## Limitations of databases w.r.t. DL

### Employee

<table>
<thead>
<tr>
<th>Name</th>
<th>Role</th>
<th>Nationality</th>
<th>Supervises</th>
</tr>
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<tbody>
<tr>
<td>Fausto</td>
<td>Professor</td>
<td>Italian</td>
<td>Rui</td>
</tr>
<tr>
<td>Rui</td>
<td>Student</td>
<td>Chinese</td>
<td>Bisu</td>
</tr>
<tr>
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<td>Student</td>
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- No negation
- No disjunction
- Ambiguous support for incomplete information (null values)

- The database represents a *single model*.
- Hence, inference is just model checking.
Defining a TBox and ABox for a database

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**Employee**

**TBox** = \{Professor \sqsubseteq Employee, Student \sqsubseteq Employee\}

**ABox** = \{Professor(Fausto), Student(Rui), Student(Bisu), Nationality(Fausto, Italian), Nationality (Rui, Chinese), Nationality (Bisu, Indian), Supervises(Fausto, Rui), Supervises(Rui, Bisu)\}
An ER conceptual schema can be expressed as a DL theory. The models of the DL theory correspond to the legal database states of the ER schema. Reasoning services, such as satisfiability of a schema or of a logical implication, can be performed by the corresponding DL theory. A DL theory allows for a greater expressivity than the original ER schema, in terms of full disjunction and negation and entity definitions by means of both necessary and sufficient conditions.
TBox = 
\[
\begin{align*}
\text{Person} & \equiv \text{Manager} \sqcup \text{Employee}, \\
\text{Manager} & \equiv \text{Person} \sqcap \neg \text{Employee}, \\
\text{Employee} & \equiv \text{Person} \sqcap \exists \text{Income}^{-1}.\text{Dollar-quantity} \sqcap \exists \text{Location}^{-1}.\text{City} \\
\text{Dollar-quantity} & \equiv \text{Quantity} \\
\text{City} & \equiv \exists \text{Is-part}^{-1}.\text{Region}
\end{align*}
\]
We can think to a database as a DL theory with one model

ABox services are generally applied to resolve a query

Complexity may go up to CO-NP complete
## How to use ABox Reasoning Services

<table>
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<tr>
<th>ABox Service</th>
<th>Description</th>
<th>Query</th>
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<td>Instance retrieval</td>
<td>Given a concept C, retrieve all the instances a which satisfy C w.r.t. the ABox A.</td>
<td>$A \models C$</td>
</tr>
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</table>
| Instance checking      | Check whether an assertion $C(a)$ is entailed by the ABox, i.e. check whether $a$ belongs to $C$. | $A \models C(a)$  
$A \models R(a,b)$ |

**NOTE:** this means that before answering we need to expand the ABox (w.r.t. the TBox) and reason on the identified model
Reasoning services over an ABox w.r.t. an acyclic TBox can be reduced to checking an expanded ABox.

We define the expansion of an ABox $A$ with respect to $T$ as the ABox $A'$ that is obtained from $A$ by replacing each concept assertion $C(a)$ with the assertion $C'(a)$, with $C'$ the expansion of $C$ with respect to $T$.

$A$ is consistent with respect to $T$ iff its expansion $A'$ is consistent.

$A$ is consistent iff $A$ is satisfiable, i.e. non contradictory.
TBox = \{\text{Horse} \sqsubseteq \text{Animal}, \text{Mule} \sqsubseteq \text{Animal}\}

ABox = \{\text{Horse}(\text{Furia}), \text{Parent}(\text{Speedy}, \text{Furia})\}

NL Query: Is Furia an animal?

DL Query: \( T, A \models \text{Animal}(\text{Furia}) \)

\textbf{YES}, in fact the ABox can be expanded as follows:
ABox = \{\text{Horse}(\text{Furia}), \text{Animal}(\text{Furia}), \text{Parent}(\text{Speedy}, \text{Furia})\}
TBox = \{\text{Horse} \sqsubseteq \text{Animal} \sqcap \neg \text{Mule}, \text{Mule} \sqsubseteq \text{Animal}\}
ABox = \{\text{Horse}(\text{Furia}), \text{Parent}(\text{Speedy}, \text{Furia})\}

NL Query: Is Furia a mule?

DL Query: T, A \models \text{Animal}(\text{Furia})

NO, in fact the ABox can be expanded as follows:
ABox = \{\text{Horse}(\text{Furia}), \text{Animal}(\text{Furia}), \neg \text{Mule}(\text{Furia}), \text{Parent}(\text{Speedy}, \text{Furia})\}
TBox = \{\text{Horse} \sqsubseteq \text{Animal}, \text{Mule} \sqsubseteq \text{Animal}\}
ABox = \{\text{Horse}(\text{Furia}), \text{Parent}(\text{Speedy}, \text{Furia})\}

NL Query: Is Furia a mule?

DL Query: T, A \models \text{Mule}(\text{Furia})

NO (BY CLOSED WORLD ASSUMPTION), in fact the ABox can be expanded as follows:
ABox = \{\text{Horse}(\text{Furia}), \text{Animal}(\text{Furia}), \text{Parent}(\text{Speedy}, \text{Furia})\}

If we drop closed world assumption the answer should be I DO NOT KNOW
TBox = \{\text{Horse} \sqsubseteq \text{Animal}, \text{Mule} \sqsubseteq \text{Animal}\}
ABox = \{\text{Horse}(\text{Speedy}), \text{Horse}(\text{Furia}), \text{Parent}(\text{Speedy}, \text{Furia})\}

NL Query: Is there any animal which is not both a horse and a mule, and is parent of a horse?

DL Query: $T, A \models \exists \text{Parent. Horse} \sqcap \neg (\text{Horse} \sqcap \text{Mule})$
\[\text{i.e. is the formula satifiable?}\]
TBox = \{\text{Horse} \sqsubseteq \text{Animal}, \text{Mule} \sqsubseteq \text{Animal}\}

ABox = \{\text{Horse(Speedy)}, \text{Horse(Furia)}, \text{Parent(Speedy, Furia)}\}

Is \ \exists \text{Parent.Horse} \sqcap \neg (\text{Horse} \sqcap \text{Mule}) \text{ satifiable?}

\square\text{-rule} \quad A' = \{ \exists \text{Parent.Horse}(x), \neg (\text{Horse} \sqcap \text{Mule})(x) \}

\exists\text{-rule} \quad A' = \{\text{Horse(Furia)}, \text{Parent(Speedy, Furia)}, (\neg \text{Horse} \sqcup \neg \text{Mule})(x)\}

\sqcup\text{-rule} \quad A' = \{\text{Horse(Furia)}, \text{Parent(Speedy, Furia)}, \neg \text{Horse(Speedy)}\} \text{ inconsistent or}

\quad \text{or}

\quad A' = \{\text{Horse(Furia)}, \text{Parent(Speedy, Furia)}, \neg \text{Mule(Speedy)}\} \text{ consistent}
NL Query: Does John have a female friend loving a not female?

DL Query: $\Gamma \models \exists \text{FRIEND.}(\text{Female} \sqcap (\exists \text{LOVES.}\neg\text{Female}))(\text{john})$
NL Query: Does John have a female friend loving a male?

DL Query: $\Gamma_1 \models \exists FRIEND.(\text{Female} \sqcap (\exists \text{LOVES. Male}))(\text{john})$
Provide the answer for the queries

\[ \Gamma \models \text{ENROLLED(Mary, cs221)} \]
\[ \Gamma \models \text{Grad(peter)} \]
\[ \Gamma \models \text{Grad(Susan)} \]
\[ \Gamma \models \exists \text{ENROLLED.Grad (ee282)} \]
\[ \Gamma \models \forall \text{TEACHES. IntermediateCourse(bob)} \]