Logics for Data and Knowledge Representation

Logical properties of foundational relations: a (non-exhaustive) comparison between FOL and DL on a computational ontology of parthood, componenthood, and containment.

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Aim

- Provide an insight on how we can specify the semantics of foundational relations using logics, and why an expressive logic such as FOL is important in describing the properties of entities in an ontology.
- Provide an insight on the impact of different logics such as DL and FOL on the description of foundational relations.
- Taken From Thomas Bittner, Maureen Donnelly: Computational ontologies of parthood, componenthood, and containment. IJCAI 2005: 382-387
- Further developments: Thomas Bittner, Maureen Donnelly: Logical properties of foundational relations in bio-ontologies. Artificial Intelligence in Medicine 39(3): 197-216 (2007)

Aim of Bittner & Donnelly

- Parthood, componenthood, and containment relations are commonly assumed in biomedical ontologies and terminology systems, but are not usually clearly distinguished from another.
- clarify distinctions between these relation as well as principles governing their interrelations;
- develop a theory of these relations in first order predicate logic and then discuss how description logics can be used to capture some important aspects of the first order theory.

Examples

- Joe's head is (a proper) part of Joe's body;
- Joe's brain is (a proper) part of Joe's head;
- the left part of Joe's body is (a proper) part of Joe's body;
- The air in Joe's lung is contained in Joe's lung;
- Joe's lung is contained in Joe's thorax;
- Joe's limbs are composed of Joe's right leg, Joe's left leg, Joe's right arm and Joe's left arm;
- Joe's trunk is composed of Joe's thorax and Joe's abdomen.

Are parthood, componenthood, and containment simply different names for the same relation or are they different relations?

If they are different what is the relation between them?

What is an Ontology?

- The study of the most general characteristics that anything must have in order to count as a (certain kind of) being or entity.
- Ontology (capital "O")
  - a philosophical discipline
- an ontology (lowercase "o")
  - a specific artifact designed with the purpose of expressing the intended meaning of a vocabulary

Taken from N. Guarino, C. Welty; "Conceptual Modeling and Ontological Analysis".

http://www.cs.vassar.edu/~weltyc/aaai-2000/tsld001.htm

What is an Ontology?

- A shared vocabulary:
- Plus . . . A characterization of the intended meaning of that vocabulary

. . . i.e., an ontology accounts for the commitment of a language to a certain conceptualization

> "An ontology is a specification of a conceptualization" [Gru95]

Taken from N. Guarino, C. Welty; "Conceptual Modeling and Ontological Analysis".

http://www.cs.vassar.edu/~weltyc/aaai-2000/tsld001.htm

In our example

- Shared vocabulary: the relations parthood, componenthood, and containment;
- What do we mean by parthood, componenthood, and containment?

...i.e., a well defined ontology accounts for the commitment of our language to the specific conceptualization of these three relations.

How?

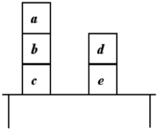
Capturing the intended meaning using logic

- An ontology consisting of just a vocabulary is not very informative;
- Unintended interpretations need to be excluded.
- $Apple(x)$:
  - this is an apple;
  - this is Apple.

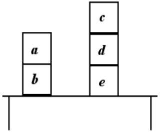
Ontology vs. Language & Data

- An ontology describes the structural parts of a conceptualization independently of:
  - The vocabulary used (i.e., the language used);
  - the actual different situations

“Student” and “Studente” can refer to the same conceptualization;



Scene 1: blocks on a table



Scene 2: a different arrangement of blocks

Can refer to the same conceptualization

$\langle\{a, b, c, d, e, table\}, \{on, above, free\}\rangle$

Ontology vs. Data

- The distinction between the structural parts of a conceptualization and the actual situation is represented in description Logic by means of the distinction between $T - box$ and $A - box$:
  - $T - box$: structural part of the conceptualization;
  - $A - box$: actual situation
- Ontologists often refer to the $T - box$ as to the ontology and to the $A - box$ as to the database.

Formal Ontological Analysis (an aproximation)

- Goal: to characterise entities by means of formal properties and relations.
- Why logic?
  - rigours;
  - general.

Aim of Bittner & Donnelly

- Parthood, componenthood, and containment relations are commonly assumed in biomedical ontologies and terminology systems, but are not usually clearly distinguished from another.
- clarify distinctions between these relation as well as principles governing their interrelations;
- develop a theory of these relations in first order predicate logic and then discuss how description logics can be used to capture some important aspects of the first order theory.

My aim

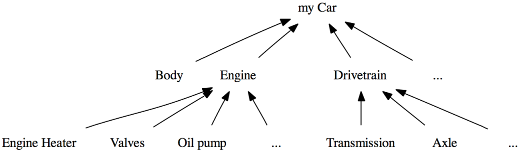
- Provide an example of how the rigours and general instrument of (first order) logic can help in specifying the semantics of foundational relations (parthood, componenthood, and containment)
- Exemplify how to specify the semantics of these foundational relations using different types of formal deductive systems:
  - first-order logic (FOL);
  - description logic (DL).
- I do not aim at discussing the adequacy of the ontological analysis of parthood, componenthood, and containment presented in the paper.

(Proper) Parthood

- Intuitively, proper parthood relations determine the general part-whole structure of an object.
- The left side of my car is a proper part-of my car
- The upper part of my body is a proper part-of my body

Componenthood

- Intuitively, a component of an object is a proper part of that object which has a complete bona de boundary (i.e., boundary that correspond discontinuities in reality) and a distinct function.
- My car has components, for example, its engine, its oil pump, its wheels, etc.



Containment

- Intuitively containment is here understood as a relation which holds between disjoint material objects when one object (the containee) is located within a space partly or wholly enclosed by the container.
- My car is a container. It contains the driver in the seat area and a tool box and a spare-tire in its trunk.

Related Relations

- All components of my car are parts of my car, but my car has also parts (e.g., its left part) that are not components.

  Being a component implies being a part of
  (Being a part of does not imply being a component)

- If the left side of my tool box is proper part of my toolbox and the toolbox is contained in the boot of my car, then the left side of my toolbox is contained in my car.

  If x is proper part of y and y is contained in z
  then x is contained in z

Similar Aspects

- All three relations are transitive and asymmetric.
- Examples of containment:
  - The screw-driver is contained in my tool box and the tool box is contained in the trunk of my car, therefore the screw-driver is contained in the trunk of my car.
  - If the tool box is contained in the trunk of my car, then the trunk of my car is not contained in the tool box.
- Since they are similar they are not always clearly distinguished in bio-medical ontologies such as GALEN or SNOMED.

Different Aspects

- There can be a container with a single containee (e.g., the screw-driver is the only tool in my tool box) but no object can have single proper part.
- Two components share a component only when one is a sub-component of the other. Instead, the left half of my car and the bottom half of my car share the bottom left part of my car but they are not proper parts of each other.

Why a (formal) ontology?

- To make explicit the semantics of these three terms.
- By explicating the distinct properties of proper parthood, componenthood and containment relations.
- That is, to specify the meaning of terms such as proper-part-of, component-of, and contained-in in a certain conceptualization.

The Work-plan (outline of the lecture)

1. Characterize important properties of binary relations and see how they apply to parthood, componenthood, and containment;
2. Use these properties to provide a formal theory of parthood, componenthood, and containment in FOL;
3. Study how to formulated the same theory in description logic.

Preliminaries

- Three relations: *contained-in*; *component-of*; *proper-part-of*.
- An R-structure $\langle \Delta, R \rangle$ consists of a non-empty domain Δ and a non-empty binary relation $R \subseteq (\Delta \times \Delta)$
- $R(x, y)$ indicates that R holds between x and y.
- We introduce 3 relations based on R:

(1) $R_{=} =_{df} R(x, y)$ or $x = y$

(2) $R_O =_{df} \exists z \in \Delta$ such that $R_{=}(z, x)$ and $R_{=}(z, y)$

(3) $R_i =_{df} R(x, y)$ and $(\neg \exists z \in \Delta$ such that $R(x, z)$ and $R(z, y))$

Note: For a given R-structure, the three relations may be empty or identical to R.

Properties of binary relations

Property	*Description*
reflexive	$\forall x \in \Delta, R(x,x)$
irreflexive	$\forall x \in \Delta, \neg R(x,x)$
symmetric	$\forall x, y \in \Delta,$ if $R(x,y)$ then $R(y,x)$
asymmetric	$\forall x, y \in \Delta,$ if $R(x,y)$ then $\neg R(y,x)$
transitive	$\forall x, y, z \in \Delta,$ if $R(x,y)$ and $R(y,z)$ then $R(x,z)$
intransitive	$\forall x, y, z \in \Delta,$ if $R(x,y)$ and $R(y,z)$ then $\neg R(x,z)$
up-discrete	$\forall x, y \in \Delta,$ if $R(x,y)$ then $R_i(x,y)$ or $\exists z \in \Delta$ s.t. $R(x,z)$ and $R_i(z,y)$
dn-discrete	$\forall x, y \in \Delta,$ if $R(x,y)$ then $R_i(x,y)$ or $\exists z \in \Delta$ s.t. $R_i(x,z)$ and $R(z,y)$
discrete	up-discrete and dn-discrete
dense	$\forall x, y \in \Delta,$ if $R(x,y)$ then $\exists z \in \Delta$ s.t. $R(x,z)$ and $R(z,y)$
WSP	$\forall x, y \in \Delta,$ if $R(x,y)$ then $\exists z \in \Delta$ s.t. $R(z,y)$ and $\neg R_O(z,x)$
NPO	$\forall x, y \in \Delta,$ if $R_O(x,y)$ then $x = y$ or $R(x,y)$ or $R(y,x)$
NSIP	$\forall x, y \in \Delta,$ if $R_i(x,y)$ then $\exists z \in \Delta$ s.t. $R_i(z,y)$ and $\neg x = z$
SIS	$\forall x, y, z \in \Delta,$ if $R_i(x,y)$ and $R_i(x,z)$ then $y = z$

Properties of our relations

Property	*Description*
reflexive	$\forall x \in \Delta, R(x, x)$
irreflexive	$\forall x \in \Delta, \neg R(x, x)$
symmetric	$\forall x, y \in \Delta,$ if $R(x, y)$ then $R(y, x)$
asymmetric	$\forall x, y \in \Delta,$ if $R(x, y)$ then $\neg R(y, x)$
transitive	$\forall x, y, z \in \Delta,$ if $R(x, y)$ and $R(y, z)$ then $R(x, z)$
intransitive	$\forall x, y, z \in \Delta,$ if $R(x, y)$ and $R(y, z)$ then $\neg R(x, z)$

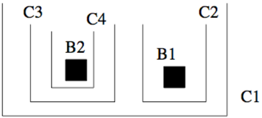
- The identity relation is reflexive, symmetric and transitive;
- R_O is symmetric, R_i is intransitive, and $R_=$ is reflexive;
- On their respective domains *contained-in*; *component-of*; *proper-part-of* are asymmetric and transitive;

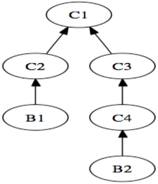
Properties of our relations

Property	*Description*
up-discrete	$\forall x, y \in \Delta,$ if $R(x, y)$ then $R_i(x, y)$ or $\exists z \in \Delta$ s.t. $R(x, z)$ and $R_i(z, y)$
dn-discrete	$\forall x, y \in \Delta,$ if $R(x, y)$ then $R_i(x, y)$ or $\exists z \in \Delta$ s.t. $R_i(x, z)$ and $R(z, y)$
discrete	up-discrete and dn-discrete
dense	$\forall x, y \in \Delta,$ if $R(x, y)$ then $\exists z \in \Delta$ s.t. $R(x, z)$ and $R(z, y)$

- *contained-in* and *component-of* are discrete;
- *proper-part-of* is dense.

Containment is discrete





$$\Delta_C = \{C_1, C_2, C_3, C_4, B_1, B_2\}$$

- if x is *contained-in* y then either
  (a) x is immediately contained in y or
  (b) there exists a z such that x is immediately contained in z and z is contained in y, or
  (c) there exists a z such that x is contained in z and z is immediately contained in y.
- Similarly for componenthood.

Proper part-hood is dense

- Due to the existence of fiat parts (parts which lack a complete bona fide boundary).
- Consider my car and its proper parts. My car does not have an immediate proper part Whatever proper part x we chose, there exists another slightly bigger proper part of my car that has x as a proper part.

Properties of binary relations

Property	*Description*
WSP	$\forall x, y \in \Delta,$ if $R(x, y)$ then $\exists z \in \Delta$ s.t. $R(z, y)$ and $\neg R_O(z, x)$
NPO	$\forall x, y \in \Delta,$ if $R_O(x, y)$ then $x = y$ or $R(x, y)$ or $R(y, x)$
NSIP	$\forall x, y \in \Delta,$ if $R_i(x, y)$ then $\exists z \in \Delta$ s.t. $R_i(z, y)$ and $\neg x = z$
SIS	$\forall x, y, z \in \Delta,$ if $R_i(x, y)$ and $R_i(x, z)$ then $y = z$

- WSP = weak supplementation property;
- NPO = no partial overlap;
- NSIP = no single immediate predecessor;
- SIS = single immediate successor.

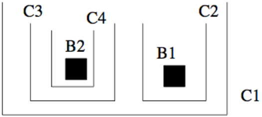
Weak supplementation property

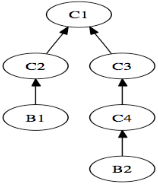
- *proper-part-of* is proper parthood on the domain of spatial objects;
- *proper-part-of*$_O$ is the overlap relation;
- The WPS tells us that if x is a proper part of y then there exists a proper part z of y that does not overlap x.
- Example: since the left side of my car is a proper part of my car there is some proper part of my car (e.g., the right side of my car) which does not overlap with the left side of my car.

Weak supplementation property

- *component-of* is componenthood on the domain of artifacts;
- *component-of*$_O$ is the relation of sharing a component;
- The WPS tell us tells us that if x is a component of y then there exists a component z of y such that z and x do not have a common component.
- Example: since the engine of my car is a component of my car there is some component of my car (e.g., the body of my car) which does not have a component in common with the engine.

Weak supplementation property

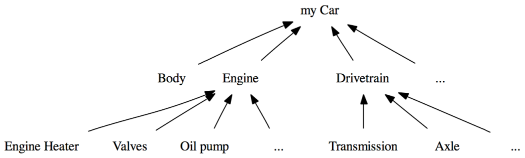




$$\Delta_C = \{C_1, C_2, C_3, C_4, B_1, B_2\}$$

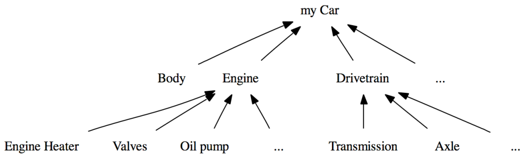
- *contained-in* defined over Δ_C does not satisfy the WPS

No partial overlap



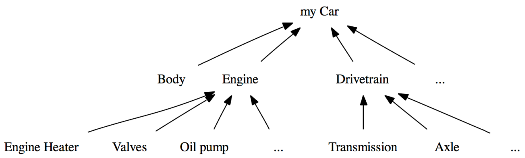
- *component-of* in the diagram above satisfies the NPO property
- *proper-part-of* in the spatial domain does not have the NPO property
- (Counter-)Example: the left half and the bottom half of my car overlap partially.

Single immediate successor



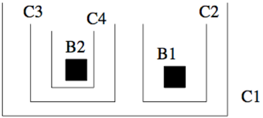
- *component-of* in the diagram above satisfies the SIS property
- containment often does not satisfy the SIS property
- Example: the tool box in the trunk of my car is also contained in my car. My car and the trunk of my car are distinct immediate containers for my tool box.

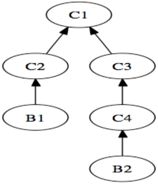
No single immediate predecessor



- *component-of* in the diagram above satisfies the NSIS property

No single immediate predecessor





- *contained-in* in the diagram above does not satisfy the NSIS property

Relations about these properties

- NPO implies SIS;
- if R is finite and has the SIS then it has the NPO;
- if R is up-discrete and NPO then it also has the SWP iff it has the NSIP;
- if R is reflexive then R_i is empty.

Useful R-Structures

R-structure	Property
Partial Ordering (PO)	asymmetric, transitive
Discrete PO	PO + discrete
Parthood structure	PO + WSP + Dense
Component-of structure	PO + WSP + NPO + Discrete

Parthood-containment-component (PCC) structure

R-structure	Property
Partial Ordering (PO)	asymmetric, transitive
Discrete PO	PO + discrete
Parthood structure	PO + WSP + Dense
Component-of structure	PO + WSP + NPO + Discrete

- $(\Delta, \mathbf{PP}, \mathbf{CntIn}, \mathbf{CmpOf})$ is a parthood-containment-component structure iff:
  1. $(\Delta, \mathbf{PP})$ is a parthood structure;
  2. $(\Delta, \mathbf{CntIn})$ is a discrete PO;
  3. $(\Delta, \mathbf{CmpOf})$ is a component-of structure;
  4. if $\mathbf{CntIn}(x, y)$ and $\mathbf{PP}(y, z)$ then $\mathbf{CntIn}(x, z)$
  5. if $\mathbf{PP}(x, y)$ and $\mathbf{CntIn}(y, z)$ then $\mathbf{CntIn}(x, z)$
  6. if $\mathbf{CmpOf}(x, y)$ then $\mathbf{PP}(x, y)$

But, where is logic?

Until now we have performed our analysis using the language of mathematics (that is, identifying the structures defined by our relations.

Formalising a parthood-containment-component structure in Logic

- First order logic with equality (identity);
- $PP, CntIn$, and $CmpOf$ are three predicates whose intended interpretation are the relations **PP**, **CntIn** and **CmpOf** of a parthood-containment-component structure

A theory for PP

Definitions

$$
\begin{array}{ll}
(D_{PP_=}) & PP_=(x,y) \equiv PP(x,y) \vee x = y \\
(D_{PP_O}) & PP_O(x,y) \equiv \exists z.PP_=(z,x) \wedge PP_=(z,y)
\end{array}
$$

Axioms

$$
\begin{array}{lll}
(APP1) & PP(x,y) \supset \neg PP(y,x) & (Asymmetry) \\
(APP2) & (PP(x,y) \wedge PP(y,z)) \supset PP(x,z) & (Transitivity) \\
(APP3) & PP(x,y) \supset \exists z.(PP(z,y) \wedge \neg PP_O(z,x)) & (WSP) \\
(APP4) & PP(x,y) \supset \exists z.(PP(x,z) \wedge PP(z,y)) & (Density)
\end{array}
$$

- The theory that includes $(APP1)$–$(APP3)$ is known as Basic Mereology [Sim87]
- Models of the theory which contain $(APP1)$–$(APP4)$ are part-hood structures.

A theory for *CmpOf*

Definitions

$$
\begin{array}{ll}
(D_{CmpOf_{=}}) & CmpOf_{=}(x,y) \equiv CmpOf(x,y) \lor x = y \\
(D_{CmpOf_{O}}) & CmpOf_{O}(x,y) \equiv \exists z.CmpOf_{=}(z,x) \land CmpOf_{=}(z,y) \\
(D_{CmpOf_{i}}) & CmpOf_{i}(x,y) \equiv CmpOf(x,y) \land (\neg\exists z.(CmpOf(x,z) \land CmpOf(z,y))
\end{array}
$$

Axioms

$$
\begin{array}{lll}
(ACP1) & (CmpOf(x,y) \land CmpOf(y,z)) \supset CmpOf(x,z) & (Transitivity) \\
(ACP2) & CmpOf(x,y) \supset PP(x,y) & (P\ 6\ of\ PCC\ structure) \\
(ACP3) & CmpOf(x,y) \supset (CmpOf_{i}(x,y) \lor & \\
 & \quad (\exists z.CmpOf_{i}(x,z) \land CmpOf(z,y)) \land & \\
 & \quad\quad \exists z.CmpOf(x,z) \land CmpOf_{i}(z,y)) & (\text{Discreteness}) \\
(ACP4) & CmpOf_{O}(x,y) \supset (CmpOf_{=}(x,y) \lor CmpOf(y,x)) & (NPO) \\
(ACP5) & CmpOf_{i}(x,y) \supset (\exists z.CmpOf_{i}(z,y) \land \neg z = x) & (NSIP)
\end{array}
$$

A theory for *CmpOf*

Theorems

$(TCP1)$	$CmpOf(x,y) \supset \neg CmpOf(y,x)$	$(Asymmetry)$
$(TCP2)$	$(CmpOf(x,y) \wedge CmpOf(y,z)) \supset CmpOf(x,z)$	$(Transitivity)$
$(TCP3)$	$CmpOf(x,y) \supset \exists z.(CmpOf(z,y) \wedge \neg CmpOf_O(z,x))$	(WSP)
$(TCP4)$	$CmpOf_i(x,z_1) \wedge CmpOf_i(x,z_2) \supset z_1 = z_2$	$(N2DS)$

Properties inferred as Theorems from $(ACP1)$–$(ACP5)$:

- Asymmetry is derived from $(ACP1)$ and $(ACP2)$;
- Transitivity is derived from $(ACP1)$–$(ACP3)$;
- WSP and N2DS are derived from the entire theory $(ACP1)$–$(ACP5)$

N2DS $=$ No two distinct immediate successors

Axioms for *CmpOf*

Definitions

$(D_{CmpOf_=})$	$CmpOf_=(x,y) \equiv CmpOf(x,y) \lor x = y$
(D_{CmpOf_O})	$CmpOf_O(x,y) \equiv \exists z.CmpOf_=(z,x) \land CmpOf_=(z,y)$
(D_{CmpOf_i})	$CmpOf_i(x,y) \equiv CmpOf(x,y) \land (\neg\exists z.(CmpOf(x,z) \land CmpOf(z,y))$

Axioms

$(ACP1)$	$(CmpOf(x,y) \land CmpOf(y,z)) \supset CmpOf(x,z)$	(*Transitivity*)
$(ACP2)$	$CmpOf(x,y) \supset PP(x,y)$	(*P6 of PCC structure*)
$(ACP3)$	$CmpOf(x,y) \supset (CmpOf_i(x,y) \lor$ $(\exists z.CmpOf_i(x,z) \land CmpOf(z,y)) \land$ $\exists z.CmpOf(x,z) \land CmpOf_i(z,y))$	(Discreteness)
$(ACP4)$	$CmpOf_O(x,y) \supset (CmpOf_=(x,y) \lor CmpOf(z,x))$	(*NPO*)
$(ACP5)$	$CmpOf_i(x,y) \supset (\exists z.CmpOf_i(z,y) \land \neg z = x)$	(*NSIP*)

- Models of the theory which contain $(ACP1)$–$(ACP5)$ are the component-of structures.

Axioms for *CntIn*

Definitions

$$
\begin{array}{ll}
(D_{CntIn_{=}}) & CntIn_{=}(x,y) \equiv CntIn(x,y) \vee x = y \\
(D_{CntIn_{O}}) & CntIn_{O}(x,y) \equiv \exists z.CntIn_{=}(z,x) \wedge CntIn_{=}(z,y) \\
(D_{CntIn_{i}}) & CntIn_{i}(x,y) \equiv CntIn(x,y) \wedge (\neg\exists z.(CntIn(x,z) \wedge v(z,y))
\end{array}
$$

Axioms

$$
\begin{array}{lll}
(ACT1) & CntIn(x,y) \supset \neg CntIn(y,x) & (Asymmetry) \\
(ACT2) & (CntIn(x,y) \wedge CntIn(y,z)) \supset CntIn(x,z) & (Transitivity) \\
(ACT3) & CntIn(x,y) \supset (CntIn_{i}(x,y) \vee & \\
 & \quad (\exists z.CntIn_{i}(x,z) \wedge CntIn(z,y)) \wedge & \\
 & \quad\quad \exists z.CntIn(x,z) \wedge CntIn_{i}(z,y)) & \text{(Discreteness)} \\
(ACT4) & PP(x,y) \wedge CntIn(y,z) \supset CntIn(x,z) & \text{(P4 of PCC structure)} \\
(ACT5) & CntIn(x,y) \wedge PP(y,z) \supset CntIn(x,z) & \text{(P5 of PCC structure)}
\end{array}
$$

- Models of the theory which contain $(ACT1)$–$(ACT5)$ are the component-of structures.

The formal theory

- FO-PCC is the theory containing axioms $(APP1)$–$(APP4)$, $(ACP1)$–$(ACP5)$ and $(ACT1)$–$(ACT5)$;
- Parthood-containment-component structures are models of this theory;
- Via reasoning we can:
  - infer properties on data (e.g., using transitivity);
  - check constraints (e.g., check if all the data comply with asymmetry);
  - check the meaning of terms in ontology integration;

    EXAMPLE: assume that another ontology has a symbol $<<$ *in its terminology. Is this just a rewriting of PP?*
    HINT for solution: Are the logical properties of these two predicates identical?
- Reasoning is nice but reasoning in FOL is undecidable. So?

Expressing FO-PCC in DL

- Description Logics (DLs) are significantly less powerful than first order logic
- but have (relatively) nice computational properties.
- Bittner and Donnelly investigate to what extent and how FO-PCC can be approximated by a theory expressed in a description logic
- They define ad hoc DLs for formulating properties of parthood, componenthood and containment relations.

Expressivity needs

	PP	*CmpOf*	*CntIn*
(def-=)	✓	✓	✓
(def-O)	✓	✓	✓
(def-i)		✓	✓
(Asymmetry)	✓	✓	✓
(Transitivity)	✓	✓	✓
(NPO)		✓	
(WSP)	✓	✓	
(dense)	✓		
(discrete)		✓	✓
(Symmetric)			
(Intransitivity)			
(SIS)		✓	
(NSIP)		✓	

The description logic $\mathcal{L}_{WSP}$: Syntax

Alphabet

The alphabet Σ of $\mathcal{L}_{WSP}$ is composed of:

- Σ_C: Concept names
- Σ_R: Role names (Includes `Id`)
- Σ_I: Individual names

Grammar

Concept	$C := A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \exists R.C \mid \forall R.C \mid = 1R$
Role	$S := R \mid \neg S \mid S_1 \sqcap S_2 \mid S_1 \sqcup S_2 \mid S_1 \circ S_2 \mid S^-$
Definition	$A \doteq C$
Subsumption	$C \sqsubseteq C$
Assertion	$C(a) \mid S(a, b)$

The description logic $\mathcal{L}_{WSP}$: Semantics

Definition

Dl interpretation A DL interpretation $\mathcal{I}$ is pair $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ where:

- $\Delta^{\mathcal{I}}$ is a non empty set called interpretation domain
- $\cdot^{\mathcal{I}}$ is an interpretation function of the alphabet Σ such that
  - $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, every concept name is mapped into a subset of the interpretation domain
  - $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, every role name is mapped into a binary relation on the interpretation domain
  - $o^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ every individual is mapped into an element of the interpretation domain.

The description logic $\mathcal{L}_{WSP}$: Semantics

Interpretation of Concepts

$$
\begin{aligned}
\top^{\mathcal{I}} &= \Delta \\
\bot^{\mathcal{I}} &= \emptyset \\
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(\exists R.C)^{\mathcal{I}} &= \{d \in \Delta^{\mathcal{I}} \mid \text{exists } d', \langle d, d' \rangle \in R^{\mathcal{I}} \text{ implies } d' \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} &= \{d \in \Delta^{\mathcal{I}} \mid \text{forall } d', \langle d, d' \rangle \in R^{\mathcal{I}} \text{ and } d' \in C^{\mathcal{I}}\} \\
(=1R)^{\mathcal{I}} &= \{d \in \Delta^{\mathcal{I}} \mid |\{d' \mid (d, d') \in R^{\mathcal{I}}\}| = 1\}
\end{aligned}
$$

The description logic $\mathcal{L}_{WSP}$: Semantics

Interpretation of Roles

$$
\begin{aligned}
(\neg R)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus R^{\mathcal{I}} \\
(R_1 \sqcap R_2)^{\mathcal{I}} &= R_1^{\mathcal{I}} \cap R_2^{\mathcal{I}} \\
(R_1 \sqcup R_2)^{\mathcal{I}} &= R_1^{\mathcal{I}} \cup R_2^{\mathcal{I}} \\
(R_1 \circ R_2)^{\mathcal{I}} &= \{(d, d') \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid \text{exists } d'' \text{s.t.} \\
&\qquad (d, d'') \in R_1^{\mathcal{I}} \text{ and } (d'', d') \in R_2^{\mathcal{I}}\} \\
(\mathtt{Id})^{\mathcal{I}} &= \{(d, d) in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid d \in \Delta^{\mathcal{I}}\} \\
(R^{-})^{\mathcal{I}} &= \{(d', d) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid (d, d') \in R^{\mathcal{I}}\}
\end{aligned}
$$

Expressing FO-PCC in $\mathcal{L}_{WSP}$

- To express $(APP1)$–$(APP4)$, $(ACP1)$–$(ACP5)$ and $(ACT1)$–$(ACT5)$ in $\mathcal{L}_{WSP}$ we first have to encode the relevant properties of relations

$$
\begin{array}{rl}
(def - i) & R_i \sqsubseteq R \sqcap \neg(R \circ R) \\
(Asymmetry) & R^- \sqsubseteq \neg R \\
(Transitivity) & R \circ R \sqsubseteq R \\
(Intransitivity) & R \circ R \sqsubseteq \neg R \\
(NPO) & R^- \circ R \sqsubseteq R \sqcup \mathtt{Id} \sqcup R^- \\
(WSP) & R^- \sqsubseteq R^- \circ \neg((R^- \sqcup \mathtt{Id}) \circ (R \sqcup \mathtt{Id})) \\
(dense) & (R \sqcap \neg\mathtt{Id}) \sqsubseteq R \circ R \\
(discrete) & R \sqsubseteq R_i \sqcup (R \circ R_i \sqcap R_i \circ R) \\
(SIS) & \exists R_i.\top \sqsubseteq = 1 R_i.\top \\
(NSIP) & = 1 R_i^-.\top \sqsubseteq \bot \\
(Reflexive) & \mathtt{Id} \sqsubseteq R \\
(Symmetric) & R^- \sqsubseteq R \\
(Irreflexive) & \mathtt{Id} \sqsubseteq \neg R
\end{array}
$$

Expressing FO-PCC in $\mathcal{L}_{WSP}$

- By using the encoding in the previous slide we can express in $\mathcal{L}_{WSP}$ all the axioms $(APP1)$–$(APP4)$, $(ACP1)$–$(ACP5)$ and $(ACT1)$–$(ACT5)$ of FO-PCC
- but … $\mathcal{L}_{WSP}$ is undecidable!

. . . And so?

- It is important to identify less complex sub-languages of $\mathcal{L}_{WSP}$ that are still sufficient to state axioms distinguishing parthood, componenthood, and containment relations.
- Otherwise the DL version of FO-PCC would have no computational advantages over the first order theory.

The language $\mathcal{L}$

Remove from $\mathcal{L}_{WSP}$

- Concept expressions:
  - Disjunction $C \sqcup D$;
  - Negation $\neg C$;
- Role expressions:
  - Conjunction $R_1 \sqcap R_2$;
  - Disjunction $R_1 \sqcup R_2$;
  - Negation $\neg R$;
  - Identity `Id`;
  - We restrict role composition so that it only appears in acyclic role terminologies with expressions of the form:

$$R \circ R \sqsubseteq R$$
$$R \circ S \sqsubseteq R$$
$$S \circ R \sqsubseteq R$$

The description logic $\mathcal{L}$

Grammar

Concept	$C := A \mid C \sqcap C \mid \exists R.C \mid \forall R.C \mid = 1R$
Role	$S := R \mid S_1 \circ S_2 \mid S^-$
Definition	$A \doteq C$
Subsumption	$C \sqsubseteq C$
Assertion	$C(a) \mid S(a,b)$

with the restriction to role composition described in the previous slide.

with the usual semantics.

Expressing FO-PCC in $\mathcal{L}$

- $\mathcal{L}$ is decidable ... but what do we lose?
- We can express:
  - Transitivity ($R \circ R \sqsubseteq R$)
  - The relations between *PP*, *CntIn* and *CmpOf*

$$\begin{aligned} \mathit{CmpOf} &\sqsubseteq \mathit{PP} \\ \mathit{PP} \circ \mathit{CntIn} &\sqsubseteq \mathit{CntIn} \\ \mathit{CntIn} \circ \mathit{PP} &\sqsubseteq \mathit{CntIn} \end{aligned}$$

- We cannot express:
  - Asymmetry
  - WSP
  - NPO
  - R_i in terms of R
  - Irreflexivity.

Expressing FO-PCC in $\mathcal{L}$

- We can express:
  - SIS ($\exists R_i.\top \sqsubseteq = 1 R_i.\top$)
  - SISP ($= 1 R_i^-.\top \sqsubseteq \bot$)
- but we cannot express that R_i is a sub-relation of R
- $\mathcal{L}$ is decidable, but we lose too much!

The description logic $\mathcal{L}_{\neg\mathtt{Id}\sqcup}$

Remove from $\mathcal{L}_{WSP}$

- Concept expressions:
  - Disjunction $C \sqcup D$;
  - Negation $\neg C$;
- Role expressions:
  - Disjunction $R_1 \sqcup R_2$;
  - Role Negation $\neg R$ is restricted to role names;
  - Role composition is restricted so that it only appears in acyclic role terminologies with expressions of the form:

$$R \circ R \sqsubseteq R$$
$$R \circ S \sqsubseteq R$$
$$S \circ R \sqsubseteq R$$

The description logic $\mathcal{L}_{\neg\mathtt{Id}\sqcup}$

Grammar

$$
\begin{array}{rl}
\text{Concept} & C := A | C \sqcap C | \exists R.C | \forall R.C | = 1R \\
\text{Role} & S := R | \neg S | S_1 \sqcup S_2 | S_1 \circ S_2 | S^{-} \\
\text{Definition} & A \doteq C \\
\text{Subsumption} & C \sqsubseteq C \\
\text{Assertion} & C(a) | S(a, b)
\end{array}
$$

with the restriction to role composition and role negation described in the previous slide.

with the usual semantics.

Expressing FO-PCC in $\mathcal{L}_{\neg\mathtt{Id}\sqcup}$

- in addition to what we can express in $\mathcal{L}$, we can also express:
    - Irreflexivity ($\mathtt{Id} \sqsubseteq \neg R$)
    - Intransitivity ($R \circ R \sqsubseteq \neg R$)
    - Asymmetry ($R^{-} \sqsubseteq \neg R$)
    - NPO ($R^{-} \circ R \sqsubseteq R \sqcup \mathtt{Id} \sqcup R^{-}$)
- but still no:
    - WSP
    - R_i in terms of R
- whether $\mathcal{L}_{\neg\mathtt{Id}\sqcup}$ is decidable, is still an open problem!

Conclusions

- How to investigate the formal properties of parthood, componenthood and containment relations.
- first order logic has the expressive power required to distinguish important properties of these relations
- DLs seem not appropriate for formulating complex interrelations between relations.
- A way out.
  A computational ontology consists of two components:
  - a DL based ontology that enables automatic reasoning and constrains meaning as much as possible, and
  - a FOL ontology that serves as meta-data and makes explicit properties of relations that cannot be expressed in computationally efficient DLs.

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