Università di Trento, Laurea Specialistica in Informatica

# LOGICS FOR DATA AND KNOWLEDGE REPRESENTATION
## Solutions of Exercises, Exam Session II - Monday 20-07-2009

**SURNAME: .................................. NAME: .................................. N. .........**

**1.** 1. What is the "expressiveness" of a representation language? Provide some examples across the logics we have seen so far.

2. What are the main steps to model a piece of world in terms of logical modelling? Explain.

**Solution:** See slides. $\dashv$

**2.** What Venn diagram models the extension of the following proposition?

$(C \to A) \wedge (C \to B) \wedge \neg (A \wedge B)$.

**Solution:** The required Venn diagram is shown in Figure 1.

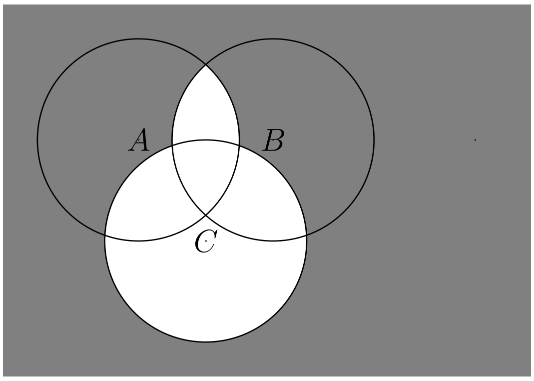


Figure 1: The extension of $(C \to A) \wedge (C \to B) \wedge \neg (A \wedge B)$.

$\dashv$

**3.** (Adapted from Barwise and Etchemendy, 1993) Read the following text:

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

1. Can you prove that the unicorn is mythical? yes ☐ no ☐

2. How about magical? Horned?

**Solution:** 1. No. From the first two statements, we see that if it is mythical, then it is immortal; otherwise it is mammal. So it must be either immortal or a mammal, and thus horned. 2. From 1 we know that the unicorn must be either immortal or a mammal, and thus horned. That means it is also magical. However, we can't deduce anything about whether it is mythical. $\dashv$

**4.** Is every existentially quantified sentence in first-order logic true in any model that contains exactly one object? Justify your answer. yes ☐ no ☐

**Solution:** No. A simple counterexample is the formula $\exists x \; (P(x) \wedge \neg P(x))$. $\dashv$

**5.** Translate into the description logic $\mathcal{ALN}$ the following proposition: "*Veal-parmesan is a meat dish with ingredient veal and exactly 9 ingredients*." (Specify meaning of concepts and roles.)

**Solution:** We define a $\mathcal{ALN}$ TBox containing these two axioms:

$\mathsf{VealParmesan} \equiv \mathsf{MeatDish} \sqcap \exists \mathsf{hasIngredient.Veal} \sqcap = 9\, \mathsf{hasIngredient}.\top$,
$= 9\, \mathsf{hasIngredient}.\top \equiv\ \geq 9\, \mathsf{hasIngredient}.\top \sqcap \leq 9\, \mathsf{hasIngredient}.\top$. $\dashv$

**6.** Represent the following propositions in an appropriate DL (define a DL KB if needed).

1. Every person has exactly one birthplace, which must be a location.
2. Paolo is a person.
3. All persons know only other persons.
4. Paolo knows John.
5. All parents of a person are adult.

**Solution:** We model each sentence by terminological axioms as follows.

1. $\mathsf{Person} \sqsubseteq (\leq 1\, \mathsf{birthPlace}) \sqcap (\geq 1\, \mathsf{birthPlace})$ (in short: $\mathsf{Person} \sqsubseteq (= 1\, \mathsf{birthPlace})$)
   $\mathsf{Person} \sqsubseteq \forall \mathsf{birthPlace.Location}$.
2. $\mathsf{Person(PAOLO)}$.
3. $\mathsf{Person} \sqsubseteq \forall \mathsf{know.Person}$.
4. $\mathsf{knows(PAOLO, JOHN)}$.
5. $\mathsf{Person} \sqsubseteq \forall \mathsf{parent.Adult}$.

$\dashv$

**7.** Let TBox $\mathcal{T}$ be the following set of axioms about documents secured according to internal or external policy on members of an university.

$\{\mathsf{ICT} \sqsubseteq \mathsf{U}, \mathsf{Student} \sqsubseteq \mathsf{ICT}, \mathsf{Faculty} \sqsubseteq \mathsf{ICT}, \mathsf{PhD} \sqsubseteq \mathsf{Student}, \mathsf{Teach} \sqsubseteq \mathsf{Faculty}, \mathsf{Student(Paolo)}, \mathsf{DIT} \sqsubseteq \mathsf{O}, \mathsf{Public} \sqsubseteq \mathsf{DIT}, \mathsf{Internal} \sqsubseteq \mathsf{DIT}, \mathsf{Internal} \equiv \neg\, \mathsf{Public}, \mathsf{ICT} \sqsubseteq \exists \mathsf{read.DIT}, \mathsf{Student} \sqsubseteq \forall \mathsf{read.Public}\}$.

1. Is $\mathcal{T} \models \mathsf{PhD} \sqsubseteq \forall \mathsf{read.Public} \sqcup \neg\, \mathsf{DIT(Paolo)}$? yes ☐ no ☐
2. Is $\mathcal{T} \models \mathsf{ICT} \sqcap \mathsf{DIT} \sqsubseteq \bot$? yes ☐ no ☐

**Solution:** 1. Yes. Simple set-theoretic argument from models. Precisely, we must verify that for all interpretations $\mathcal{I}$, $\mathsf{PhD}^{\mathcal{I}} \subseteq (\forall \mathsf{read.Public}^{\mathcal{I}} \cup \neg\, \mathsf{DIT(Paolo)}^{\mathcal{I}}$.
From $\mathcal{T}$ we know that for all interpretations $\mathcal{I}$,
$\mathsf{PhD}^{\mathcal{I}} \subseteq \mathsf{Student}^{\mathcal{I}}$ and
$\mathsf{Student}^{\mathcal{I}} \subseteq \forall \mathsf{read.Public}^{\mathcal{I}}$.
Hence $\mathsf{PhD}^{\mathcal{I}} \subseteq \forall \mathsf{read.Public}^{\mathcal{I}} \subseteq (\forall \mathsf{read.Public}^{\mathcal{I}} \cup \neg\, \mathsf{DIT(Paolo)}^{\mathcal{I}}$ for all interpretations $\mathcal{I}$, QED.

2. No. To see this we proceed by contradiction. Assume $\mathcal{T} \models \mathsf{ICT} \sqcap \mathsf{DIT} \sqsubseteq \bot$. That is, for all interpretations $\mathcal{I}$ of $\mathcal{T}$, $\mathsf{DIT}^{\mathcal{I}} \cap \mathsf{ICT}^{\mathcal{I}} = \emptyset$ (*).

Let's define an interpretation $\mathcal{I}'$ of 'DIT' as follows: $\mathsf{DIT}^{\mathcal{I}'} = \{\mathsf{Paolo}\}$. Observe that $\mathcal{I}'$ is an interpretation of $\mathcal{T}$. Also observe that by axioms $\mathsf{Student} \sqsubseteq \mathsf{ICT}$ and $\mathsf{Student(Paolo)}$ of $\mathcal{T}$ we have $\mathsf{ICT(Paolo)}$. It follows that $\mathsf{DIT}^{\mathcal{I}'} \cap \mathsf{ICT}^{\mathcal{I}'} = \{\mathsf{Paolo}\} \neq \emptyset$. This contradicts (*). $\dashv$

**8.** Prove the following equivalences.

1. $\neg\,(C \sqcup D) \equiv \neg\, C \sqcap \neg\, D$
2. $\neg\, \exists R.C \equiv \forall R.\neg\, C$

**Solution:** 1. For all DL interpretations $(\Delta, I)$, we have the following:
$I(\neg\,(C \sqcup D)) =$
$= \Delta \setminus I(C \sqcup D)$
$= \Delta \setminus (I(C) \cup I(D))$

$= (\Delta \setminus I(C)) \cap (\Delta \setminus I(D))$
$= I(\neg C) \cap I(\neg D)$
$= I(\neg C \sqcap \neg D)$.

2. For all DL interpretations $(\Delta, I)$, we have the following:
$I(\neg \exists R.C) =$
$= \Delta \setminus I(\exists R.C)$
$= \Delta \setminus \{a \in \Delta \mid \text{for some } b \in \Delta, (a,b) \in I(R) \text{ and } b \in I(C)\}$
$= \{a \in \Delta \mid \text{not for some } b \in \Delta, (a,b) \in I(R) \text{ and } b \in I(C)\}$
$= \{a \in \Delta \mid \text{for all } b \in \Delta, \text{not } (a,b) \in I(R) \text{ and } b \in I(C)\}$
$= \{a \in \Delta \mid \text{for all } b \in \Delta, \text{if } (a,b) \in I(R) \text{ then } b \notin I(C)\}$
$= \{a \in \Delta \mid \text{for all } b \in \Delta, \text{if } (a,b) \in I(R) \text{ then } b \in I(\neg C)\}$.
$= I(\forall R.\neg C)$. ⊣

**9.** How can you represent the following propositions in default logic (Reiter)?

1. 'A person's hometown is almost always that of his/her spouse.'
2. 'A person's hometown is almost always where his/her employer is located.'

**Solution:** 1.

$$\frac{\mathsf{spouse}(\mathsf{x},\mathsf{y}) \wedge \mathsf{hometown}(\mathsf{y}) = \mathsf{z} \quad : M\mathsf{hometown}(\mathsf{x}) = \mathsf{z}}{\mathsf{hometown}(\mathsf{x}) = \mathsf{z}}$$

2.

$$\frac{\mathsf{employer}(\mathsf{x},\mathsf{y}) \wedge \mathsf{location}(\mathsf{y}) = \mathsf{z} \quad : M\mathsf{hometown}(\mathsf{x}) = \mathsf{z}}{\mathsf{hometown}(\mathsf{x}) = \mathsf{z}}$$

⊣

**10.** Let default theory $\Delta = (D, W)$ be defined as follows.

$$D = \left\{\frac{A \quad : M\exists x P(x)}{\exists x P(x)}, \frac{: MA}{A}, \frac{: M\neg A}{\neg A}\right\} \qquad W = \{A \rightarrow \exists x P(x)\}.$$

Define $\Delta$'s extensions, if any. Motivate your answer.

**Solution:** $\Delta$ has two extensions: $E_1 = \mathit{Cn}(W \cup \{\neg A\})$ and $E_2 = \mathit{Cn}(W \cup \{A\})$. ⊣