Discrete-Time Signals: Time-Domain Representation

- Signals represented as sequences of numbers, called **samples**
- Sample value of a typical signal or sequence denoted as $x[n]$ with n being an integer in the range $-\infty \leq n \leq \infty$
- $x[n]$ defined only for integer values of n and undefined for noninteger values of n
- Discrete-time signal represented by $\{x[n]\}$

Discrete-Time Signals: Time-Domain Representation

- Discrete-time signal may also be written as a sequence of numbers inside braces:

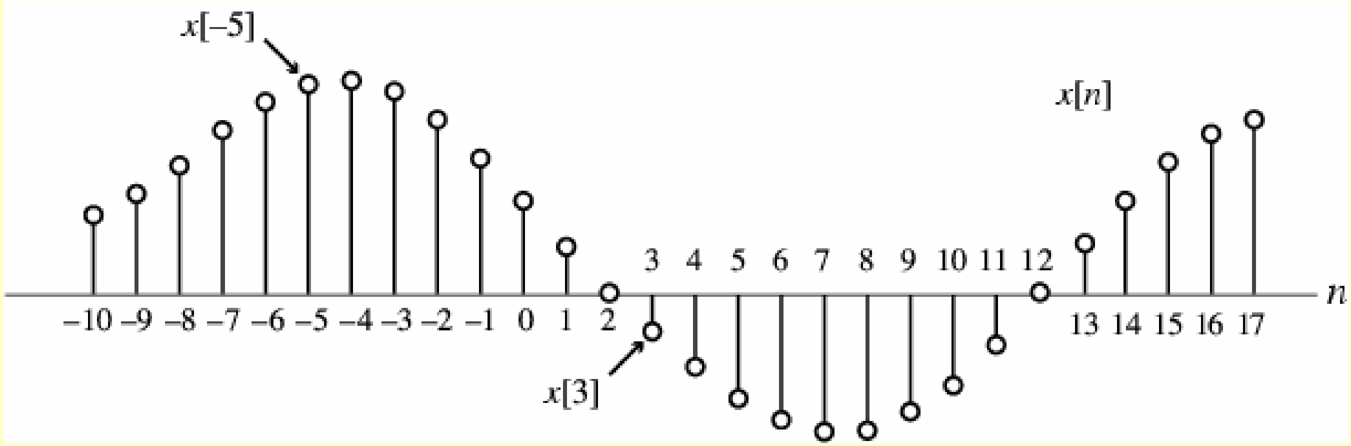
$$\{x[n]\} = \{\ldots, -0.2, \underset{\uparrow}{2.2}, 1.1, 0.2, -3.7, 2.9, \ldots\}$$

- In the above, $x[-1] = -0.2$, $x[0] = 2.2$, $x[1] = 1.1$, etc.

- The arrow is placed under the sample at time index $n = 0$

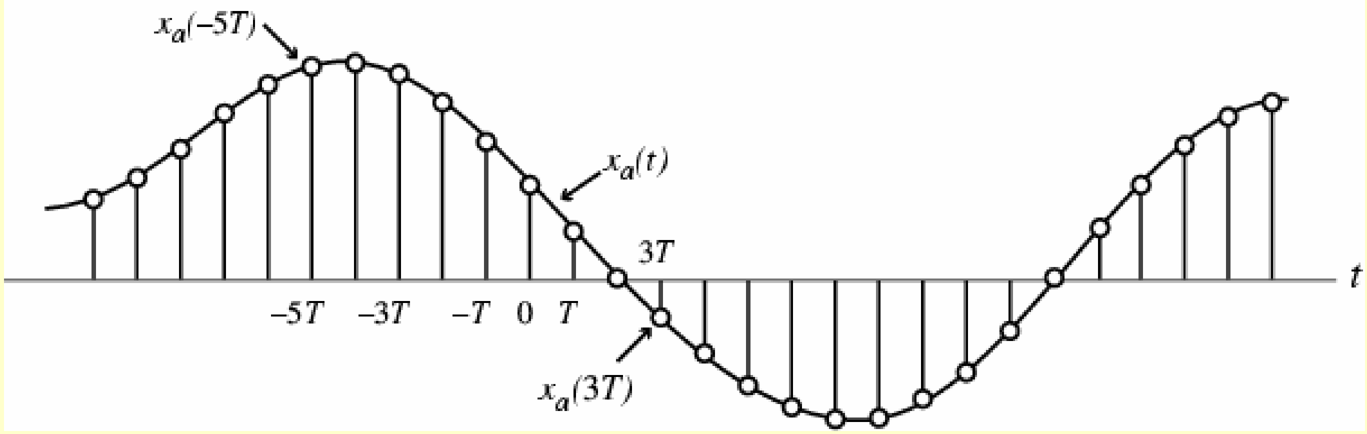
Discrete-Time Signals: Time-Domain Representation

- Graphical representation of a discrete-time signal with real-valued samples is as shown below:



Discrete-Time Signals: Time-Domain Representation

- In some applications, a discrete-time sequence $\{x[n]\}$ may be generated by periodically sampling a continuous-time signal $x_a(t)$ at uniform intervals of time



Discrete-Time Signals: Time-Domain Representation

- Here, n-th sample is given by

$$x[n] = x_a(t)\Big|_{t=nT} = x_a(nT), \; n = \ldots, -2, -1, 0, 1, \ldots$$

- The spacing T between two consecutive samples is called the **sampling interval** or **sampling period**

- Reciprocal of sampling interval T, denoted as F_T, is called the **sampling frequency**:

$$F_T = \frac{1}{T}$$

Discrete-Time Signals: Time-Domain Representation

- Two types of discrete-time signals:

  - **Sampled-data signals** in which samples are continuous-valued

  - **Digital signals** in which samples are discrete-valued

- Signals in a practical digital signal processing system are digital signals obtained by quantizing the sample values either by **rounding** or **truncation**

Discrete-Time Signals: Time-Domain Representation

- A discrete-time signal may be a **finite-length** or an **infinite-length sequence**
- Finite-length (also called **finite-duration** or **finite-extent**) sequence is defined only for a finite time interval: $N_1 \leq n \leq N_2$

  where $-\infty < N_1$ and $N_2 < \infty$ with $N_1 \leq N_2$
- **Length** or **duration** of the above finite-length sequence is $N = N_2 - N_1 + 1$

Discrete-Time Signals: Time-Domain Representation

- Example - $x[n] = n^2, -3 \le n \le 4$ is a finite-length sequence of length $4-(-3)+1=8$

$y[n] = \cos 0.4n$ is an infinite-length sequence

Discrete-Time Signals: Time-Domain Representation

- A length-N sequence is often referred to as an N**-point sequence**

- The length of a finite-length sequence can be increased by **zero-padding**, i.e., by **appending** it **with zeros**

Discrete-Time Signals: Time-Domain Representation

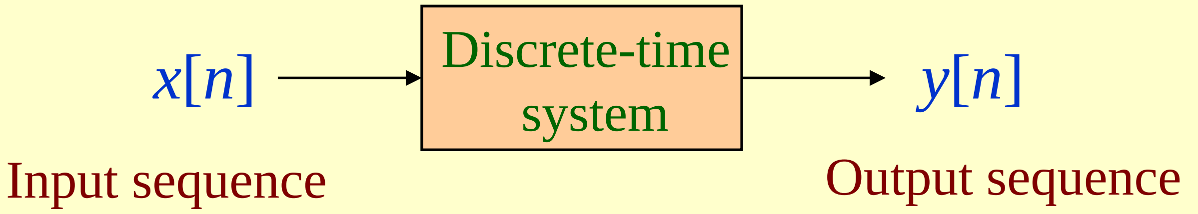
- Example -

$$x_e[n] = \begin{cases} n^2, & -3 \le n \le 4 \\ 0, & 5 \le n \le 8 \end{cases}$$

is a finite-length sequence of length 12 obtained by zero-padding $x[n] = n^2, -3 \le n \le 4$ with 4 zero-valued samples

Operations on Sequences

- A single-input, single-output discrete-time system operates on a sequence, called the **input sequence**, according some prescribed rules and develops another sequence, called the output sequence, with more desirable properties

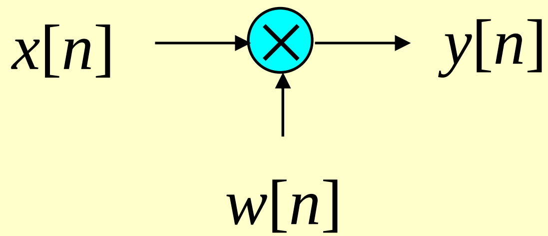


Operations on Sequences

- For example, the input may be a signal corrupted with additive noise
- Discrete-time system is designed to generate an output by removing the noise component from the input
- In most cases, the operation defining a particular discrete-time system is composed of some **basic operations**

Basic Operations

- **Product (modulation)** operation:
  - Modulator

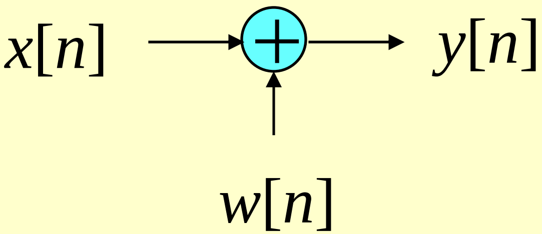


$$y[n] = x[n] \cdot w[n]$$

- An application is in forming a finite-length sequence from an infinite-length sequence by multiplying the latter with a finite-length sequence called an **window sequence**
- Process called **windowing**

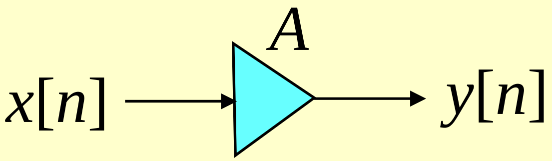
Basic Operations

- **Addition** operation:
  - Adder



$$y[n] = x[n] + w[n]$$

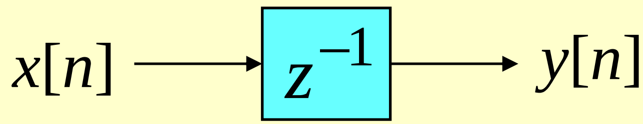
- **Multiplication** operation
  - Multiplier



$$y[n] = A \cdot x[n]$$

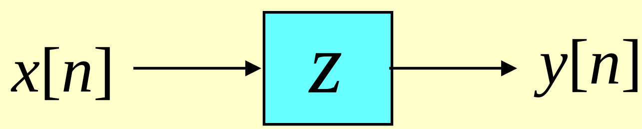
Basic Operations

- **Time-shifting** operation: $y[n] = x[n-N]$
  where N is an integer
- If $N > 0$, it is **delaying** operation
  - Unit delay



$y[n] = x[n-1]$

- If $N < 0$, it is an **advance** operation



$y[n] = x[n+1]$

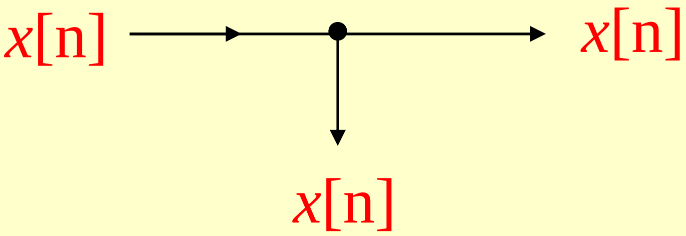
  - Unit advance

Basic Operations

- **Time-reversal (folding)** operation:

$$y[n] = x[-n]$$

- **Branching** operation: Used to provide multiple copies of a sequence



Basic Operations

- Example - Consider the two following sequences of length 5 defined for $0 \leq n \leq 4$:

$$\{a[n]\} = \{3 \quad 4 \quad 6 \quad -9 \quad 0\}$$

$$\{b[n]\} = \{2 \quad -1 \quad 4 \quad 5 \quad -3\}$$

- New sequences generated from the above two sequences by applying the basic operations are as follows:

Basic Operations

$$\{c[n]\} = \{a[n] \cdot b[n]\} = \{6 \quad -4 \quad 24 \quad -45 \quad 0\}$$

$$\{d[n]\} = \{a[n] + b[n]\} = \{5 \quad 3 \quad 10 \quad -4 \quad -3\}$$

$$\{e[n]\} = \frac{3}{2}\{a[n]\} = \{4.5 \quad 6 \quad 9 \quad -13.5 \quad 0\}$$

- As pointed out by the above example, operations on two or more sequences can be carried out if all sequences involved are of same length and defined for the same range of the time index n

Basic Operations

- However if the sequences are not of same length, in some situations, this problem can be circumvented by appending zero-valued samples to the sequence(s) of smaller lengths to make all sequences have the same range of the time index
- Example - Consider the sequence of length 3 defined for $0 \le n \le 2$: $\{f[n]\} = \{-2 \;\; 1 \;\; -3\}$

Basic Operations

- We cannot add the length-3 sequence $\{f[n]\}$ to the length-5 sequence $\{a[n]\}$ defined earlier
- We therefore first append $\{f[n]\}$ with 2 zero-valued samples resulting in a length-5 sequence $\{f_e[n]\} = \{-2 \;\; 1 \;\; -3 \;\; 0 \;\; 0\}$
- Then

$$\{g[n]\} = \{a[n]\} + \{f_e[n]\} = \{1 \quad 5 \quad 3 \quad -9 \quad 0\}$$

Classification of Sequences: Energy and Power Signals

- Total **energy** of a sequence $x[n]$ is defined by

$$\mathcal{E}_{\mathrm{x}} = \sum_{n=-\infty}^{\infty} \left| x[n] \right|^2$$

- An infinite length sequence with finite sample values may or may not have finite energy
- A finite length sequence with finite sample values has finite energy

Classification of Sequences: Energy and Power Signals

- The **average power** of an aperiodic sequence is defined by

$$P_x = \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} \left| x[n] \right|^2$$

- Define the **energy** of a sequence $x[n]$ over a finite interval $-K \le n \le K$ as

$$\mathcal{E}_{x,K} = \sum_{n=-K}^{K} \left| x[n] \right|^2$$

Classification of Sequences: Energy and Power Signals

- Then

$$P_x = \lim_{K\to\infty} \frac{1}{2K+1} \mathcal{E}_{x.K}$$

- The **average power** of a periodic sequence $\tilde{x}[n]$ with a period N is given by

$$P_x = \frac{1}{N} \sum_{n=0}^{N-1} \left|\tilde{x}[n]\right|^2$$

- The average power of an infinite-length sequence may be finite or infinite

Classification of Sequences: Energy and Power Signals

- Example - Consider the causal sequence defined by

$$x[n] = \begin{cases} 3(-1)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

- Note: $x[n]$ has infinite energy
- Its average power is given by

$$P_x = \lim_{K\to\infty} \frac{1}{2K+1}\left(9\sum_{n=0}^{K} 1\right) = \lim_{K\to\infty} \frac{9(K+1)}{2K+1} = 4.5$$

Classification of Sequences: Energy and Power Signals

- An infinite energy signal with finite average power is called a **power signal**

  Example - A periodic sequence which has a finite average power but infinite energy

- A finite energy signal with zero average power is called an **energy signal**

  Example - A finite-length sequence which has finite energy but zero average power

Other Types of Classifications

- A sequence $x[n]$ is said to be **bounded** if

$$|x[n]| \le B_x < \infty$$

- <u>Example</u> - The sequence $x[n] = \cos 0.3\pi n$ is a bounded sequence as

$$|x[n]| = |\cos 0.3\pi n| \le 1$$

Other Types of Classifications

- A sequence $x[n]$ is said to be **absolutely summable** if

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty$$

- Example - The sequence

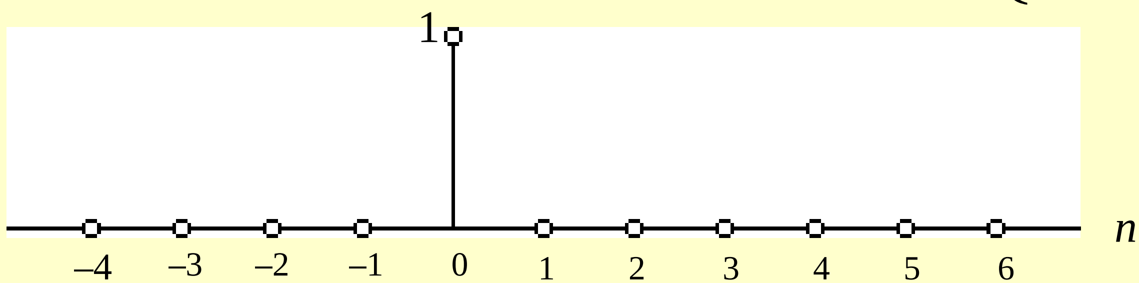
$$y[n] = \begin{cases} 0.3^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

is an absolutely summable sequence as

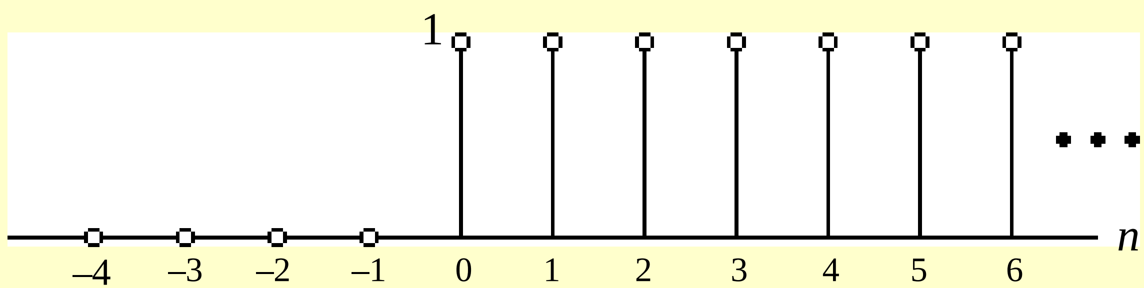
$$\sum_{n=0}^{\infty} |0.3^n| = \frac{1}{1-0.3} = 1.42857 < \infty$$

Basic Sequences

- **Unit sample sequence** - $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$



- **Unit step sequence** - $\mu[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$



Basic Sequences

- **Real sinusoidal sequence** -

$$x[n] = A\cos(\omega_o n + \phi)$$

where A is the **amplitude**, ω_o is the **angular frequency**, and ϕ is the **phase** of $x[n]$

Example -