogni sottoalbero con un arco orientato.

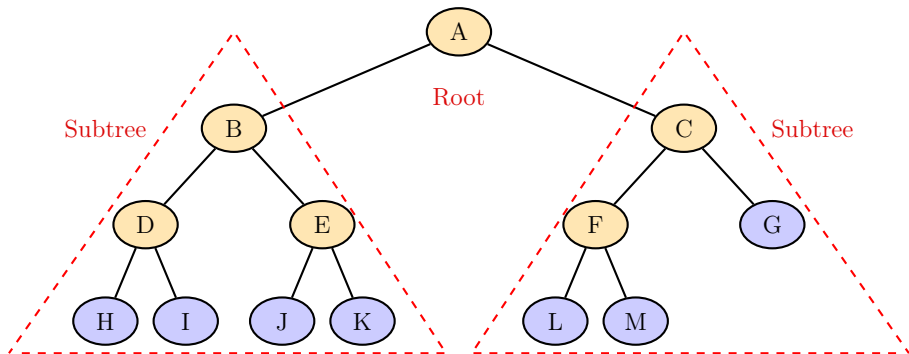


Terminologia



- A è la **radice**
- B, C sono radici dei sottoalberi
- D, E sono **fratelli**
- D, E sono **figli** di B
- B è il **padre** di D, E
- I nodi viola sono **foglie**
- Gli altri nodi sono **nodi interni**

Terminology (English)



- A is the tree **root**
- B, C are roots of their subtrees
- D, E are **siblings**
- D, E are **children** of B
- B is the **parent** of D, E
- Purple nodes are **leaves**
- The other nodes are **internal nodes**

Terminologia

Profondità nodi (Depth)

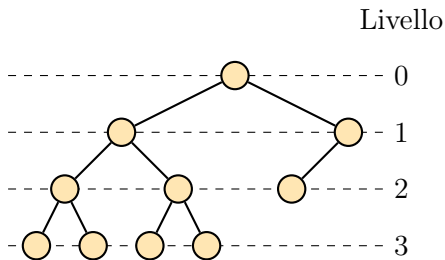
La lunghezza del cammino semplice dalla radice al nodo (misurato in numero di archi)

Livello (Level)

L'insieme di nodi alla stessa profondità

Altezza albero (Height)

La profondità massima della sue foglie



Altezza di questo albero = 3

Sommario

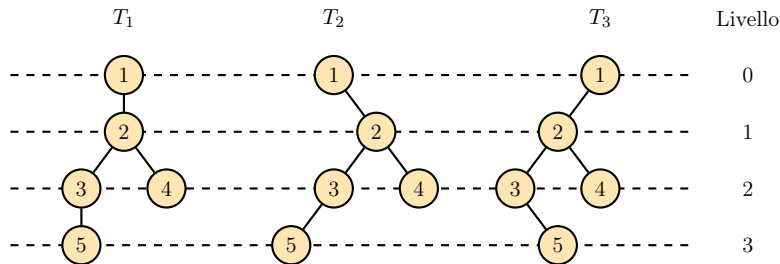
- 1 Introduzione
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- 3 Alberi generici
 - Visite
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Albero binario

Albero binario

Un **albero binario** è un albero radicato in cui ogni nodo ha al massimo due figli, identificati come figlio **sinistro** e figlio **destro**.

Nota: Due alberi T e U che hanno gli stessi nodi, gli stessi figli per ogni nodo e la stessa radice, sono distinti qualora un nodo u sia designato come figlio sinistro di v in T e come figlio destro di v in U .



Specifica (Albero binario)

TREE

% Costruisce un nuovo nodo, contenente v , senza figli o genitori

Tree(ITEM v)

% Legge il valore memorizzato nel nodo

ITEM read()

% Modifica il valore memorizzato nel nodo

write(ITEM v)

% Restituisce il padre, oppure **nil** se questo nodo è radice

TREE parent()

Specifica (Albero binario)

TREE

% Restituisce il figlio sinistro (destra) di questo nodo; restituisce **nil**
se assente

TREE left()

TREE right()

% Inserisce il sottoalbero radicato in *t* come figlio sinistro (destra)
di questo nodo

insertLeft(TREE t)

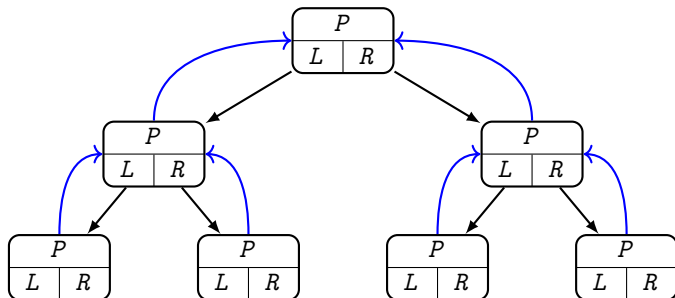
insertRight(TREE t)

% Distrugge (ricorsivamente) il figlio sinistro (destra) di questo
nodo

deleteLeft()

deleteRight()

Memorizzare un albero binario



Campi memorizzati nei nodi

- *parent*: reference al nodo padre
- *left*: reference al figlio sinistro
- *right*: reference al figlio destro

Implementazione

TREE

Tree(ITEM *v*)

```

TREE t = new TREE
t.parent = nil
t.left = t.right = nil
t.value = v
return t

```

insertLeft(TREE *T*)

```

if left == nil then
  T.parent = this
  left = T

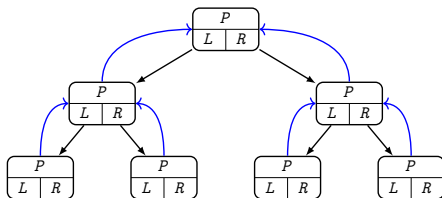
```

insertRight(TREE *T*)

```

if right == nil then
  T.parent = this
  right = T

```



Implementazione

TREE

deleteLeft()

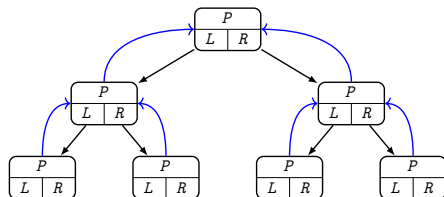
```

if left ≠ nil then
    left.deleteLeft()
    left.deleteRight()
    delete left
    left = nil
  
```

deleteRight()

```

if right ≠ nil then
    right.deleteLeft()
    right.deleteRight()
    delete right
    right = nil
  
```



Visite di alberi

Visita di un albero / ricerca

Una strategia per analizzare (visitare) tutti i nodi di un albero.

Visita in profondità Depth-First Search (DFS)

- Per visitare un albero, si visita ricorsivamente ognuno dei suoi **sottoalberi**
- Tre varianti: pre/in/post visita (**pre/in/post order**)
- Richiede uno **stack**

Visita in ampiezza Breadth First Search (BFS)

- Ogni **livello** dell'albero viene visitato, uno dopo l'altro
- Si parte dalla radice
- Richiede una **queue**

Depth-First Search

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
    % pre-order visit of  $t$ 
```

```
    print  $t$ 
```

```
    dfs( $t$ .left())
```

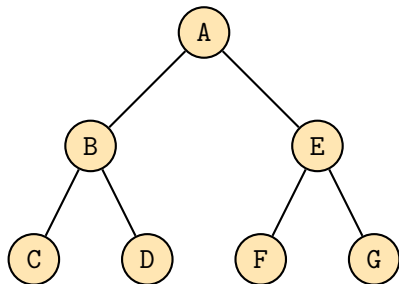
```
    % in-order visit of  $t$ 
```

```
    print  $t$ 
```

```
    dfs( $t$ .right())
```

```
    % post-order visit of  $t$ 
```

```
    print  $t$ 
```



Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
    % pre-order visit of  $t$ 
```

```
    print  $t$ 
```

```
    dfs( $t$ .left())
```

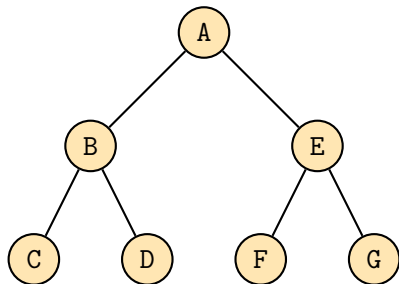
```
% in-order visit of  $t$ 
```

```
print  $t$ 
```

```
    dfs( $t$ .right())
```

```
% post-order visit of  $t$ 
```

```
print  $t$ 
```



Sequence: **A**

Stack: **A**

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

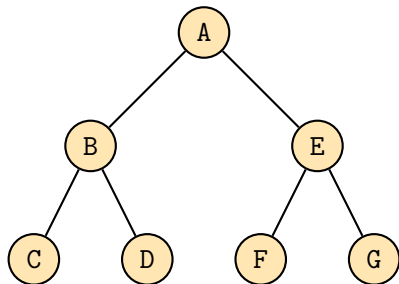
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: A B

Stack: A B

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

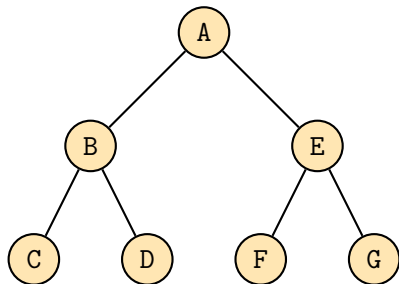
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: A B C

Stack: A B C

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t  $\neq$  nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

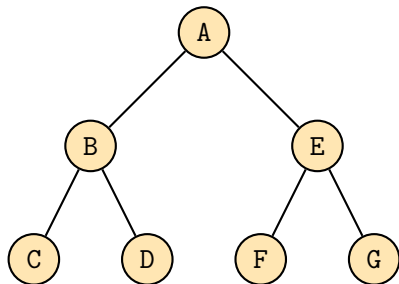
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: **A B C**

Stack: **A B**

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

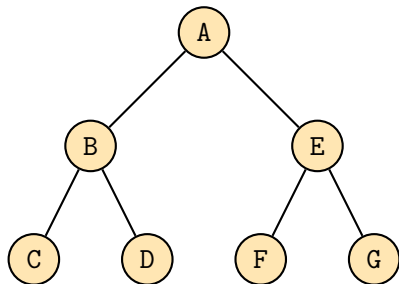
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: A B C D

Stack: A B D

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

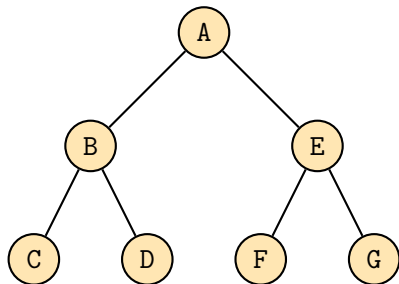
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: A B C D

Stack: A B

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

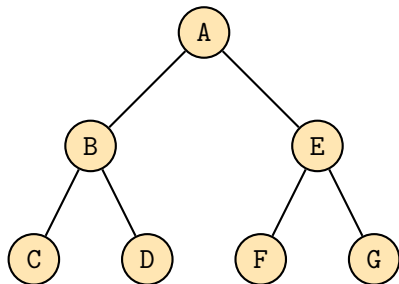
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: A B C D

Stack: A

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

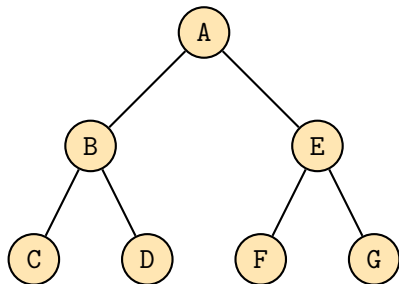
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: **A B C D E**

Stack: **A E**

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
  % pre-order visit of  $t$ 
```

```
  print  $t$ 
```

```
  dfs( $t$ .left())
```

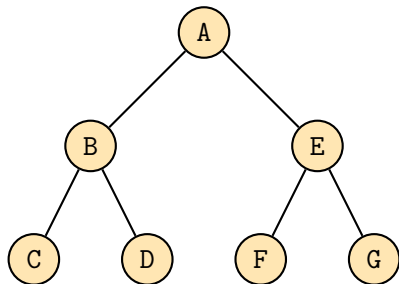
```
% in-order visit of  $t$ 
```

```
print  $t$ 
```

```
  dfs( $t$ .right())
```

```
% post-order visit of  $t$ 
```

```
print  $t$ 
```



Sequence: **A B C D E F**

Stack: **A E F**

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

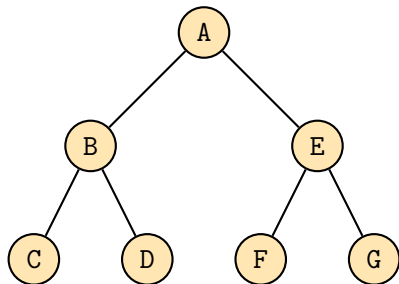
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: **A B C D E F**

Stack: **A E**

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

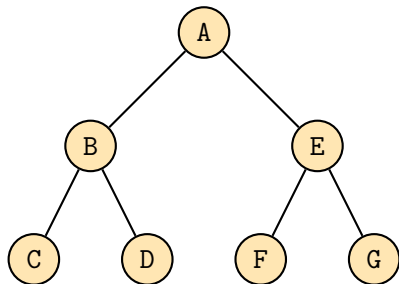
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: **A B C D E F G**

Stack: **A E G**

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

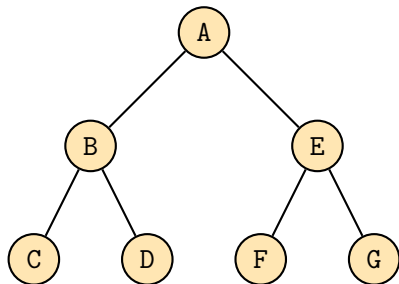
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: **A B C D E F G**

Stack: **A E**

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

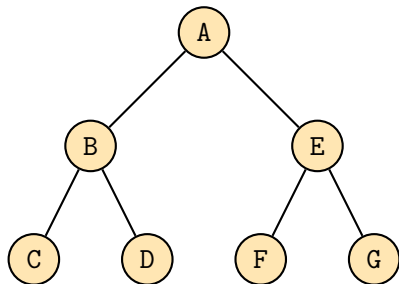
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: **A B C D E F G**

Stack: **A**

Depth-First Search - Pre-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

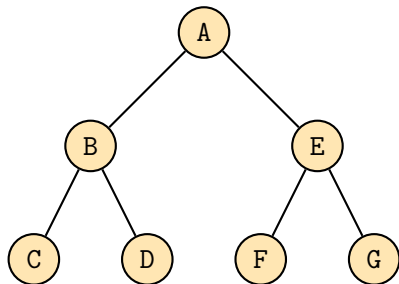
```
% in-order visit of t
```

```
print t
```

```
  dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: **A B C D E F G**

Stack:

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
% pre-order visit of t
```

```
print t
```

```
dfs(t.left())
```

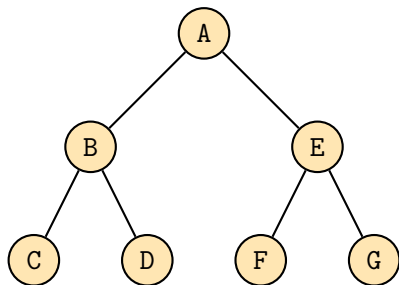
```
% in-order visit of t
```

```
print t
```

```
dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence:

Stack: **A**

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
% pre-order visit of t
```

```
print t
```

```
dfs(t.left())
```

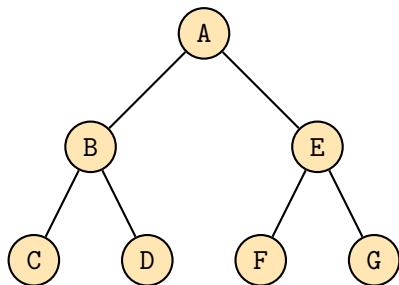
```
% in-order visit of t
```

```
print t
```

```
dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence:

Stack: **A B**

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

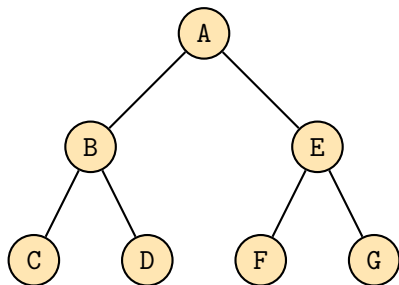
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C

Stack: A B C

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
% pre-order visit of t
```

```
print t
```

```
dfs(t.left())
```

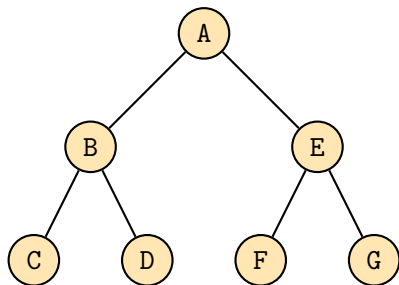
```
% in-order visit of t
```

```
print t
```

```
dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: C B

Stack: A B

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

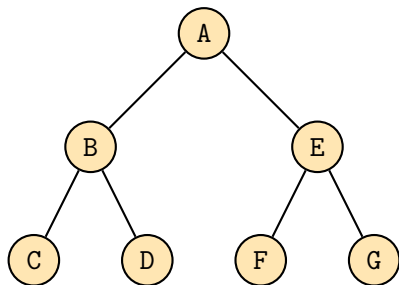
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C B D

Stack: A B D

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

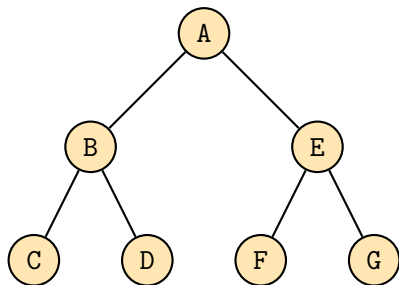
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C B D

Stack: A B

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

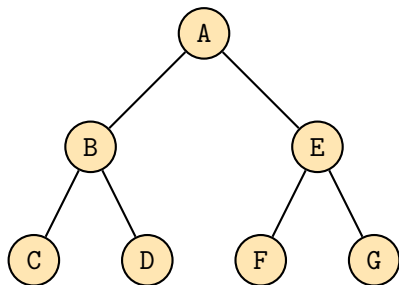
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C B D A

Stack: A

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

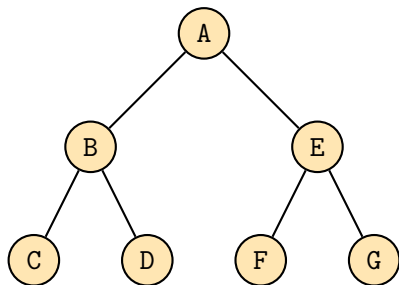
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C B D A

Stack: A E

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
% pre-order visit of t
```

```
print t
```

```
dfs(t.left())
```

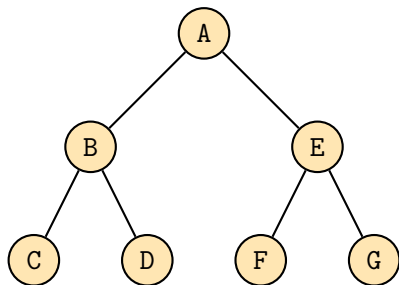
```
% in-order visit of t
```

```
print t
```

```
dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: **C B D A F**

Stack: **A E F**

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

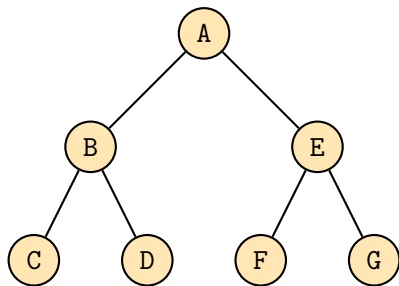
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C B D A F E

Stack: A E

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

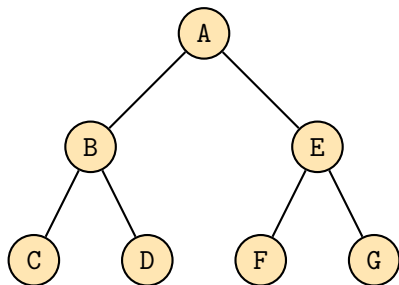
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C B D A F E G

Stack: A E G

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

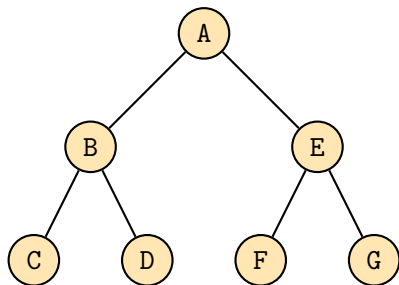
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C B D A F E G

Stack: A E

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
% pre-order visit of t
```

```
print t
```

```
dfs(t.left())
```

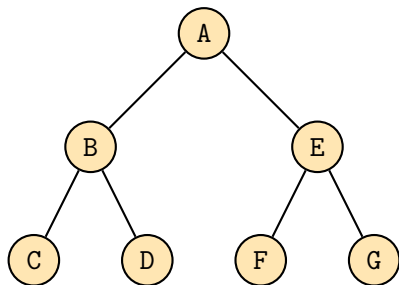
```
% in-order visit of t
```

```
print t
```

```
dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: **C B D A F E G**

Stack: **A**

Depth-First Search - In-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
% pre-order visit of t
```

```
print t
```

```
dfs(t.left())
```

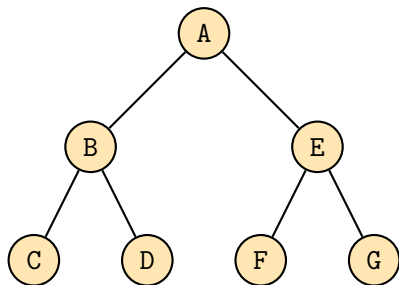
```
% in-order visit of t
```

```
print t
```

```
dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: **C B D A F E G**

Stack:

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

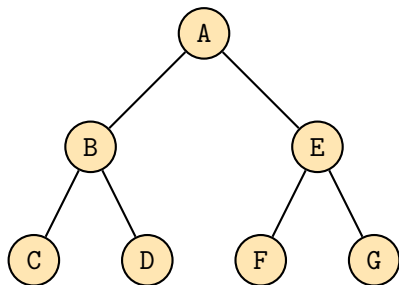
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence:

Stack: **A**

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

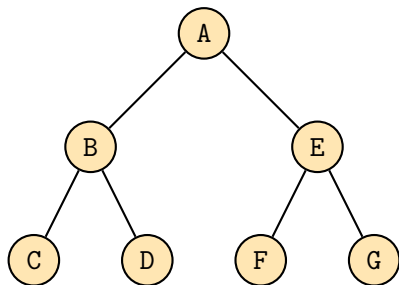
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence:

Stack: A B

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
% pre-order visit of t
```

```
print t
```

```
dfs(t.left())
```

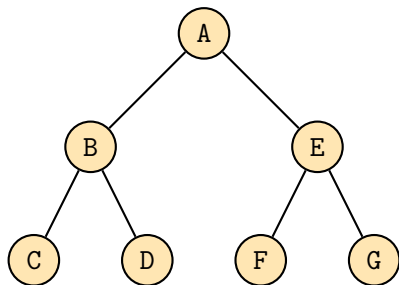
```
% in-order visit of t
```

```
print t
```

```
dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: **C**

Stack: **A B C**

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

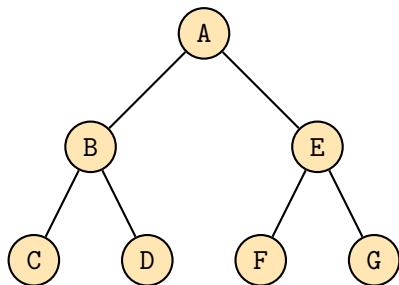
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C

Stack: A B

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

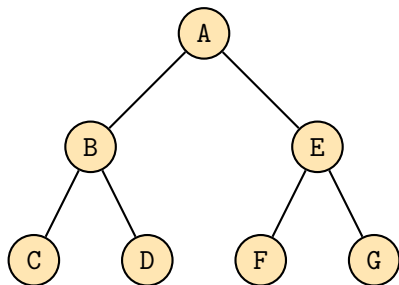
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C D

Stack: A B D

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

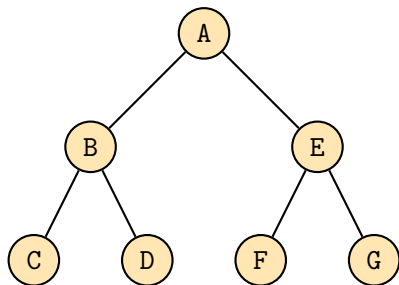
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C D B

Stack: A B

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

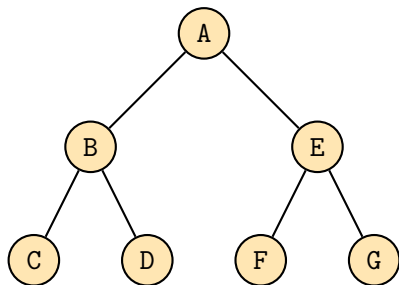
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C D B

Stack: A

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

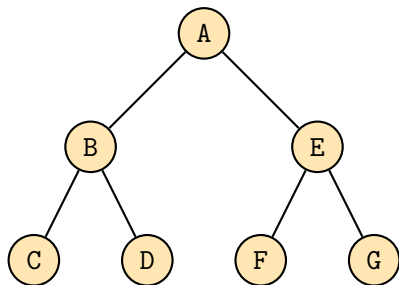
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C D B

Stack: A E

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

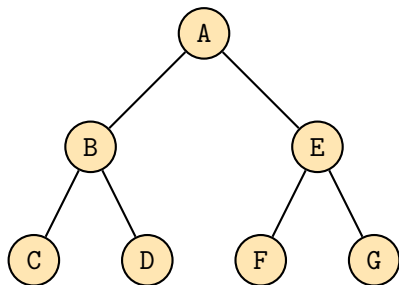
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C D B F

Stack: A E F

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

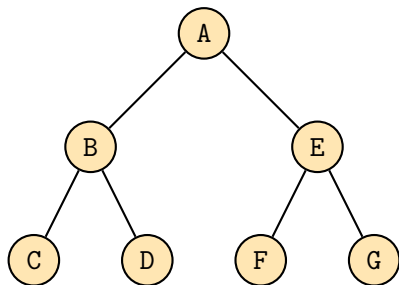
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C D B F

Stack: A E

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
% pre-order visit of t
```

```
print t
```

```
dfs(t.left())
```

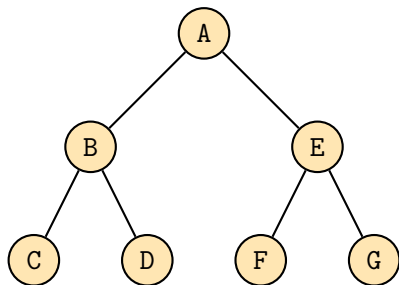
```
% in-order visit of t
```

```
print t
```

```
dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: C D B F G

Stack: A E G

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
% pre-order visit of t
```

```
print t
```

```
dfs(t.left())
```

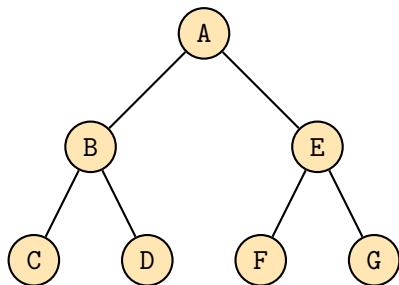
```
% in-order visit of t
```

```
print t
```

```
dfs(t.right())
```

```
% post-order visit of t
```

```
print t
```



Sequence: C D B F G E

Stack: A E

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

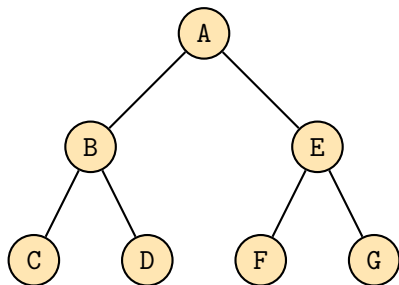
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



Sequence: C D B F G E A

Stack: A

Depth-First Search - Post-Order

```
dfs(TREE t)
```

```
if t ≠ nil then
```

```
  % pre-order visit of t
```

```
  print t
```

```
  dfs(t.left())
```

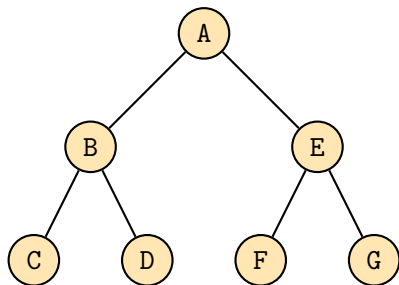
```
  % in-order visit of t
```

```
  print t
```

```
  dfs(t.right())
```

```
  % post-order visit of t
```

```
  print t
```



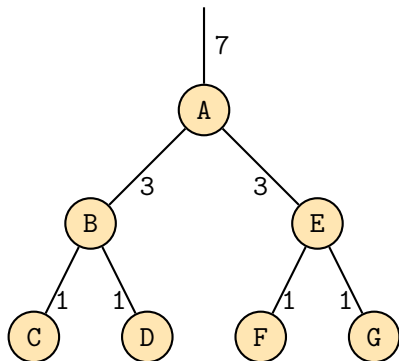
Sequence: C D B F G E A

Stack:

Esempi di applicazione

Contare nodi – Post-visita

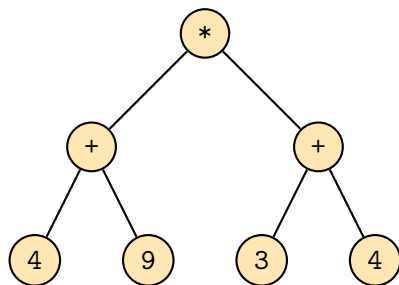
```
int count(TREE T)
if T == nil then
    return 0
else
     $C_\ell = \text{count}(T.\text{left}())$ 
     $C_r = \text{count}(T.\text{right}())$ 
    return  $C_\ell + C_r + 1$ 
```



Esempi di applicazione

Stampare espressioni – In-visita

```
int printExp(TREE T)
if T.left() == nil and
  T.right() == nil then
  | print T.read()
else
  | print "("
  | printExp(T.left())
  | print T.read()
  | printExp(T.right())
  | print ")"
```



$((4+9) * (3+4))$

Costo computazionale

Il costo di una visita di un albero contenente n nodi è $\Theta(n)$, in quanto ogni nodo viene visitato al massimo una volta.

Sommario

- 1 Introduzione
 - Esempi
 - Definizioni
- 2 Alberi binari
 - Introduzione
 - Implementazione
 - Visite
- 3 Alberi generici
 - Visite
 - Implementazione

Specifica (Albero generico)

TREE

% Costruisce un nuovo nodo, contenente v , senza figli o genitori

Tree(ITEM v)

% Legge il valore memorizzato nel nodo

ITEM read()

% Modifica il valore memorizzato nel nodo

write(ITEM v)

% Restituisce il padre, oppure **nil** se questo nodo è radice

TREE parent()

Specifica (Albero generico)

TREE

% Restituisce il primo figlio, oppure **nil** se questo nodo è una foglia

TREE leftmostChild()

% Restituisce il prossimo fratello, oppure **nil** se assente

TREE rightSibling()

% Inserisce il sottoalbero *t* come primo figlio di questo nodo

insertChild(TREE *t*)

% Inserisce il sottoalbero *t* come prossimo fratello di questo nodo

insertSibling(TREE *t*)

% Distruggi l'albero radicato identificato dal primo figlio

deleteChild()

% Distruggi l'albero radicato identificato dal prossimo fratello

deleteSibling()

Esempio: Class Node (Java 8)

```
package org.w3c.dom;
public interface Node {

    /** The parent of this node. */
    public Node    getParentNode();

    /** The first child of this node. */
    public Node    getFirstChild()

    /** The node immediately following this node. */
    public Node    getNextSibling()

    /** Inserts the node newChild before the existing child node refChild. */
    public Node    insertBefore(Node newChild, Node refChild)

    /** Adds the node newChild to the end of the list of children of this node. */
    public Node    appendChild(Node newChild)

    /** Removes the child node indicated by oldChild from the list of children. */
    public Node    removeChild(Node oldChild)

    [...]
}
```

Depth-First Search

```
dfs(TREE t)
```

```
if  $t \neq \text{nil}$  then
```

```
  % pre-order visit of node  $t$ 
```

```
  print  $t$ 
```

```
  TREE  $u = t.\text{leftmostChild}()$ 
```

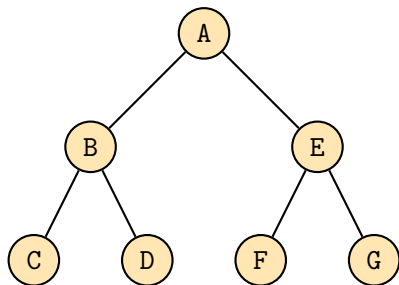
```
  while  $u \neq \text{nil}$  do
```

```
    | dfs( $u$ )
```

```
    |  $u = u.\text{rightSibling}()$ 
```

```
  % post-order visit of node  $t$ 
```

```
  print  $t$ 
```



Breadth-First Search

```
bfs(TREE t)
```

```
QUEUE Q = Queue()
```

```
Q.enqueue(t)
```

```
while not Q.isEmpty() do
```

```
    TREE u = Q.dequeue()
```

```
    % visita per livelli nodo u
```

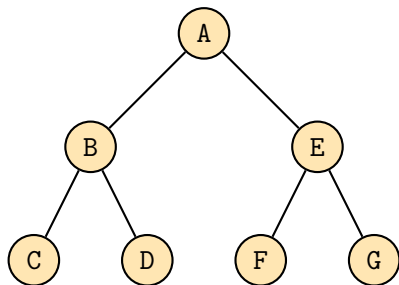
```
    print u
```

```
    u = u.leftmostChild()
```

```
    while u ≠ nil do
```

```
        Q.enqueue(u)
```

```
        u = u.rightSibling()
```



Sequence:

Queue: **A**

Breadth-First Search

```
bfs(TREE t)
```

```
QUEUE Q = Queue()
```

```
Q.enqueue(t)
```

```
while not Q.isEmpty() do
```

```
    TREE u = Q.dequeue()
```

```
    % visita per livelli nodo u
```

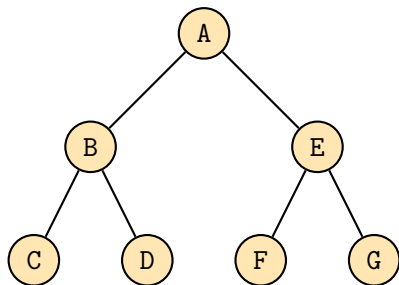
```
    print u
```

```
    u = u.leftmostChild()
```

```
    while u ≠ nil do
```

```
        Q.enqueue(u)
```

```
        u = u.rightSibling()
```



Sequence: **A**

Queue: **B E**

Breadth-First Search

```
bfs(TREE t)
```

```
QUEUE Q = Queue()
```

```
Q.enqueue(t)
```

```
while not Q.isEmpty() do
```

```
    TREE u = Q.dequeue()
```

```
    % visita per livelli nodo u
```

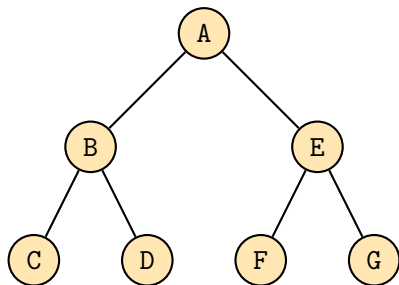
```
    print u
```

```
    u = u.leftmostChild()
```

```
    while u ≠ nil do
```

```
        Q.enqueue(u)
```

```
        u = u.rightSibling()
```



Sequence: A B

Queue: E C D

Breadth-First Search

```
bfs(TREE t)
```

```
QUEUE Q = Queue()
```

```
Q.enqueue(t)
```

```
while not Q.isEmpty() do
```

```
    TREE u = Q.dequeue()
```

```
    % visita per livelli nodo u
```

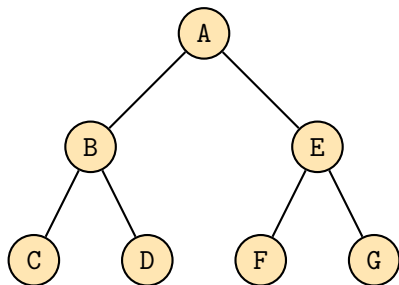
```
    print u
```

```
    u = u.leftmostChild()
```

```
    while u ≠ nil do
```

```
        Q.enqueue(u)
```

```
        u = u.rightSibling()
```



Sequence: A B E

Queue: C D F G

Breadth-First Search

```
bfs(TREE t)
```

```
QUEUE Q = Queue()
```

```
Q.enqueue(t)
```

```
while not Q.isEmpty() do
```

```
    TREE u = Q.dequeue()
```

```
    % visita per livelli nodo u
```

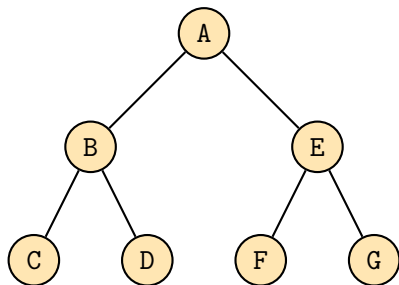
```
    print u
```

```
    u = u.leftmostChild()
```

```
    while u ≠ nil do
```

```
        Q.enqueue(u)
```

```
        u = u.rightSibling()
```



Sequence: A B E C

Queue: D F G

Breadth-First Search

```
bfs(TREE t)
```

```
QUEUE Q = Queue()
```

```
Q.enqueue(t)
```

```
while not Q.isEmpty() do
```

```
    TREE u = Q.dequeue()
```

```
    % visita per livelli nodo u
```

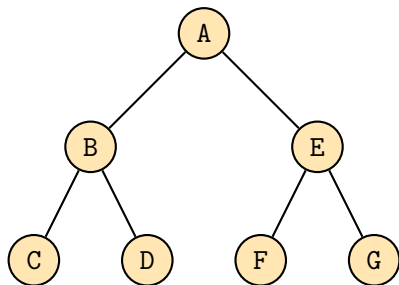
```
    print u
```

```
    u = u.leftmostChild()
```

```
    while u ≠ nil do
```

```
        Q.enqueue(u)
```

```
        u = u.rightSibling()
```



Sequence: A B E C D

Queue: F G

Breadth-First Search

```
bfs(TREE t)
```

```
QUEUE Q = Queue()
```

```
Q.enqueue(t)
```

```
while not Q.isEmpty() do
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```
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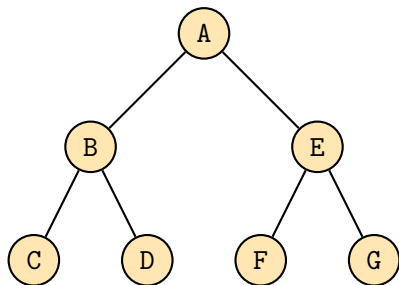
```
    print u
```

```
    u = u.leftmostChild()
```

```
    while u ≠ nil do
```

```
        Q.enqueue(u)
```

```
        u = u.rightSibling()
```



Sequence: A B E C D F

Queue: G

Breadth-First Search

```
bfs(TREE t)
```

```
QUEUE Q = Queue()
```

```
Q.enqueue(t)
```

```
while not Q.isEmpty() do
```

```
    TREE u = Q.dequeue()
```

```
    % visita per livelli nodo u
```

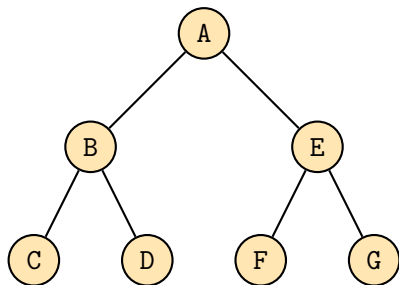
```
    print u
```

```
    u = u.leftmostChild()
```

```
    while u ≠ nil do
```

```
        Q.enqueue(u)
```

```
        u = u.rightSibling()
```



Sequence: **A B E C D F G**

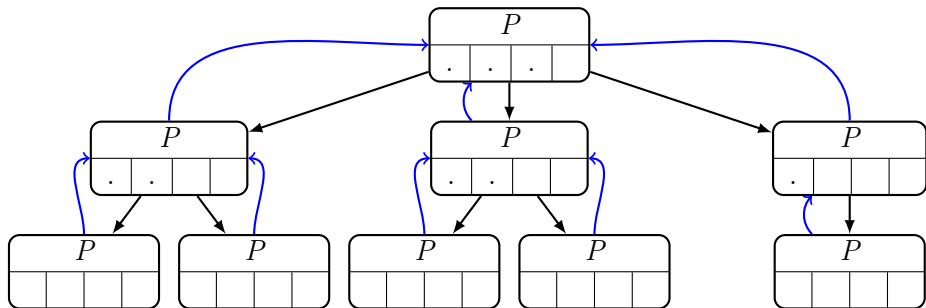
Queue:

Memorizzazione

Esistono diversi modi per memorizzare un albero, più o meno indicati a seconda del numero massimo e medio di figli presenti.

- Realizzazione con vettore dei figli
- Realizzazione primo figlio, prossimo fratello
- Realizzazione con vettore dei padri

Realizzazione con vettore dei figli

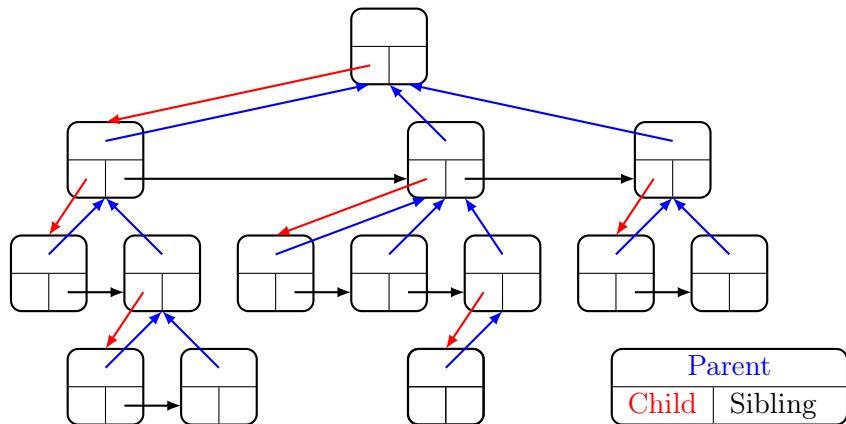


Campi memorizzati nei nodi

- *parent*: reference al nodo padre
- **Vettore dei figli**: a seconda del numero di figli, può comportare una discreta quantità di spazio sprecato

Realizzazione basata su Primo figlio, prossimo fratello

Implementato come una lista di fratelli



Implementazione

```

TREE
TREE parent                                % Reference al padre
TREE child                                 % Reference al primo figlio
TREE sibling                               % Reference al prossimo fratello
ITEM value                                 % Valore memorizzato nel nodo

Tree(ITEM v)                               % Crea un nuovo nodo
┌   TREE t = new TREE
│   t.value = v
│   t.parent = t.child = t.sibling = nil
└   return t

insertChild(TREE t)
┌   t.parent = self
│   t.sibling = child                       % Inserisce t prima dell'attuale primo figlio
└   child = t

insertSibling(TREE t)
┌   t.parent = parent
│   t.sibling = sibling                   % Inserisce t prima dell'attuale prossimo fratello
└   sibling = t

```

Implementazione

TREE

deleteChild()

```
TREE newChild = child.rightSibling()
delete(child)
child = newChild
```

deleteSibling()

```
TREE newSibling = sibling.rightSibling()
delete(sibling)
sibling = newSibling
```

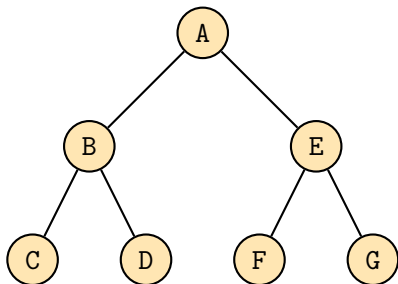
delete(TREE t)

```
TREE u = t.leftmostChild()
while u ≠ nil do
  TREE next = u.rightSibling()
  delete(u)
  u = next
delete t
```

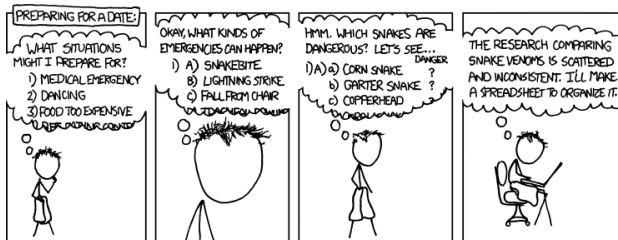
Realizzazione con vettore dei padri

L'albero è rappresentato da un vettore i cui elementi contengono il valore associato al nodo e l'indice della posizione del padre nel vettore.

1	A	0
2	B	1
3	E	1
4	C	2
5	D	2
6	F	3
7	G	3



DFS (<https://xkcd.com/>)



I REALLY NEED TO STOP USING DEPTH-FIRST SEARCHES.