MATHEMATICAL LOGIC: final Exam – 1st February 2013

NAME: ………………………………….. SURNAME: …………………………………..

STUDENT ID: ……………………………….

1. [1 PT each] Say whether the following statements are true or false:

1) Model checking is the service that finds a model satisfying a given proposition P	□ **True** □ **False**
2) The semantics of the arcs in lightweight ontologies is given by the union of the formulas along the path from the root to the node	□ **True** □ **False**
3) In syntactic matching between two ontologies, a similarity measure is computed between the concepts at nodes	□ **True** □ **False**
4) The pair <N, +> is a relational structure	□ **True** □ **False**

2. [2 PT] Explain the main differences between *syntactic* and *semantic* matching, in particular in terms of input and output

See slides; it is mainly in the fact that the former compares strings returning a [0,1] confidence value while the latter compares concepts at nodes, thus returning semantic relations such as equivalence, subsumption and disjointness in output.

3. [2 PT] Provide the definition of *lightweight ontology*

See slides (the student is supposed to provide its definition as a graph <N, E, C> and emphasize the notion of concept at node and corresponding constraints on the nodes).

MATHEMATICAL LOGIC: final Exam – 1st February 2013

NAME: ………………………………….. SURNAME: …………………………………..

STUDENT ID: ……………………………….

4. [2 PT] Provide the formal definition of *entailment w.r.t. a set of formulas* in PL

$\Gamma \vDash \psi$

where $\Gamma = \{\theta_1, ..., \theta_n\}$ is a finite set of propositions

For all v such that $v \vDash \theta_i$ for all θ_i in Γ then $v \vDash \psi$

5. [2 PT] Provide the formal definition of (a) *being true in a model* and (b) *satisfiability* for a proposition P in ClassL

(a) P is true under σ if $\sigma \vDash P$, where $\sigma \vDash P$ iff $\sigma(P) \neq \emptyset$

(b) P is satisfiable if there is a class valuation σ such that $\sigma \vDash P$

6. [2 PT] Explain what is the goal of the *refutation tableau* in the PL calculus

A refutation tableau is used to check unsatisfiability or validity of a formula. The formula is first negated and then the goal is to check whether all the branches are closed.

MATHEMATICAL LOGIC: final Exam – 1st February 2013

NAME: ………………………………….. SURNAME: …………………………………..

STUDENT ID: ……………………………….

7. **[2 PT] Say whether the PL formula P: $((A \rightarrow B) \vee \neg C) \rightarrow A$ is *satisfiable*, *unsatisfiable* or *valid*. Provide a formal rationale for the answer.**

The formula can be rewritten as: $\neg (\neg A \vee B \vee \neg C) \vee A$

That is equivalent to $(A \wedge \neg B \wedge C) \vee A$

The models in the truth table of P will be all those entries having
- **A true or**
- **A true, B false and C true (a sub-case of the first one)**

The formula P is clearly not valid since not for all truth valuations it is true. For instance it is not true for all the valuations in which A is false.

Therefore it is satisfiable, it is not unsatisfiable, it is not valid.

8. **[2 PT] Define a PL language <u>with no more than four descriptive symbols</u> and a theory for the following problem: "There are two cats. One is white and the other is black. Exactly one of them is dead. If the white cat is dead then the mice will dance".**

By defining the propositions:

W = "the white cat is dead"
B = "the black cat is dead"
C = "the mice will dance"

When can define the following theory:

$W \leftrightarrow \neg B$ (exactly one of them is dead)

$W \rightarrow C$ (if the white cat is dead the mice will dance)

MATHEMATICAL LOGIC: final Exam – 1st February 2013

NAME: ………………………………….. SURNAME: …………………………………..

STUDENT ID: ……………………………….

9. [4 PT] Given the TBox T = { $A \sqsubseteq \neg B$, $C \equiv D \sqcap A$, $E \sqsubseteq D$ } and the ABox A = { C(a), E(c), B(b) }

(a) Provide the expansion of A w.r.t. T (without normalizing T)
(b) Provide the instance retrieval of B

(a) C(a), D(a), A(a), ¬B(a), E(c), D(c), B(b)
or in other words, A ∪ {D(a), A(a), ¬B(a), D(c) }

(b) {b}

10. [2 PT] Express in FOL the transitivity of an arbitrary relation R.

$\forall x\, \forall y\, \forall z\, (R(x,y) \wedge R(y,z) \rightarrow R(x,z))$

MATHEMATICAL LOGIC: final Exam – 1st February 2013

NAME: ……………………………………..   SURNAME: ……………………………………..

STUDENT ID: ………………………………….

11. **[4 PT] Let C(x) be the statement "x has a cat" and D(x) be the statement "x has a dog". Express each of these statements in FOL using these relations. Let the domain be your classmates, indicated with CM(x).**
    a. **There is a classmate who has a cat and a dog**
    b. **All classmates have a cat or a dog**
    c. **None of your classmates has both a cat and a dog**
    d. **Mary has a dog**

(a) $\exists\ x\ (CM(x) \wedge C(x) \wedge D(x))$

(b) $\forall\ x\ (CM(x) \rightarrow (C(x) \vee D(x)))$

(c) $\neg\exists\ x\ (CM(x) \wedge C(x) \wedge D(x))$

(d) D(Mary)

12. **[2 PT] Given the database instance below and the natural language query "provide the title of the books whose price is greater than 20 and library place is Rome"**

BOOK			
TITLE	**QUANTITY**	**SUPPLIER**	**PRICE**
AAA	5	Mondadori	10
BBB	10	Mondadori	20
CCC	20	Atlas	25
DDD	25	Mondadori	15

LIBRARY	
BOOK	**PLACE**
AAA	ROME
BBB	ROME
CCC	PARIS
DDD	ROME

a. **Provide the corresponding formula in Domain Relational Calculus**
b. **Provide corresponding answer set**

$\gamma = \exists y\ \exists z\ \exists w\ (Book(x,y,z,w) \wedge Library(x, Rome) \wedge (w > 20)$

the answer set is empty

13. [3 PT] Using the tableaux calculus, determine whether the set of PL formulas {¬A, B, A ↔ ¬B} is satisfiable. Explicitly mark branches as closed or open.

This is equivalent to check for A ∧ B ∧ (¬A ∨ ¬B) ∧ (B ∨ A)

The corresponding tableaux is:

```
            ¬A
             |
             B
             |
         (¬A ∨ ¬B)
             |
          (B ∨ A)
         /       \
       ¬A         ¬B
      /   \      /   \
     B     A    B     A
    (o)   (c)  (c)   (c)
```

Since not all branches are closed, the set is satisfiable.