C-OWL: Contextualizing Ontologies

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Abstract. Ontologies are *shared* models of a domain that encode a view
which is common to a set of different parties. Contexts are *local* mod-
els that encode a party's subjective view of a domain. In this paper we
show how ontologies can be contextualized, thus acquiring certain useful
properties that a pure shared approach cannot provide. We say that an
ontology is contextualized or, also, that it is a *contextual ontology*, when
its contents are kept local, and therefore not shared with other ontologies,
and mapped with the contents of other ontologies via explicit (context)
mappings. The result is *Context OWL (C-OWL)*, a language whose syn-
tax and semantics have been obtained by extending the OWL syntax
and semantics to allow for the representation of contextual ontologies.

1   Introduction

The aim of the Semantic Web is to make information on the World Wide Web
more accessible using machine-readable meta-data. In this context, the need for
explicit models of semantic information (terminologies and background knowl-
edge) in order to support information exchange has been widely acknowledged by
the research community. Several different ways of describing information seman-
tics have been proposed and used in applications. However we can distinguish
two broad approaches which follow somehow opposite directions:

Ontologies are *shared* models of some domain that encode a view which is
   common to a set of different parties [15];
Contexts are *local* (where *local* is intended here to imply *not shared*) models
   that encode a party's view of a domain [11, 12, 9].

Thus, ontologies are best used in applications where the core problem is the
use and management of common representations. Many applications have been
developed, for instance in bio-informatics [7], or for knowledge management pur-
poses inside organizations [6]. Contexts, instead, are best used in those applica-
tions where the core problem is the use and management of local and autonomous
representations with a need for a limited and controlled form of globalization
(or, using the terminology used in the context literature, maintaining *locality*

still guaranteeing semantic *compatibility* among representations [9]). Examples of uses of contexts are the classifications of documents [4], distributed knowledge management [1], the development and integration of catalogs [8, 2], peer-to-peer applications with a large degree of *autonomy* of the peer nodes but still with a strong need of *coordination* [13, 16] (with autonomy and coordination being the behavioral counterpart of the semantic need of locality and compatibility).

Contexts and ontologies have both strengths and weaknesses. It can be argued that the strengths of ontologies are the weaknesses of contexts and vice versa. On the one hand, the use of ontologies enables the parties to communicate and exchange information. Shared ontologies define a common understanding of specific terms, and thus make it possible to communicate between systems on a semantic level. On the weak side, ontologies can be used only as long as consensus about their contents is reached. Furthermore, building and maintaining (!) them may become arbitrarily hard, in particular in a very dynamic, open and distributed domain like the Web. On the other hand, contexts encode not shared interpretation schemas of individuals or groups of individuals. Contexts are easier to define and to maintain. They can be constructed with no consensus with the other parties, or only with the limited consensus which makes it possible to achieve the desired level of communication and only with the "relevant" parties. On the weak side, since contexts are local to parties, communication can be achieved only by constructing explicit mappings among the elements of the contexts of the involved parties; and extending the communication to new topics and/or new parties requires the explicit definition of new mappings.

Depending on their attitude, from an epistemological point of view, some people would argue that ontologies are all we need, while others would argue the exact contrary, namely that contexts are all we need. Our attitude in this paper is quite pragmatical. We believe that ontologies and contexts have both some advantages and that, therefore, they should be integrated in the representational infrastructure of the Semantic Web. Thus, on the one hand, the intended meaning of terms provided by parties which are willing to share information can be more easily captured with an ontology (or a set of shared ontologies). On the other hand, multiple ontologies (or sets or shared ontologies) which contain information which *should not* be integrated (an obvious example being information which is mutually inconsistent) should be contextualized. We say that an ontology is contextualized, or that it is a *contextual ontology*, when its contents are kept local (and therefore not shared with other ontologies) and are put in relation with the contents of other ontologies via explicit mappings.

Our approach in this paper is as follows. We take the notion of ontology as the core representation mechanism for representing information semantics. To this end, we start from the standard Web ontology language OWL [14]. Notice that from OWL we inherit the possibility to have shared ontologies. We show, providing some motivating examples, that OWL cannot model certain situations (Section 4). Finally, we provide an extension of OWL, that we call *Context OWL* (*C-OWL*), which allows us to deal with all the examples of Section 4. C-OWL

integrates in a uniform way the, somehow orthogonal, key architectural features of contexts and ontologies and the consequent semantic level differences.

The main technical contributions of this paper are the following:

1. We provide a (somewhat synthetic) description of OWL and its semantics, restating Patel-Schneider and Hayes' semantics [15], in a formal framework more adequate to be extended (adapted) with a contextualized interpretation. These are the contents of Section 3.
2. We modify the OWL semantics to make it able to deal with the motivating examples reported in Section 4. These are the contents of Section 5.
3. We define the C-OWL syntax by taking the OWL syntax and by adding *bridge rules*, which allow to relate, at the syntactic and at the semantic level, concepts, roles and individuals in different ontologies. We call a set of bridge rules between two ontologies a *context mapping*. Thus a *contextual ontology* is an OWL ontology embedded in a space of other OWL ontologies and related to them via context mappings. We define the C-OWL semantics by taking the modified OWL semantics, as defined in Section 5. These are the contents of Section 6.

The semantics of C-OWL is obtained by modifying the OWL semantics [15] using the ideas and notions originally developed in the semantics of context (the, so called, Local Models Semantics [9, 10]). Bridge rules were originally defined in [11] and further studied in [9, 10, 4, 3]. The bridge rules proposed in this paper were first defined in [5]. Finally the constructs for representing bridge rules have been taken from the context markup language CTxML [4].

2 Ontologies vs. Contexts, or globalize vs. localize

At the architectural level, the crucial difference between the notions of context and ontology is in how mappings among multiple models are constructed:

- in OWL, mappings are not part of the language. The ability of combining models is restricted to the import of complete models and to the use of the imported elements by direct reference. Via the import mechanism, a set of local models is *globalized* in a unique shared model (which, however, keeps track of the original distinctions). It is assumed that references to external statements are only made for statements from imported models, however, this is strictly speaking not required. As a consequence, mappings rather implicitly exist in terms of mutual use of statements across models. Further, there are two ways of treating external statements: we can either treat the referred statement as a single fact with no further implications that the one it directly encodes, or we can use the complete model containing the fact as additional knowledge. This latter view is the one adopted in OWL, and it is the one we will consider in the following.
- in context-based approaches, local models are kept *localized*. A limited and completely controlled form of globalization is obtained by using explicit mappings. In this approach, mappings are regarded as projections of a local

representation onto another, and are first class modelling elements with a unique identity. In other words, also mappings are viewed as part of a local representation. This view makes it possible to have multiple alternative mappings between the same pair of contexts, and to define mappings in one direction that differ from the mappings in the opposite direction.

This different bias towards localization/globalization, and the consequent very different treatment of mappings lead to important semantic differences. OWL is mainly inspired by the Tarskian style semantics of propositional description logics. A model theoretic semantics is provided by mapping the elements of existing models into an abstract domain, where concepts are represented by sets, relation by sets of tuples and instances by elements of that domain. When reasoning is performed across different models, then these models are assumed to share the interpretation domain. Thus, as a consequence, the mappings between two models become part of the overall model and define constraints on the elements of the original two models.

The situation is quite different when we move to contexts. In the Local Models Semantics, each context uses a local set of models and a local domain of interpretation. Relations between these local interpretation domains are established by *domain relations* which explicitly codify how elements in one domain map into elements of the other domain. Domain relations are indexed by source and target domain, making them irreversible and non-transitive; and bridge rules modify only the target context, leaving the source unaffected.

3   A global semantics for OWL

According to [15], an OWL ontology is a set of annotated *axioms* and *facts*, plus import references to other ontologies. OWL ontologies can be referenced by means of a URI. Ontologies can also have annotations that can be used to record authorship and other information associated with an ontology. Since annotation directives have no effect on the semantics of OWL ontologies in the abstract syntax, we ignore them. We concentrate on the OWL-DL fragment of OWL. This language is equivalent to the SHOIQ(D+) DL, i.e., SHIQ(D+) extended with an equivalent of the oneOf constructor. The proposed framework can be restricted or generalized to OWL-lite and OWL-full, respectively.

Let I be a set of indexes, standing for a set of URI's of onotlogies. For instance I contains `http://www.w3.org/2002/07/owl`. Let also $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{O}$ be the sets of strings that can used to denote concepts, roles and individuals respectively. The disjoint union of $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{O}$ is denoted with $\mathbb{L}$.

Definition 1 (OWL Ontology). *An* OWL Ontology *(or simply an* ontology*) is a pair $\langle i, O_i \rangle$, where $i \in I$ and $O_i = \langle T_i, A_i \rangle$ where T and A are a T-box and an A-box respectively in the SHOIQ(D+) description logic on $\mathbb{L} \cup (I \times \mathbb{L})$. $\langle i, O_i \rangle$ is an ontology with index i.*

Suppose that $C, D, E, F \in \mathbb{C}$ and $r, s \in \mathbb{R}$. The following are examples of concepts that can appear in O_i.

$$C, \quad i{:}C, \quad C \sqcap D, \quad j{:}E, \quad C \sqcap (j{:}E), \quad \exists r.C \sqcup D, \quad \exists(j{:}s).C \sqcup (j{:}F) \quad (1)$$

Every expression occurring in O_i without an index is intended to be in the language defined by O_i, L_i. The expressions appearing in O_i with indexes j are supposed to be defined in O_j; therefore they appear in O_j without index. We introduce the notions of *local language* and *foreign language*.

Definition 2 (Local language). *A local concept, w.r.t. i, is an element of $\mathbb{C}$ that appears in O_i either without indexes or with index equal to i.* Local roles *and* local individuals *are defined analogously. The set of local concepts, local roles, and local individuals w.r.t. i are denoted by $\mathbb{C}_i$, $\mathbb{R}_i$, and $\mathbb{O}_i$. The* local language *to i, $\mathbb{L}_i$, is the disjoint union of them.*

Local objects of a language L_i are also called *i-objects*. For notational convenience, in the following we always use the colon notation. Thus, for instance, that local concepts $C \in \mathbb{C}_i$ of an ontology O_i are written as $i : C$. A *foreign concept*, or equivalently a *non local concept*, w.r.t. $i \in I$, is a concept that appears in O_i but is defined in some ontology O_j. Foreign concepts are referred with the notation $j{:}c$. An analogous definition can be given for roles and individuals.

Definition 3 (Foreign language). *For any $j \neq i$, a j-foreign concept w.r.t. i is an element of $\mathbb{C}$ that appears in O_i with index j. j-foreign roles and j-foreign individuals are defined analogously. The j-foreign language w.r.t. i is the disjoint union of them.*

Among the concepts described in (1), C and D are local concepts w.r.t. i and r is a local role (w.r.t. i), while E and is a j-foreign concept and s is a j-foreign role. By means of foreign concepts, roles and individuals, two ontologies can refer to the *same* semantic object defined in a third ontology.

Definition 4 (OWL space). *An OWL space is a family of ontologies $\{\langle i, O_i \rangle\}_{i \in I}$ such that every O_i is an ontology, and for each $i \neq j$, the j-foreign language of O_i is contained in the local language of O_j.*

Moving to semantics, the idea is now to restate the semantics in [15] making explicit reference to the notions of local and foreign language. This distinction, crucial for the work developed in the next section, is not made in [15].

The semantics for OWL spaces defined in [15] is based on the intuition that, in OWL, as in RDF, a data type denotes the set of data values that is the value space for the data type. Concepts denote sets of individuals. Properties relate individuals to other information, and are divided into two disjoint groups, data-valued properties and individual-valued properties. Data-valued properties relate individuals to data values; individual-valued properties relate individuals to other individuals.

In the following we assume that any domain we introduce (denoted by Δ possibly with indexes) contains the union of the value spaces of the OWL data types and Unicode strings.

Definition 5 (OWL interpretation [15]). *An* OWL interpretation *for the OWL space* $\{\langle i, O_i \rangle\}_{i \in I}$, *is a pair* $\mathcal{I} = \langle \Delta^{\mathcal{I}}, (.)^{\mathcal{I}} \rangle$, *where* $\Delta^{\mathcal{I}}$, *contains a non-empty set of objects (the resources) and* $(.)^{\mathcal{I}}$ *is a function such that*

1. $\mathcal{I}(i, C) \subseteq \Delta^{\mathcal{I}}$ *for any* $i \in I$ *and* $C \in \mathbb{C}_i$;
2. $\mathcal{I}(i, r) \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ *for any* $i \in I$ *and* $r \in \mathbb{R}_i$;
3. $\mathcal{I}(i, o) \in D^{\mathcal{I}}$ *for any* $i \in I$ *and* $o \in \mathbb{O}_i$;

Notice that $(.)^{\mathcal{I}}$ can be extended to all the complex descriptions of SHIQ(D+) as usual. Statements contained in the A-box and the T-box (i.e., facts and axioms) of an ontology O of an OWL space $\{\langle i, O_i \rangle\}_{i \in I}$ can be verified/falsified by an interpretation according the axioms written in [15].

We call the above interpretation, a *global interpretation*, to emphasize the fact that language is interpreted against a global domain. We call the overall approach, the *global semantics* approach to OWL.

Definition 6 (OWL axiom and fact satisfiability [15]). *Given an* OWL interpretation $\mathcal{I}$ *for* $\{\langle i, O_i \rangle\}_{i \in I}$, $\mathcal{I}$ *satisfies a fact or an axiom* ϕ *of the* O_i *according to the rules defined in the table "Interpretation of Axioms and Facts" of* [15]. *An OWL interpretation* $\mathcal{I}$ *satisfies an OWL space* $\{\langle i, O_i \rangle\}_{i \in I}$, *if* $\mathcal{I}$ *satisfies each axiom and fact of* O_i, *for any* i.

Notice that we do not give any interpretation of the possibility for O_i to import another ontology O_j. However, from the logical point of view, importing O_j into O_i can be thought of as duplicating all the statements of O_j in O_i.

4 Motivating Examples

We provide some examples which cannot be represented with the current syntax and semantics of OWL. These examples show the need to enrich ontologies with the capability to cope with:

1. *the directionality of information flow*: we need to keep track of the source and the target ontology of a specific piece of information;
2. *Local domains*: we need to give up the hypothesis that all ontologies are interpreted in a single global domain;
3. *Context mappings*: we need to be able to state that two elements (concepts, roles, individuals) of two ontologies, though being (extensionally) different, are *contextually* related, for instance because they both refer to the same object in the world.

Example 1 (Directionality). Consider two ontologies O_1 and O_2 and suppose that O_2 is an extension of O_1, i.e., O_2 imports O_1 and adds it some new axiom. Directionality is fulfilled if the axioms added to O_2 should not affect what is stated in O_1. Consider the case where O_1 contains the axioms $A \sqsubseteq B$ and $C \sqsubseteq D$; furthermore, suppose that O_2 contains the axiom $B \sqsubseteq C$. We would like to derive $A \sqsubseteq D$ in O_2 but not in O_1.

Let us see how the global semantics behaves in this case. Let $\{\langle 1, O_1\rangle, \langle 2, O_2\rangle\}$ be the OWL space containing O_1 and O_2. Let $A, B, C,$ and B be 1 local concepts. Suppose that O_1 contains the axioms $A \sqsubseteq B$ and $C \sqsubseteq D$. Suppose that O_2 imports O_1, this implies that O_2 contains $1\!:\!A \sqsubseteq 1\!:\!B$ and $1\!:\!C \sqsubseteq 1\!:\!D$. Finally suppose that O_2 contains the extra axiom $1 : B \sqsubseteq 1 : C$. We have that any interpretation of $\{\langle 1, O_1\rangle, \langle 2, O_2\rangle\}$, should be such that $(1\!:\!A)^{\mathcal{I}} \subseteq (1\!:\!B)^{\mathcal{I}} \subseteq (1\!:\!C)^{\mathcal{I}} \subseteq (1\!:\!D)^{\mathcal{I}}$; and therefore $(1\!:\!A)^{\mathcal{I}} \subseteq (1\!:\!D)^{\mathcal{I}}$. This means that $1\!:\!A \sqsubseteq 1\!:\!D$ is a logical consequence of the statements contained in the OWL space and, therefore, that directionality is not fulfilled.

Example 2 (A special form of directionality: the propagation of inconsistency). Consider the previous example and suppose that O_2 contains also the following two facts: $1 : A(a)$ and $1 : \neg D(a)$. O_2 is inconsistent, but we want to avoid the propagation of inconsistency to O_1. However, this is not possible as the fact that there is no interpretation that satisfies the axioms in O_2, automatically implies that there is no interpretation for the whole OWL space, either.

Example 3 (Local domains). Consider the ontology O_{WCM} of a worldwide organization on car manufacturing. Suppose that O_{WCM} contains the "standard" description of a car with its components. The domain of interpretation of this ontology is the totality of cars with their components. Clearly such a domain should be abstract enough so that it has not be changed whenever a new car appears on the earth. For instance, the extension of the concept car in this ontology will not be the set of actual physical cars in circulation, rather it will be some set of abstract objects for generic cars. Suppose also that this ontology contains two individual constants Diesel and Petrol for Diesel engine and petrol engine, and an axiom stating that a car has only one engine which is either Diesel or petrol, and that these two engines are different.

$$\mathsf{Car} \sqsubseteq (\exists 1)\mathsf{hasEngine}.\{\mathsf{Diesel}, \mathsf{Petrol}\} \tag{2}$$
$$\mathsf{Diesel} \neq \mathsf{Petrol} \tag{3}$$

A car manufacturing company, say Ferrari, decides to adopt the WCM standard and imports it in its ontology, O_{Ferrari}, used to describe Ferrari's production. Since Ferrari produces many petrol engines, a set of local constants for engines are added, eg. F23 and F34i, as well as the axiom, stating that the engine of a Ferrari is either an F23 or an F34i

$$\mathsf{Ferrari} \sqsubseteq (\mathsf{WCM}\!:\!\mathsf{car} \sqcap (\exists 1)(\mathsf{WCM}\!:\!\mathsf{hasEngine}).\{\mathsf{F23}, \mathsf{F23}\}) \tag{4}$$
$$\mathsf{F23} \neq \mathsf{F34i} \tag{5}$$

According to the global semantics we have that any interpretation of the OWL space containing O_{WCM} and O_{Ferrari} is such that, either $(\mathsf{F23})^{\mathcal{I}_{\mathsf{Ferrari}}} = (\mathsf{Diesel})^{\mathcal{I}_{\mathsf{WCM}}}$ or $(\mathsf{F34i})^{\mathcal{I}_{\mathsf{Ferrari}}} = (\mathsf{Diesel})^{\mathcal{I}_{\mathsf{WCM}}}$, which is not what we want as Ferrari produces only petrol engines. The main problem here is the diversity of the domains between O_{Ferrari} and O_{WCM}. Indeed since in O_{WCM} it is not possible to list all the existing engines, this domain is abstracted to two abstract objects called Petrol

and Diesel. In O_{Ferrari}, instead, this enumeration is possible and, therefore, the domain is more specific. Ultimately, the two domains are at two different level of abstraction and cannot be merged in a single one.

Example 4 (Context mappings). Suppose we have an ontology O_{FIAT} describing cars from a manufacturing point of view, and a completely independent ontology O_{Sale} describing cars from a car vendor point of view. The two concepts of car defined in the two ontologies, (that can be referred by $\mathsf{Sale\!:\!Car}$ and $\mathsf{FIAT\!:\!Car}$) are very different and it makes no sense for either ontology to import the concept of car from the other. The two concepts are not extensionally equivalent and the instances of $\mathsf{FIAT\!:\!Car}$ do not belong to $\mathsf{Sale\!:\!Car}$ and vice-versa. On the other hand the two concepts describe the same real-world class of objects from two different points of view, and there can be many reasons for wanting to integrate this information. For instance one might need to build a new concept which contains (some of) the information in $\mathsf{Sale\!:\!Car}$ and in $\mathsf{FIAT\!:\!Car}$. This connection cannot be stated via OWL axioms, as, for instance

$$\mathsf{Sale\!:\!Car} \equiv \mathsf{FIAT\!:\!Car}$$

implies that

$$\mathsf{Car}^{\mathcal{I}_{\mathsf{Sale}}} = \mathsf{Car}^{\mathcal{I}_{\mathsf{FIAT}}}$$

i.e., that the two classes coincide at the instance level.

In this example, the problem is not only at the semantic level. As the following section will show, handling this example requires an extension of the OWL syntax.

5   A semantics for contextual ontologies

In this section we incrementally extend/modify the OWL global semantics, and in the last subsection, also its syntax, in order to be able to model the above examples.

5.1   Directionality

We modify the definition of interpretation given above according to the intuition described in [3]. The main idea is that we split a global interpretation into a family of (local) interpretations, one for each ontology. Furthermore, we allow for an ontology to be locally inconsistent, i.e., not to have a local interpretation. In this case we associate to O_i a special "interpretation" $\mathcal{H}$, called a *hole*, that verifies any set of axioms, possibly contradictory.

Definition 7 (Hole). *A Hole is a pair $\langle \Delta^{\mathcal{H}}, (.)^{\mathcal{H}} \rangle$, such that $\Delta^{\mathcal{H}}$ is a nonempty set and $(.)^{\mathcal{H}}$ is a function that maps every constant of $\mathbb{O}_i$ into an element of $\Delta^{\mathcal{H}}$, every concept of $\mathbb{C}_i$ in the whole $\Delta^{\mathcal{H}}$ and every role of $\mathbb{R}_i$ into the set $\Delta^{\mathcal{H}} \times \Delta^{\mathcal{H}}$. $\mathcal{H}$ is called a hole on Δ^H.*

Analogously to what done in [3], the function $(.)^{\mathcal{H}}$ can be extended to complex descriptions and complex roles in the obvious way.

Definition 8 (Satisfiability in a hole). $\mathcal{H}$ *satisfies all the axioms and facts, i.e., if ϕ is an axiom or a fact, $\mathcal{H} \models \phi$.*

Definition 9 (OWL interpretation with holes). *An* OWL interpretation with holes *for the OWL space $\{\langle i, O_i \rangle\}_{i \in I}$, is a family $\mathcal{I} = \{\mathcal{I}_i\}_{i \in I}$, where each $\mathcal{I}_i = \langle \Delta^{\mathcal{I}_i}, (.)^{\mathcal{I}_i} \rangle$, called the* local interpretation *of O_i, is either an interpretation of L_i on $\Delta^{\mathcal{I}_i}$, or it is a hole for L_i on $\Delta^{\mathcal{I}_i}$, and for all $i \in I$, each $\Delta^{\mathcal{I}_i}$ coincides an are equal to a set denoted by $\Delta^{\mathcal{I}}$.*

Each $(.)^{\mathcal{I}_i}$ can be extended in the usual way to interpret local descriptions. Foreign descriptions are interpreted by the combination of the different $(.)^{\mathcal{I}_i}$ for each $i \in I$. In particular for any concept, role or individual of the alphabet L_j, $(.)^{\mathcal{I}_i}$ can be extended to be the same as $(.)^{\mathcal{I}_j}$. Namely:

$$(j{:}x)^{\mathcal{I}_i} = (x)^{\mathcal{I}_j} \tag{6}$$

which can intuitively be read as, "the meaning of the j-foreign concept $j : x$ occurring in O_i is the same as the meaning of x occurring in O_j". Since all interpretations share the same domain, this semantics is well founded. Namely, the interpretation of j-foreign concepts in i are contained in the domain of i, $\Delta^{\mathcal{I}_i}$. In the following we give some examples of $(.)^{\mathcal{I}_i}$, for which we suppose that $C, D \in \mathbb{C}_i$ and $r \in \mathbb{R}_i$ and $D, F \in \mathbb{C}_j$ and $s \in \mathbb{R}_j$.

$$C^{\mathcal{I}_i} = \begin{cases} \text{Any subset of } \Delta^{\mathcal{I}_i} \text{ if } \mathcal{I}_i \neq \mathcal{H}_i \\ \Delta^{\mathcal{I}} \text{ otherwise} \end{cases}$$

$$(C \sqcap D)^{\mathcal{I}_i} = \begin{cases} (C)^{\mathcal{I}_i} \cap (D)^{\mathcal{I}_i} \text{If } \mathcal{I}_i \neq \mathcal{H}_i \\ \Delta^{\mathcal{I}} \text{ otherwise} \end{cases}$$

$$(j : E)^{\mathcal{I}_i} = \begin{cases} (E)^{\mathcal{I}_j} \text{If } \mathcal{I}_i \neq \mathcal{H}_i \\ \Delta^{\mathcal{I}} \text{ otherwise} \end{cases}$$

$$(C \cap j : E)^{\mathcal{I}_i} = \begin{cases} (C)^{\mathcal{I}_i} \cap (E)^{\mathcal{I}_j} \text{If } \mathcal{I}_i \neq \mathcal{H}_i \\ \Delta^{\mathcal{I}} \text{ otherwise} \end{cases}$$

Definition 10 (Axiom satisfiability). *Given an* OWL interpretation with holes, $\mathcal{I}$ *for $\{\langle i, O_i \rangle\}_{i \in I}$, $\mathcal{I}$ satisfies a fact or an axiom ϕ of the O_i, in symbols $\mathcal{I} \models i : \phi$ if $\mathcal{I}_i \models \phi$. An* OWL interpretation $\mathcal{I}$ *satisfies an* OWL space $\{\langle i, O_i \rangle\}_{i \in I}$, *if $\mathcal{I}$ satisfies each axiom and fact of O_i for each i.*

Notice that interpretations with hole can behave differently in different ontologies. Thus, for instance, the same axiom can be satisfied in an ontology and not satisfied in another. Consider for instance the OWL interpretation with holes $\{\mathcal{I}_1, \mathcal{I}_2, \mathcal{H}_3\}$, where $\mathcal{I}_1$ and $\mathcal{I}_2$ are not holes. Suppose that $(A)^{\mathcal{I}_1} \not\subseteq (B)^{\mathcal{I}_2}$. Then we have that $1{:}A \sqsubseteq 1{:}B$ is not satisfied if it occurs in O_2, while it is satisfied if it occurs in O_3.

If every $\mathcal{I}_i$ is not a hole, the interpretation given of Definition 9 coincides with the global interpretation, as defined in Section 3. Let us analyze what happens when some $\mathcal{I}_i$ is a hole.

Example 5 (Examples 1 and 2 formalized). Consider the OWL interpretation with holes, $\mathcal{I} = \{\mathcal{I}_1, \mathcal{I}_2\}$ defined as follows

1. $\Delta^{\mathcal{I}_1} = \{a, b, c, d\}$, $A^{\mathcal{I}_1} = \{a\}$, $B^{\mathcal{I}_1} = \{a, b\}$, $C^{\mathcal{I}_1} = \{c\}$, $D^{\mathcal{I}_1} = \{c, d\}$,
2. $\Delta^{\mathcal{I}_2} = \{a, b, c, d\}$, and $\mathcal{I}_2 = \mathcal{H}_2$, i.e. $\mathcal{I}_2$ is a hole.

$\mathcal{I}$ is an interpretation for the OWL space containing O_1 and O_2, since

1. $\mathcal{I}_1 \models A \sqsubseteq B$, $\mathcal{I}_1 \models C \sqsubseteq D$, and $\mathcal{I}_1 \not\models A \sqsubseteq D$, by construction of $\mathcal{I}_1$,
2. $\mathcal{I}_2 \models 1{:}A \sqsubseteq 1{:}B$, $\mathcal{I}_2 \models 1{:}B \sqsubseteq 1{:}C$, and $\mathcal{I}_2 \models 1{:}C \sqsubseteq 1{:}D$, because $\mathcal{I}_2$ is a hole.

Notice that $\mathcal{I}$ is an interpretation that satisfies O_2 (i.e., $1{:}A \sqsubseteq 1{:}B$ $1{:}B \sqsubseteq 1{:}C$, and $1{:}C \sqsubseteq 1{:}D)$], without making $A \sqsubseteq D$ true in O_1.

To formalize Example 2, we consider the same interpretation as above. This interpretation satisfies any axiom in O_2 ($\mathcal{I}_2$ is a hole) still keeping O_1 consistent ($\mathcal{I}_1$ is an interpretation which is not a hole and which satisfies O_1).

5.2 Local domains

The OWL semantics described in the previous section assumes the existence of a unique shared domain, namely, that each ontology describes the properties of the whole universe. In many cases this is not true as, for instance, an ontology on cars is not supposed to speak about medicines, or food. The idea here is to associate to each ontology a *local domain*. Local domains may overlap as we have to cope with the case where two ontologies refer to the same object.

Definition 11 (OWL interpretation with local domains). *An* OWL interpretation with local domains *for the OWL space* $\{\langle i, O_i \rangle\}_{i \in I}$*, is a family* $\mathcal{I} = \{\mathcal{I}_i\}_{i \in I}$*, where each* $\mathcal{I}_i = \langle \Delta^{\mathcal{I}_i}, (.)^{\mathcal{I}_i} \rangle$*, called the* local interpretation *of* O_i*, is either an interpretation of* L_i *on* $\Delta^{\mathcal{I}_i}$*, or a hole.*

Definition 11 is obtained from Definition 9 simply by dropping the restriction on domain equality. The interpretation $(.)^{\mathcal{I}_i}$ is extended to complex concepts, roles, and individuals, in the usual way. We have to take care, however, that j-foreign concepts, roles, and individuals used in O_i could be interpreted (by the local interpretation $\mathcal{I}_j$) in a (set of) object(s) which are not in the local domain $\Delta^{\mathcal{I}_i}$. Indeed, to deal with this problem, we have to impose that any expression occurring in O_i should be interpretable in the local domain $\Delta^{\mathcal{I}_i}$. As a consequence, we restrict the interpretation of any foreign concept $C \in \mathbb{C}_j$, any foreign role $r \in \mathbb{R}_j$ and any foreign individual $a \in \mathbb{O}_j$ as follows:

1. $(j{:}C)^{\mathcal{I}_i} = (C)^{\mathcal{I}_j} \cap \Delta^{\mathcal{I}_i}$
2. $(j{:}r)^{\mathcal{I}_i} = (r)^{\mathcal{I}_j} \cap (\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}})$

3. $(j\!:\!a)^{\mathcal{I}_i} = (a)^{\mathcal{I}_j}$

Notice that point 3 above implicitly imposes that if a j-foreign constant $j\!:\!a$ is used in the ontology O_i, then its interpretation in j, i.e., $a^{\mathcal{I}_j}$, must be contained in the domain $\Delta^{\mathcal{I}_i}$. Let us now see how we can deal with Example 3.

Example 6 (Example 3 formalized). Consider the OWL interpretation with local domains, $\mathcal{I} = \{\mathcal{I}_{\mathsf{WCM}}, \mathcal{I}_{\mathsf{Ferrari}}\}$ for the OWL space containing O_{WCM} and O_{Ferrari}. Suppose that Δ_{WCM} contains two constants d_1 and d_2, which are the interpretations of petrol and diesel, respectively. Suppose that $\Delta_{\mathsf{Ferrari}}$ contains two constants d_1 and d_3 which are the interpretations of $\mathsf{F23}$ and $\mathsf{F34i}$, respectively. Suppose also that $(\mathsf{hasAngine})^{\mathcal{I}_{\mathsf{WCM}}} = \{\langle c_1, d_1 \rangle, \langle c_2, d_2 \rangle\}$, where c_1 and c_2 are object for cars in both Δ_{WCM} and $\Delta_{\mathsf{Ferrari}}$. If can be verified that the axioms (2–5) are verified by the interpretation $\mathcal{I}$ and that it is the case that no Ferrari engine is a diesel.

5.3   Context mappings

We have concepts, roles and individuals local to different ontologies and domains of interpretation. A context mapping allows us to state that a certain property holds between elements of two different ontologies. Thus, for instance, in Example 4, one possible mapping could allow us to say that the class Car in the ontology O_{FIAT} contains the same cars as (or, as we say, is contextually equivalent to) the class of Car defined in the ontology O_{Sale}. As from Example 4 this cannot be done via local axioms within an ontology.

The basic notion towards the definition of context mappings are *bridge rules*.

Definition 12 (Bridge rules). *A* bridge rule from i to j is a statement of one of the four following forms,

$$i\!:\!x \xrightarrow{\sqsubseteq} j\!:\!y, \quad i\!:\!x \xrightarrow{\sqsupseteq} j\!:\!y, \quad i\!:\!x \xrightarrow{\equiv} j\!:\!y, \quad i\!:\!x \xrightarrow{\perp} j\!:\!y, \quad i\!:\!x \xrightarrow{*} j\!:\!y,$$

where x and y are either concepts, or individuals, or roles of the languages L_i and L_j respectively.

A mapping between two ontologies is a set of bridge rules between them.

Definition 13 (Mapping). *Given a OWL space* $\{\langle i, O_i \rangle\}_{i \in I}$ *a mapping* M_{ij} *from* O_i *to* O_j *is a set of bridge rules from* O_i *to* O_j, *for some* $i, j \in I$.

Mappings are directional, i.e., M_{ij} is not the inverse of M_{ji}. A mapping M_{ij} might be empty. This represents the impossibility for O_j to interpret any i-foreign concept into some local concept. Dually M_{ij} might be a set of bridge rules of the form $i\!:\!x \xrightarrow{\equiv} j\!:\!y$ for any element x (concept, role, and individual) of O_i. This represents the operation of mapping all of O_i into an equivalent subset of O_j. If this subset is O_j itself then this becomes the contextual mapping version of the OWL import operation. However, notice that importing O_i into O_j is not the same as mapping O_i to O_j with M_{ij}. In both cases information goes from i

to j. The difference is that, in the former case, O_j duplicates the information of i- foreign elements without any change, while, in the latter, O_j translates (via the mapping M_{ij}) the semantics of O_i into its internal (local) semantics.

Definition 14 (Context space). *A context space is a pair composed of an OWL space $\{\langle i, O_i\rangle\}_{i\in I}$ and a family $\{M_{ij}\}_{i,j\in I}$ of mappings from i to j, for each pair $i, j \in I$.*

To give the semantics of context mappings we extend the definition of OWL interpretation with local domains with the notion of *domain relation*. A domain relation $r_{ij} \subseteq \Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$ states, for each element in $\Delta^{\mathcal{I}_i}$ to which element in $\Delta^{\mathcal{I}_j}$ it corresponds to. The semantics for bridge rules from i to j can then be given with respect to r_{ij}.

Definition 15 (Interpretation for context spaces). *An interpretation for a context space $\langle \{\langle i, O_i\rangle\}_{i\in I}, \{M_{ij}\}_{ij\in I}\rangle$ is composed of a pair $\langle \mathcal{I}, \{r_{ij}\}_{i,j\in I}\rangle$. where $\mathcal{I}$ is an OWL interpretation with holes and local domains of $\{\langle i, O_i\rangle\}_{i\in I}$ and r_{ij}, the domain relation from i to j, is a subset of $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$.*

Definition 16 (Satisfiability of bridge rules[1]).

1. $\mathfrak{I} \models i{:}x \xrightarrow{\sqsubseteq} j{:}y$ *if* $r_{ij}(x^{\mathcal{I}_i}) \subseteq y^{\mathcal{I}_j}$;
2. $\mathfrak{I} \models i{:}x \xrightarrow{\sqsupseteq} j{:}y$ *if* $r_{ij}(x^{\mathcal{I}_i}) \supseteq y^{\mathcal{I}_j}$;
3. $\mathfrak{I} \models i{:}x \xrightarrow{\equiv} j{:}y$ *if* $r_{ij}(x^{\mathcal{I}_i}) = y^{\mathcal{I}_j}$;
4. $\mathfrak{I} \models i{:}x \xrightarrow{\perp} j{:}y$ *if* $r_{ij}(x^{\mathcal{I}_i}) \cap y^{\mathcal{I}_j} = \emptyset$;
5. $\mathfrak{I} \models i{:}x \xrightarrow{*} j{:}y$ $r_{ij}(x^{\mathcal{I}_i}) \cap y^{\mathcal{I}_j} \neq \emptyset$;

A interpretation for a context space is a model for it if all the bridge rules are satisfied.

When x and y are concepts, say C and D, the intuitive reading of $i{:}C \xrightarrow{\sqsubseteq} j{:}D$, is that the i-local concept C is more specific than the j-concept D. An analogous reading can be given to $i : C \xrightarrow{\sqsupseteq} j : D$. The intuitive reading of $i : C \xrightarrow{\perp} j : D$ is that C is disjoint from D. Finally, the intuitive reading of $i : C \xrightarrow{*} j : D$ is that C and D are two concepts which are compatible. When x and y are individuals, then $i : x \xrightarrow{\sqsubseteq} j : y$ states that y is a more abstract representation of the object represented by x in i (intuitively, there might be more than one x's corresponding to the same y) Vive-versa $i{:}x \xrightarrow{\sqsupseteq} j{:}y$ states that y is a less abstract (more concrete) representation of the object represented by x in i (intuitively there might be more than one y's corresponding to the same x). $i{:}x \xrightarrow{\equiv} j{:}y$ states that x and y are at the same level of abstraction. Notice that, we add $i{:}a \xrightarrow{\equiv} j{:}a$ for any individual a of Δ_i and Δ_j we reduce to the case of OWL interpretation with holes and local domains). $i{:}x \xrightarrow{\perp} j{:}y$

---

[1] In this definition, to be more homogeneous, we consider the interpretations of individuals to be sets containing a single object rather than the object itself.

states that x and y denotes completely unrelated objects. While $i : x \xrightarrow{*} j : y$ states that x and y might be related.

Example 7 (Example 4 and 3 formalized). The fact that Sale:Car describes the *same* set of objects from two different points of view, can be captured by asserting the bridge rule:

$$\mathsf{Sale : Car} \xrightarrow{\equiv} \mathsf{FIAT : Car} \tag{7}$$

The domain relation from O_{Sale} to O_{FIAT} of any contextual interpretation satisfying (7) will be such that $r_{ij}(\mathsf{Car})^{\mathcal{I}_{\mathsf{Sale}}} = (\mathsf{Car})^{\mathcal{I}_{\mathsf{FIAT}}}$.

Contextual mappings can also be used to better formalize example 3. If, for instance, we want to state that F23 and F34i are two petrol engines, we can state the following bridge rules from O_{WCM} to O_{Ferrari}.

$$\mathsf{WCM : Petrol} \xrightarrow{\sqsupseteq} \mathsf{Ferrari : F23} \tag{8}$$

$$\mathsf{WCM : Petrol} \xrightarrow{\sqsupseteq} \mathsf{Ferrari : F34i} \tag{9}$$

The domain relation form O_{WCM} to O_{Ferrari} that satisfies the above axioms is such that $r_{\mathsf{WCM,Ferrari}}(\mathsf{Petrol}^{\mathcal{I}_{\mathsf{WCM}}}) \supseteq \{\mathsf{F23}^{\mathcal{I}_{\mathsf{Ferrari}}}, \mathsf{F34i}^{\mathcal{I}_{\mathsf{Ferrari}}}\}$.

6   Context OWL

As from Section 5, in C-OWL, directionality and local domains can be dealt with by maintaining the OWL syntax unchanged and by suitably modifying the OWL semantics. In practice this means that a C-OWL model will look exactly the same as a OWL model. However, from an operational point of view, certain operations that are allowed in OWL will no longer be allowed in C-OWL. In particular, for what concerns directionality, the facts of the source domain will not be accessible in the target domain while, for what concerns local domains, each ontology will be able to use only the individuals in its local domain.

As from Section 5.3, dealing with context mappings is more complex and requires also an extension of the OWL syntax (with the proper semantics, as provided in that section). In particular, we need to add constructs for representing bridge rules and, within them, domain relations. We propose below a syntax for C-OWL which basically replicates the constructs for bridge rules defined within CTxML. CTxML has two main components:

1. contexts, which in the first proposal, as described in [4], are very simple representational devices and allow only for hierarchical classifications; and
2. mappings between contexts, namely a set of bridge rules.

C-OWL can therefore be straightforwardly obtained from CTxML by substituting the language for representing contexts in item 1 with OWL, and by keeping item 2 unchanged. As a consequence, C-OWL has the full representational power of OWL when we boil down to using ontologies, and the full
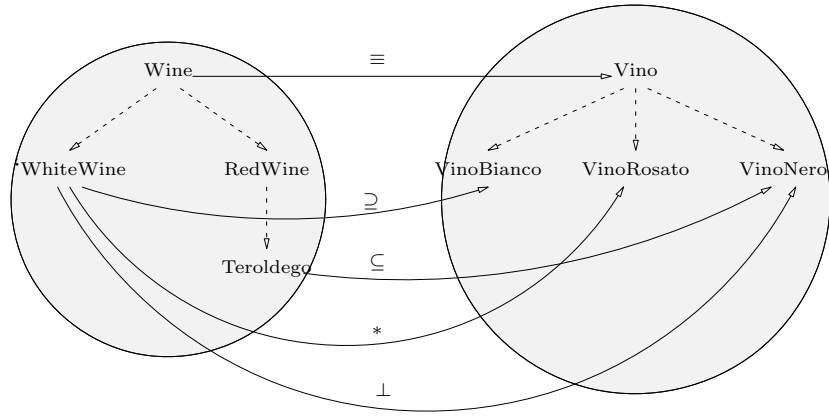
Fig. 1. A C-OWL mapping from the ontology "wine" to the ontology "vino".

representational power of CTXML when we boil down to using contextual information. The further nice property of C-OWL is that the two components are completely orthogonal and one can use the ontology or the contextual component in a totally independent manner.

A contextual ontology is therefore the pair: OWL ontology, set of C-OWL mappings, where each C-OWL mapping is a set of bridge rules with the same target ontology. A C-OWL mapping has therefore the following form:

1. A mapping identifier (URI)
2. A source context containing an OWL ontology (URI of an ontology)
3. A target context containing an OWL ontology (URI of an ontology)
4. A set of bridge rules from the local language of the source ontology to the local language of the target ontology. Each mapping is composed of three elements:
   (a) a source element, which is either a concept, a role or an individual of the source ontology;
   (b) a target element, which is either a concept, a role or an individual of the target ontology, with the restriction that this element must be of the same type as the source element.
   (c) the type of mapping which is one of $\xrightarrow{\sqsubseteq}$, $\xrightarrow{\sqsupseteq}$, $\xrightarrow{\perp}$ and $\xrightarrow{*}$.

The proposal of a precise syntax for C-OWL is out of the scope of this paper. However the XML representation of the mapping between the two ontologies about wine given in Figure 1 will look something like Figure 2 (Teroldego is a very nice Trentino red wine).

7 Conclusion

In this paper we have shown how the syntax and the semantics of OWL can be extended to deal with some problems that couldn't otherwise be dealt with.

```
;;;;; http://www.example.org/whine-to-vino.map;;;;;

<cowl:mapping>
  <rdfs:comment>Example of a mapping of wine into vino</rdfs:comment>
  <cowl:sourceOntology  rdf:resource="http://www.example.org/wine.owl"/>
  <cowl:targetOntology  rdf:resource="http://www.example.org/vino.owl"/>

  <cowl:bridgRule cowl:br-type="equiv">
     <cowl:sourceConcept rdf:resource="http://www.example.org/wine.owl#wine"/>
     <cowl:targedConcept rdf:resource="http://www.example.org/vino.owl#vino"/>
  </cowl:bridgRule>

  <cowl:bridgRule cowl:br-type="onto">
     <cowl:sourceConcept rdf:resource="http://www.example.org/wine.owl#RedWine"/>
     <cowl:targedConcept rdf:resource="http://www.example.org/vino.owl#VinoRosso"/>
  </cowl:bridgRule>

  <cowl:bridgRule cowl:br-type="into">
     <cowl:sourceConcept rdf:resource="http://www.example.org/wine.owl#Teroldego"/>
     <cowl:targedConcept rdf:resource="http://www.example.org/vino.owl#VinoRosso"/>
  </cowl:bridgRule>

  <cowl:bridgRule cowl:br-type="compat">
     <cowl:sourceConcept rdf:resource="http://www.example.org/wine.owl#WhiteWine"/>
     <cowl:targedConcept rdf:resource="http://www.example.org/vino.owl#Passito"/>
  </cowl:bridgRule>

  <cowl:bridgRule cowl:br-type="incompat">
     <cowl:sourceConcept rdf:resource="http://www.example.org/wine.owl#WhiteWine"/>
     <cowl:targedConcept rdf:resource="http://www.example.org/vino.owl#VinoNero"/>
  </cowl:bridgRule>
```

Fig. 2. A C-OWL mapping.

The result is C-OWL (Context OWL), an extended language with an enriched semantics which allows us to contextualize ontologies, namely, to localize their contents (and, therefore, to make them not visible to the outside) and to allow for explicit mappings (bridge rules) which allow for limited and totally controlled forms of global visibility.

This is only the first step and a lot of research remains to be done. The core issue at stake here is the tension between how much we should share and globalize (via ontologies) and how much we should localize with limited and totally controlled forms of globalization (via contexts).

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