

MATHEMATICAL LOGIC: final Exam – 1st February 2013

NAME: SURNAME:

STUDENT ID:

1. [1 PT each] Say whether the following statements are true or false:

1) Model checking is the service that finds a model satisfying a given proposition P	<input type="checkbox"/> True <input type="checkbox"/> False
2) The semantics of the arcs in lightweight ontologies is given by the union of the formulas along the path from the root to the node	<input type="checkbox"/> True <input type="checkbox"/> False
3) In syntactic matching between two ontologies, a similarity measure is computed between the concepts at nodes	<input type="checkbox"/> True <input type="checkbox"/> False
4) The pair $\langle \mathbb{N}, + \rangle$ is a relational structure	<input type="checkbox"/> True <input type="checkbox"/> False

2. [2 PT] Explain the main differences between *syntactic* and *semantic* matching, in particular in terms of input and output

See slides; it is mainly in the fact that the former compares strings returning a $[0,1]$ confidence value while the latter compares concepts at nodes, thus returning semantic relations such as equivalence, subsumption and disjointness in output.

3. [2 PT] Provide the definition of *lightweight ontology*

See slides (the student is supposed to provide its definition as a graph $\langle \mathbb{N}, E, C \rangle$ and emphasize the notion of concept at node and corresponding constraints on the nodes).

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4. [2 PT] Provide the formal definition of *entailment w.r.t. a set of formulas* in PL

$\Gamma \models \psi$

where $\Gamma = \{\theta_1, \dots, \theta_n\}$ is a finite set of propositions

For all v such that $v \models \theta_i$ for all θ_i in Γ then $v \models \psi$

5. [2 PT] Provide the formal definition of (a) *being true in a model* and (b) *satisfiability* for a proposition P in ClassL

(a) P is true under σ if $\sigma \models P$, where $\sigma \models P$ iff $\sigma(P) \neq \emptyset$

(b) P is satisfiable if there is a class valuation σ such that $\sigma \models P$

6. [2 PT] Explain what is the goal of the *refutation tableau* in the PL calculus

A refutation tableau is used to check unsatisfiability or validity of a formula. The formula is first negated and then the goal is to check whether all the branches are closed.

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7. [2 PT] Say whether the PL formula $P: ((A \rightarrow B) \vee \neg C) \rightarrow A$ is *satisfiable*, *unsatisfiable* or *valid*. Provide a formal rationale for the answer.

The formula can be rewritten as: $\neg (\neg A \vee B \vee \neg C) \vee A$

That is equivalent to $(A \wedge \neg B \wedge C) \vee A$

The models in the truth table of P will be all those entries having

- A true or
- A true, B false and C true (a sub-case of the first one)

The formula P is clearly not valid since not for all truth valuations it is true. For instance it is not true for all the valuations in which A is false.

Therefore it is satisfiable, it is not unsatisfiable, it is not valid.

8. [2 PT] Define a PL language with no more than four descriptive symbols and a theory for the following problem: “There are two cats. One is white and the other is black. Exactly one of them is dead. If the white cat is dead then the mice will dance”.

By defining the propositions:

W = “the white cat is dead”

B = “the black cat is dead”

C = “the mice will dance”

When can define the following theory:

$W \leftrightarrow \neg B$ (exactly one of them is dead)

$W \rightarrow C$ (if the white cat is dead the mice will dance)

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9. [4 PT] Given the TBox $T = \{ A \sqsubseteq \neg B, C \equiv D \sqcap A, E \sqsubseteq D \}$ and the ABox $A = \{ C(a), E(c), B(b) \}$

(a) Provide the expansion of A w.r.t. T (without normalizing T)

(b) Provide the instance retrieval of B

(a) $C(a), D(a), A(a), \neg B(a), E(c), D(c), B(b)$
or in other words, $A \cup \{D(a), A(a), \neg B(a), D(c)\}$

(b) $\{b\}$

10. [2 PT] Express in FOL the transitivity of an arbitrary relation R.

$\forall x \forall y \forall z (R(x,y) \wedge R(y,z) \rightarrow R(x,z))$

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11. [4 PT] Let $C(x)$ be the statement “ x has a cat” and $D(x)$ be the statement “ x has a dog”. Express each of these statements in FOL using these relations. Let the domain be your classmates, indicated with $CM(x)$.

- There is a classmate who has a cat and a dog
- All classmates have a cat or a dog
- None of your classmates has both a cat and a dog
- Mary has a dog

(a) $\exists x (CM(x) \wedge C(x) \wedge D(x))$

(b) $\forall x (CM(x) \rightarrow (C(x) \vee D(x)))$

(c) $\neg \exists x (CM(x) \wedge C(x) \wedge D(x))$

(d) $D(\text{Mary})$

12. [2 PT] Given the database instance below and the natural language query “provide the title of the books whose price is greater than 20 and library place is Rome”

BOOK			
TITLE	QUANTITY	SUPPLIER	PRICE
AAA	5	Mondadori	10
BBB	10	Mondadori	20
CCC	20	Atlas	25
DDD	25	Mondadori	15

LIBRARY	
BOOK	PLACE
AAA	ROME
BBB	ROME
CCC	PARIS
DDD	ROME

- Provide the corresponding formula in Domain Relational Calculus
- Provide corresponding answer set

$\gamma = \exists y \exists z \exists w (\text{Book}(x,y,z,w) \wedge \text{Library}(x, \text{Rome}) \wedge (w > 20))$

the answer set is empty

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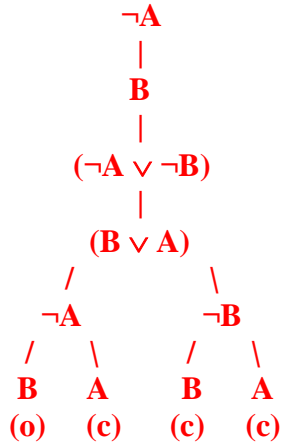
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13. [3 PT] Using the tableaux calculus, determine whether the set of PL formulas $\{\neg A, B, A \leftrightarrow \neg B\}$ is satisfiable. Explicitly mark branches as closed or open.

This is equivalent to check for $A \wedge B \wedge (\neg A \vee \neg B) \wedge (B \vee A)$

The corresponding tableaux is:



Since not all branches are closed, the set is satisfiable.