Mathematical Logics
Description Logic and Databases

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Limitations of databases w.r.t. DL

Employee			
Name	**Role**	**Nationality**	**Supervises**
Fausto	Professor	Italian	Rui
Rui	Student	Chinese	Bisu
Bisu	Student	Indian	-

- No negation
- No disjunction
- Ambiguous support for incomplete information (null values)

- The database represents a *single model.*
- Hence, inference is just model checking.

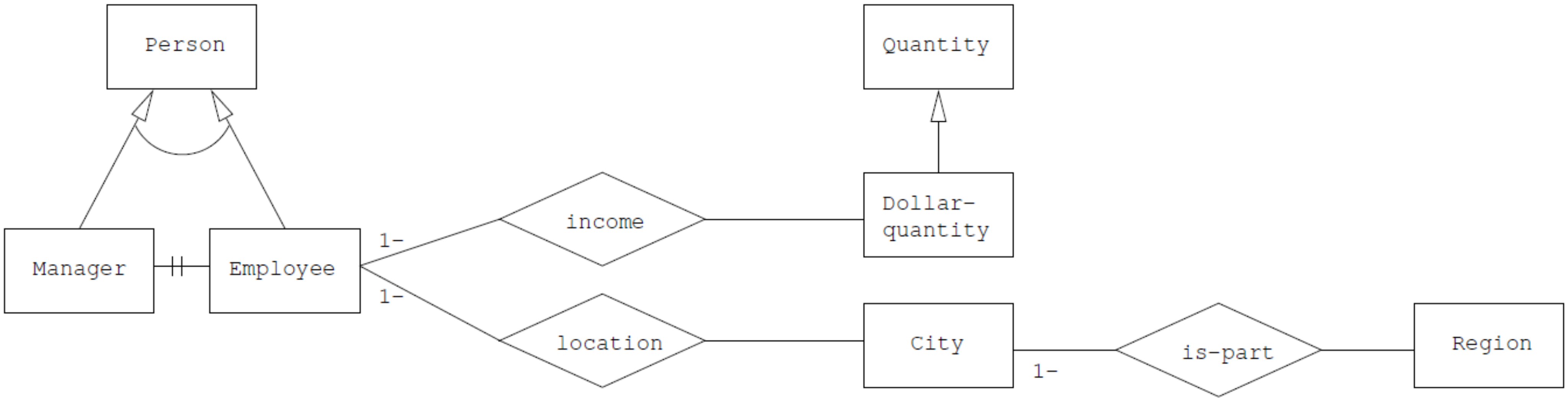
Defining a TBox and ABox for a database

Employee			
Name	**Role**	**Nationality**	**Supervises**
Fausto	Professor	Italian	Rui
Rui	Student	Chinese	Bisu
Bisu	Student	Indian	-
Individual	Class	Attribute	Relation

TBox = {Professor $\sqsubseteq$ Employee, Student $\sqsubseteq$ Employee}

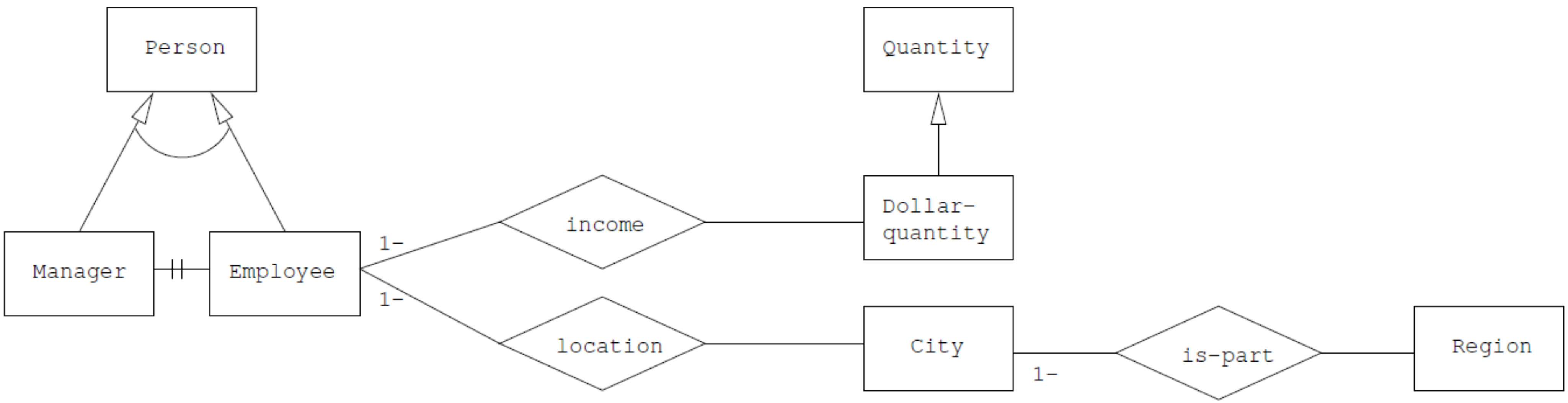
ABox = {Professor(Fausto), Student(Rui), Student(Bisu),
Nationality(Fausto, Italian), Nationality (Rui, Chinese),
Nationality (Bisu, Indian), Supervises(Fausto, Rui),
Supervises(Rui, Bisu)}

Defining DL theories for ER diagrams



- An ER conceptual schema can be expressed as a DL theory
- The models of the DL theory correspond to the legal database states of the ER schema.
- Reasoning services, such as satisfiability of a schema or of a logical implication, can be performed by the corresponding DL theory.
- A DL theory allows for a greater expressivity than the original ER schema, in terms of full disjunction and negation and entity definitions by means of both necessary and sufficient conditions.

Defining DL theories for ER diagrams



TBox = {

$\mathrm{Person} \sqsubseteq \mathrm{Manager} \sqcup \mathrm{Employee}$,

$\mathrm{Manager} \sqsubseteq \mathrm{Person} \sqcap \neg\, \mathrm{Employee}$,

$\mathrm{Employee} \sqsubseteq \mathrm{Person} \sqcap \exists \mathrm{income}^{-1}.\mathrm{Dollar\text{-}quantity} \sqcap \exists \mathrm{location}^{-1}.\mathrm{City}$

$\mathrm{Dollar\text{-}quantity} \sqsubseteq \mathrm{Quantity}$

$\mathrm{City} \sqsubseteq \exists \mathrm{is\text{-}part}^{-1}.\mathrm{Region}$

}

DL as Query Language

TBox = {}
ABox = {Child(John, Mary), Female(Mary)}

NL Query: Who are the individuals having only female children?

DL Query: $T, A \models \forall Child.Female$

Answer: {John}

- We can think to a database as a DL theory with <u>one model</u>
- <u>ABox services</u> are generally applied to resolve a query
- Complexity may go up to CO-NP complete

How to use ABox Reasoning Services

ABox Service	Description	Query
Instance retrieval	Given a concept C, retrieve all the instances *a* which satisfy C w.r.t. the ABox A.	A ⊨ C
Instance checking	Check whether an assertion C(*a*) is entailed by the ABox, i.e. check whether *a* belongs to C.	A ⊨ C(*a*) A ⊨ R(*a*,*b*)

NOTE: this means that before answering we need to expand the ABox (w.r.t. the TBox) and reason on the identified model

RECALL: Reasoning via expansion of the ABox

- Reasoning services over an ABox w.r.t. an <u>acyclic</u> TBox can be reduced to checking an expanded ABox.
- We define the expansion of an ABox A with respect to T as the ABox A' that is obtained from A by replacing each concept assertion C(*a*) with the assertion C'(*a*), with C' the expansion of C with respect to T.
- A is consistent with respect to T iff its expansion A' is consistent
- A is consistent iff A is satisfiable, i.e. non contradictory.

Answering Queries via instance checking (I)

TBox = {Horse $\sqsubseteq$ Animal, Mule $\sqsubseteq$ Animal}

ABox = {Horse(Furia), Parent(Speedy, Furia)}

NL Query: Is Furia an animal?

DL Query: T, A $\models$ Animal(Furia)

YES, in fact the ABox can be expanded as follows:

ABox = {Horse(Furia), Animal(Furia), Parent(Speedy, Furia)}

Answering Queries via instance checking (II)

TBox = {Horse $\sqsubseteq$ Animal $\sqcap$ $\neg$Mule, Mule $\sqsubseteq$ Animal}

ABox = {Horse(Furia), Parent(Speedy, Furia)}

NL Query: Is Furia a mule?

DL Query: T, A $\models$ Animal(Furia)

NO, in fact the ABox can be expanded as follows:

ABox = {Horse(Furia), Animal(Furia), $\neg$Mule(Furia),
Parent(Speedy, Furia)}

Answering Queries via instance checking (III)

TBox = {Horse $\sqsubseteq$ Animal, Mule $\sqsubseteq$ Animal}

ABox = {Horse(Furia), Parent(Speedy, Furia)}

NL Query: Is Furia a mule?

DL Query: T, A $\vDash$ Mule(Furia)

NO (BY CLOSED WORLD ASSUMPTION), in fact the ABox can be expanded as follows:

ABox = {Horse(Furia), Animal(Furia), Parent(Speedy, Furia)}

If we drop closed world assumption the answer should be I DO NOT KNOW

Answering Queries via instance retrieval: Tableaux (I)

TBox = {Horse $\sqsubseteq$ Animal, Mule $\sqsubseteq$ Animal}

ABox = {Horse(Speedy), Horse(Furia), Parent(Speedy, Furia)}

NL Query: Is there any animal which is not both a horse and a mule, and is parent of a horse?

DL Query: T, A $\models \exists$ Parent.Horse $\sqcap \neg$ (Horse $\sqcap$ Mule)

i.e. is the formula satifiable?

Answering Queries via instance retrieval: Tableaux (I)

TBox = {Horse $\sqsubseteq$ Animal, Mule $\sqsubseteq$ Animal}

ABox = {Horse(Speedy), Horse(Furia), Parent(Speedy, Furia)}

Is $\exists$Parent.Horse $\sqcap$ $\neg$ (Horse $\sqcap$ Mule) satifiable?

$\sqcap$-rule A' = { $\exists$Parent.Horse(x), $\neg$(Horse $\sqcap$ Mule)(x)}

$\exists$-rule A' = {Horse(Furia), Parent(Speedy, Furia), ($\neg$Horse $\sqcup$ $\neg$Mule)(x)}

$\sqcup$-rule A' = {Horse(Furia), Parent(Speedy, Furia), $\neg$Horse(Speedy)} inconsistent

or

A' = {Horse(Furia), Parent(Speedy, Furia), $\neg$Mule(Speedy)} consistent

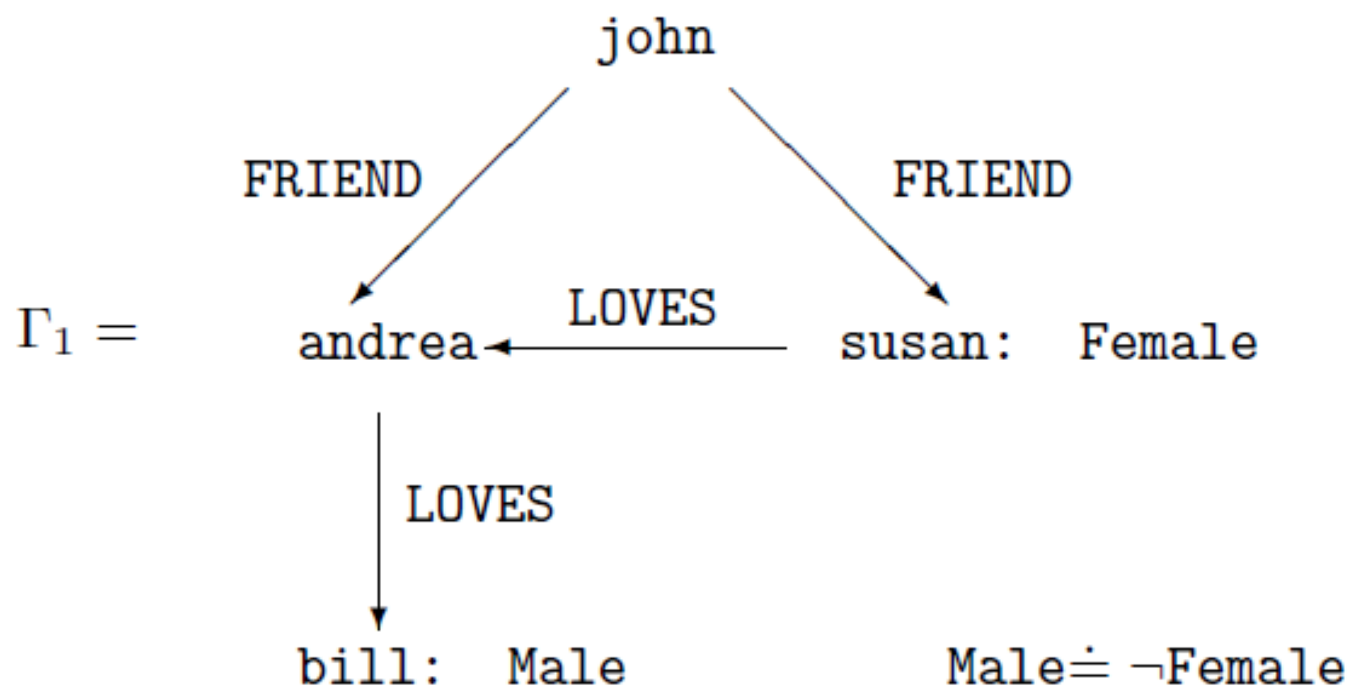
Answering Queries via graph reasoning

$$\Gamma = \begin{tikzcd} & \texttt{john} \arrow[dl, "\texttt{FRIEND}"'] \arrow[dr, "\texttt{FRIEND}"] & \\ \texttt{andrea} \arrow[d, "\texttt{LOVES}"] & & \texttt{susan:}\ \ \texttt{Female} \arrow[ll, "\texttt{LOVES}"'] \\ \texttt{bill:}\ \ \neg\texttt{Female} & & \end{tikzcd}$$

NL Query: Does John have a female friend loving a not female?

DL Query: $\Gamma \vDash \exists \mathrm{FRIEND}.(\mathrm{Female} \sqcap (\exists \mathrm{LOVES}.\neg \mathrm{Female}))(\mathrm{john})$

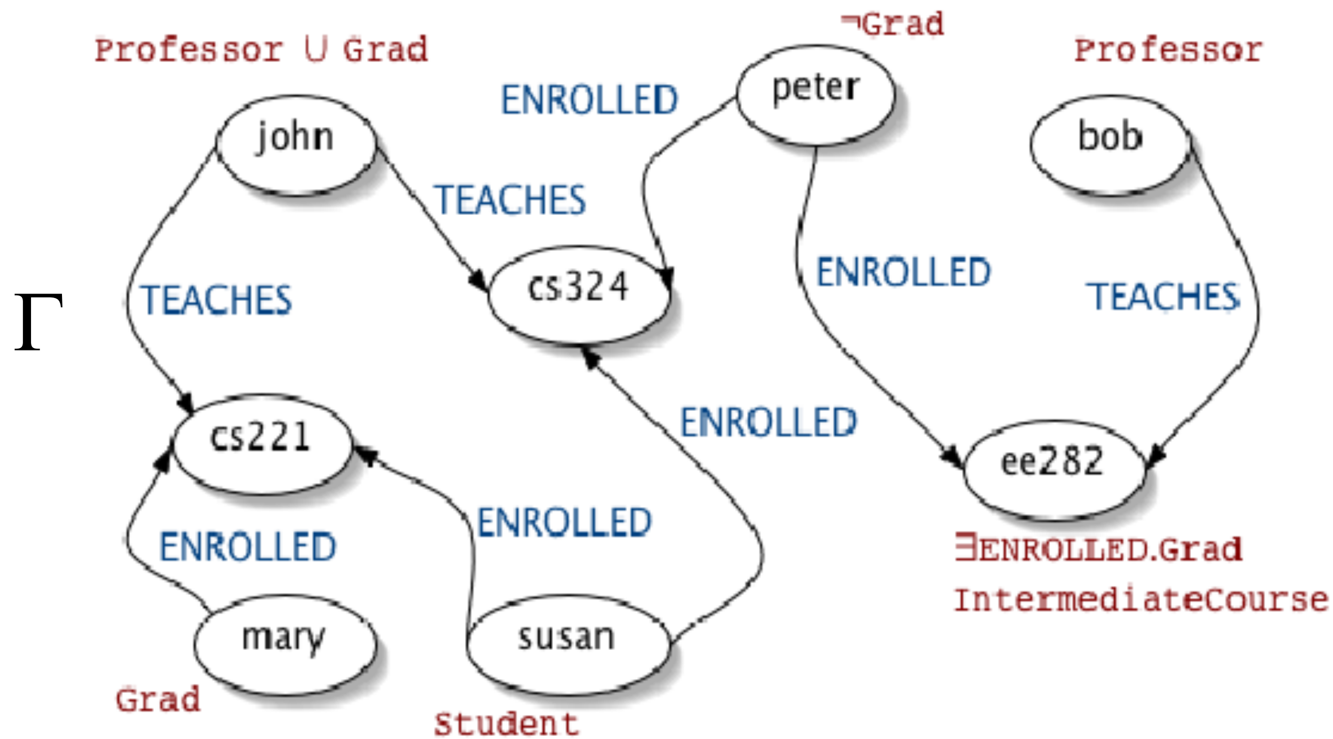
Answering Queries via graph reasoning



NL Query: Does John have a female friend loving a male?

DL Query: $\Gamma_1 \models \exists \text{FRIEND}.(\text{Female} \sqcap (\exists \text{LOVES}.\text{Male}))(\text{john})$

Provide the answer for the queries



$\Gamma \vDash$ ENROLLED(Mary, cs221)

$\Gamma \vDash$ Grad(peter)

$\Gamma \vDash$ Grad(Susan)

$\Gamma \vDash \exists$ ENROLLED.Grad (ee282)

$\Gamma \vDash \forall$ TEACHES. IntermediateCourse(bob)

$\Gamma \vDash$ Grad $\sqcap \; \exists$ TEACHES.$\top$

$\Gamma \vDash$ Student $\sqcap \; \forall$ ENROLLED.$\top$