Mathematical Logics

17 bis: Exercises on Resolution and Unification

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Substitution

Exercize

Let $\sigma = [a/x, f(b)/y, c/z]$ and $\theta = [f(f(a))/v, x/z, g(y)/x]$

- compute $\sigma \circ \theta$ and $\theta \circ \sigma$
- For every of the following formulæ, compute (i) $\phi\sigma$; (ii) $\phi\theta$; (iii) $\phi\sigma \circ \theta$; and (iv) $\phi\theta \circ \sigma$
  1. $\phi = p(x, y, z)$
  2. $\phi = p(h(v)) \vee \neg q(z, x)$
  3. $\phi = q(x, z, v) \vee \neg q(g(y), x, f(f(a)))$
- are σ and θ and their compositions idempotent?

Definition

A function $f : X \longrightarrow X$ on a set X is idempotent if and only if $f(x) = f(f(x))$

An example of idempotent function are $round(\cdot) : \mathbb{R} \longrightarrow \mathbb{R}$, that returns the closer integer $round(x)$ to a real number x.

Unification

Exercize

For every C_1, C_2 and σ, decide whether (i) σ is a unifier of C_1 and C_2; and (ii) σ is the MGU of C_1 and C_2

C_1	C_2	σ
$P(a, f(y), z)$	$Q(x, f(f(v)), b)$	$[a/x, f(b)/y, b/z]$
$Q(x, h(a, z), f(x))$	$Q(g(g(v)), y, f(w))$	$[g(g(v))/x, h(a, z)/y, x/w]$
$Q(x, h(a, z), f(x))$	$Q(g(g(v)), y, f(w))$	$[g(g(v))/x, h(a, z)/y, g(g(v))/w]$
$R(f(x), g(y))$	$R(z, g(v))$	$[a/x, f(a)/z, v/y]$

Exercize

Consider the signature $\Sigma = \langle a, b, f(\cdot,\cdot), g(\cdot,\cdot), P(\cdot,\cdot,\cdot)\rangle$ Use the algorithm from the previous lecture to decide whether the following clauses are unifiable.

1. $\{P(f(x,a), g(y,y), z), P(f(g(a,b), z), x, a)\}$
2. $\{P(x,x,z), P(f(a,a), y, y)\}$
3. $\{P(x, f(y,z), b), P(g(a,y), f(z, g(a,x)), b)\}$
4. $\{P(a, y, U), P(x, f(x,U), g(z,b))\}$

Unification of $P(f(x,a), g(y,y), z)$, $P(f(g(a,b), z), x, a)$

- $\{P(f(x,a), g(y,y), Z), P(f(g(a,b), Z), x, a)\}$
- $\sigma = [g(a,b)/x]$
- $\{P(f(x,a), g(y,y), Z), P(f(g(a,b), Z), x, a)\}\sigma =$
  $\{P(f(g(a,b),a), g(y,y), z), P(f(g(a,b), z), g(a,b), a)\}$.
- $\{P(f(g(a,b),a), g(y,y), z), P(f(g(a,b), z), g(a,b), a)\}$.
- $\sigma = [g(a,b)/x, a/z]$
- $\{P(f(g(a,b),a), g(y,y), z), P(f(g(a,b), z), g(a,b), a)\}\sigma =$
  $\{P(f(g(a,b),a), g(y,y), a), P(f(g(a,b), a), g(a,b), a)\}$
- $\{P(f(g(a,b),a), g(y,y), a), P(f(g(a,b), a), g(a,b), a)\}$
- $\sigma = [g(a,b)/x, a/z, a/y]$
- $\{P(f(g(a,b),a), g(y,y), a), P(f(g(a,b), a), g(a,b), a)\}\sigma =$
  $\{P(f(g(a,b),a), g(a,a), a), P(f(g(a,b), a), g(a,b), a)\}$
- $\{P(f(g(a,b),a), g(a,a), a), P(f(g(a,b), a), g(a,b), a)\}$
- a and b are two constants and they are not unificable. So the algorithm returns that the set of clauses are not unifiable.

Unification of $\{P(x,x,z), P(f(a,a),y,y)\}$

- $\{P(x,x,z), P(f(a,a),y,y)\}$
- $\sigma = [f(a,a)/x]$
- $\{P(x,x,z), P(f(a,a),y,y)\}\sigma =$
  $\{P(f(a,a),f(a,a),z), P(f(a,a),y,y)\}$
- $\{P(f(a,a),f(a,a),z), P(f(a,a),y,y)\}$
- $\sigma = [f(a,a)/x, f(a,a)/y]$
- $\{P(f(a,a),f(a,a),z), P(f(a,a),y,y)\}\sigma =$
  $\{P(f(a,a),f(a,a),z), P(f(a,a),f(a,a),f(a,a))\}$
- $\{P(f(a,a),f(a,a),z), P(f(a,a),f(a,a),f(a,a))\}$
- $\sigma = [f(a,a)/x, f(a,a)/y, f(a,a)/z]$
- $\{P(f(a,a),f(a,a),z), P(f(a,a),f(a,a),f(a,a))\}\sigma =$
  $\{P(f(a,a),f(a,a),f(a,a)), P(f(a,a),f(a,a),f(a,a))\}$
- the two terms are equal, so the initial terms are unifiable with the mgu equal to $\sigma = [f(a,a)/x, f(a,a)/y, f(a,a)/z]$

Unification

Exercize

Find, when possible, the MGU of the following pairs of clauses.

1. $\{q(a), q(b)\}$
2. $\{q(a, x), q(a, a)\}$
3. $\{q(a, x, f(x)), q(a, y, y,)\}$
4. $\{q(x, y, z), q(u, h(v, v), u)\}$
5. $\left\{ \begin{array}{l} p(x_1, g(x_1), x_2, h(x_1, x_2), x_3, k(x_1, x_2, x_3)), \\ p(y_1, y_2, e(y_2), y_3, f(y_2, y_3), y_4) \end{array} \right\}$

Exercize

Find the possible resolvents of the following pairs of clauses.

C	D
$\neg p(x) \vee q(x,b)$	$p(a) \vee q(a,b)$
$\neg p(x) \vee q(x,x)$	$\neg q(a, f(a))$
$\neg p(x,y,u) \vee \neg p(y,z,v) \vee \neg p(x,v,w) \vee p(u,z,w)$	$p(g(x,y),x,y)$
$\neg p(v,z,v) \vee p(w,z,w)$	$p(w,h(x,x),w)$

Solution

C	D	σ
$\neg p(x) \vee q(x,b)$	$p(a) \vee q(a,b)$	$[a/x]$
$\neg p(x) \vee q(x,x)$	$\neg q(a, f(a))$	NO
$\neg p(x,y,u) \vee \neg p(y,z,v) \vee \neg p(x,v,w) \vee p(u,z,w)$	$p(g(x',y'),x',y')$	
$\neg p(x,y,u) \vee \neg p(y,z,v) \vee \neg p(x,v,w) \vee p(u,z,w)$	$p(g(x',y'),x',y')$	
$\neg p(x,y,u) \vee \neg p(y,z,v) \vee \neg p(x,v,w) \vee p(u,z,w)$	$p(g(x',y'),x',y')$	
$\neg p(v,z,v) \vee p(w,z,w)$	$p(w',h(x',x'),w')$	

Exercize

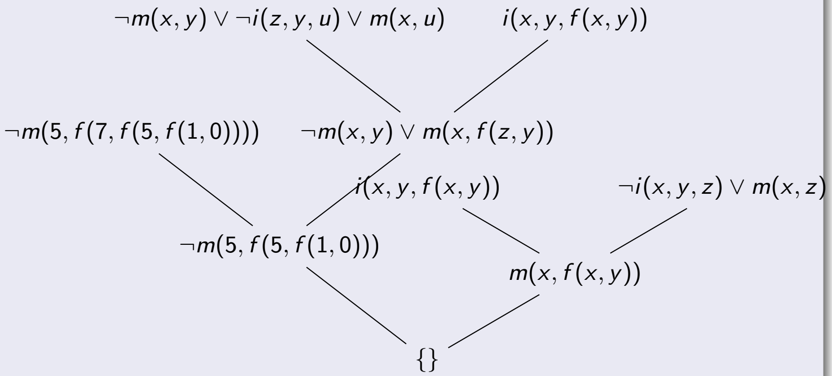
Apply resolution (with refutation) to prove that the following formula

$$\phi_5 \quad m(5, f(7, f(5, f(1, 0))))$$

is a consequence of the set

$$\begin{array}{ll} \phi_1 & \neg m(x, 0) \\ \phi_2 & \neg i(x, y, z) \vee m(x, z) \\ \phi_3 & \neg m(x, z) \vee \neg i(v, z, y) \vee m(x, y) \\ \phi_4 & i(x, y, f(x, y)) \end{array}$$

Solution


¬m(x, y) ∨ ¬i(z, y, u) ∨ m(x, u)
i(x, y, f(x, y))
¬m(5, f(7, f(5, f(1, 0))))
¬m(x, y) ∨ m(x, f(z, y))
i(x, y, f(x, y))
¬i(x, y, z) ∨ m(x, z)
¬m(5, f(5, f(1, 0)))
m(x, f(x, y))
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