Logic and Natural Language Semantics: Syntax-Semantics interface

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1. Classical Logic

$$A, \Gamma \vdash A, \Delta$$

$$\frac{A, B, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta} \; (\wedge \mathrm{L}) \qquad \frac{\Gamma \vdash \Delta, A \quad \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \wedge B} \; (\wedge \mathrm{R})$$

$$\frac{A, \Gamma \vdash \Delta \quad B, \Gamma \vdash \Delta}{A \vee B, \Gamma \vdash \Delta} \; (\vee \mathrm{L}) \qquad \frac{\Gamma \vdash \Delta, A, B}{\Gamma \vdash \Delta, A \vee B} \; (\vee \mathrm{R})$$

$$\frac{\Gamma \vdash \Delta, A \quad B, \Gamma \vdash \Delta}{A \to B, \Gamma \vdash \Delta} \; (\to \mathrm{L}) \qquad \frac{A, \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \to B} \; (\to \mathrm{R})$$

$$\frac{\Gamma \vdash \Delta, A}{\neg A, \Gamma \vdash \Delta} \; (\neg \mathrm{L}) \qquad \frac{A, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg A} \; (\neg \mathrm{R})$$

with the structural rules, Weakening, Contraction, Permutation. E.g.

Weakening $\frac{\Lambda \vdash \Sigma}{B, \Lambda \vdash \Sigma} \quad \frac{\Lambda \vdash \Sigma}{\Lambda \vdash B, \Sigma}$ Axiom: $\Gamma, A \vdash \Delta, A$

2. $\models$ and $\Vdash$

Raj: Used global assumption.

$$\Gamma \models A \;\text{ iff }\quad \forall M \;\text{ if } M \Vdash \Gamma \;\text{ then } M \Vdash A$$

If Raj wants to use "no global assumption" the takes Γ to be empty.

Us We are using "local assumption":

$$\Gamma \models A \;\text{ iff }\quad \forall M.\forall w \;\text{ if } w \Vdash \Gamma \;\text{ then } w \Vdash A$$

Hence, Γ is a set of formulas.

Then soundness and completeness give the relation between $\models$ and $\vdash$. Hence, we can just use the latter.

3. Tableaux and Sequents

Tableaux: $A \to B, A \vdash B$

	$A \to B$		prem.
	A		prem.
	$\neg B$		neg. of conc.
	$\neg A \vee B$		nnf
$\neg A$	$\mid$	B	$\vee$ rule
$\bot$		$\bot$	closed!

Sequent

$$\frac{A \vdash A \quad B \vdash B}{A \to B, A \vdash B}\ (\to L)$$

Tableaux: $A \vdash (A \to B) \to B$

	A		prem.
	$\neg((A \to B) \to B)$		neg. of con.
	$(\neg A \vee B) \wedge \neg B$		nnf
	$\neg A \vee B$		$\wedge$ rule
	$\neg B$		$\wedge$ rule
$\neg A$		B	$\vee$ rule
$\bot$		$\bot$	closed!

Sequent

$$\cfrac{\cfrac{A \vdash A \quad B \vdash B}{A \to B, A \vdash B}}{A \vdash (A \to B) \to B}$$

4. NL: Sequents

$$\frac{}{A \vdash A}\ (\text{axiom})$$

$$\frac{\Delta \vdash B \quad \Gamma[A] \vdash C}{\Gamma[(A/B \otimes \Delta)] \vdash C}\ (/L) \qquad \frac{\Gamma \otimes C \vdash B}{\Gamma \vdash B/C}\ (/R)$$

$$\frac{\Delta \vdash B \quad \Gamma[A] \vdash C}{\Gamma[(\Delta \otimes B\backslash A)] \vdash C}\ (\backslash L) \qquad \frac{C \otimes \Gamma \vdash B}{\Gamma \vdash C\backslash B}\ (\backslash R)$$

$$\frac{\Gamma[(A \otimes B)] \Rightarrow C}{\Gamma[A \bullet B] \Rightarrow C}\ (\bullet L) \qquad \frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{(\Gamma \otimes \Delta) \Rightarrow A \bullet B}\ (\bullet R)$$

4.1. For your notes

4.2. Example

[1] Check carefully how the derivation is built $s/(np\backslash s) \otimes np\backslash s \vdash s$

$$\dfrac{\dfrac{\dfrac{np \vdash np \quad s \vdash s}{np \otimes np\backslash s \vdash s}(\backslash L)}{np\backslash s \vdash np\backslash s}(\backslash R) \qquad s \vdash s}{s/(np\backslash s) \otimes (np\backslash s) \vdash s}(/L)$$

[2] In the derivation below, at each infer step put the name of the rule and highlight its corresponding main connective

$$\dfrac{np \vdash np \quad \dfrac{np \vdash np \quad \dfrac{np \vdash np \quad s \vdash s}{np \otimes np\backslash s \vdash s}(\quad)}{(np \otimes np\backslash np) \otimes np\backslash s \vdash s}(\quad)}{(np \otimes ((np\backslash np)/np \otimes np)) \otimes np\backslash s \vdash s}(\quad)$$

4.3. Exercise

Prove that

[3] $np \vdash s/(np\backslash s)$

[4] $np \otimes (QP\backslash QP)/QP \otimes s/(np\backslash s) \vdash s/(np\backslash s)$

[5] $(np\backslash s) \otimes s/(np\backslash s) \vdash s$

QP is just my abbreviation for $s/(np\backslash s)$ (to make the formula easier to read.

5. Sequents and terms

$$\frac{}{x : A \vdash x : A}\ (\text{axiom})$$

$$\frac{\Delta \vdash t : B \quad \Gamma[x : A] \vdash u : C}{\Gamma[(y : A/B \otimes \Delta)] \vdash u[x := y\ t] : C}\ (/\text{L}) \qquad \frac{\Gamma \otimes x : B \vdash t : A}{\Gamma \vdash \lambda x.t : A/B}\ (/\text{R})$$

$$\frac{\Delta \vdash t : B \quad \Gamma[x : A] \vdash u : C}{\Gamma[(\Delta \otimes y : B\backslash A)] \vdash u[x := y\ t] : C}\ (\backslash\text{L}) \qquad \frac{x : B \otimes \Gamma \vdash t : A}{\Gamma \vdash \lambda x.t : B\backslash A}\ (\backslash\text{R})$$

$$\frac{\Gamma[(x : A \otimes y : B)] \Rightarrow t : C}{\Gamma[z : A \bullet B] \Rightarrow t[x := (z)_0 \otimes (zy :=)_1] : C}\ (\bullet\text{L}) \qquad \frac{\Gamma \Rightarrow t : A \quad \Delta \Rightarrow u : B}{(\Gamma \otimes \Delta) \Rightarrow \langle t \otimes u \rangle : A \bullet B}\ (\bullet\text{R})$$

5.1. For your notes

5.2. Example with proof terms

Check carefully how the proof term is built

$$
\dfrac{y : np \vdash y : np \quad \dfrac{z : np \vdash z : np \quad \dfrac{x : np \vdash x : np \quad u : s \vdash u : s}{x : np \otimes t : np\backslash s \vdash u[u := (t\ x)] : s}(\backslash L)}{(z : np \otimes p : np\backslash np) \otimes t : np\backslash s \vdash (t\ x)[x := (p\ z)] : s}(\backslash L)}{(z : np \otimes (q : (np\backslash np)/np \otimes y : np)) \otimes t : np\backslash s \vdash t\ (p\ z)[p := (q\ y)] : s}(/L)
$$

$t\ ((q\ y)\ z)$

5.3. For you to write

Try to label your derivations [1], [3] and [4].

5.4. Exercise with proof terms

[1]

$$\dfrac{\dfrac{\dfrac{x : np \vdash x : np \quad y : s \vdash y : s}{x : np \otimes t : np\backslash s \vdash y[y := t\ x] : s}\ (\backslash L)}{t : np\backslash s \vdash \lambda x.t\ x : np\backslash s}\ (\backslash R) \qquad v : s \vdash v : s}{p : s/(np\backslash s) \otimes t : (np\backslash s) \vdash v[v := p\ \lambda x.(t\ x)] : s}\ (/L)$$

Notice: $\lambda x.t\ x =_\eta t$. Proof term: $p\ t$

[3]

$$\dfrac{\dfrac{y : np \vdash y : np \quad u : s \vdash u : s}{y : np, x : np\backslash s \vdash u[u := x\ y] : s}\ (\backslash \mathrm{L})}{y : np \vdash \lambda x.x\ y : s/(np\backslash s)}\ (/\mathrm{R})$$

Proof term: $\lambda x.x\ y$

6. Meaning representation

Take the derivation above to be a prove of the grammaticality of sentences as specified below and use the corresponding proof term to build their meaning representation.

[1] "someone left" ∈ s

[4] "mary and someone" ∈ $s/(np\backslash s)$

[1] $p : s/(np\backslash s) \otimes t : np\backslash s$, proof term: $p\ t$.

1. $s/(np\backslash s)$ stands for "someone" and $np\backslash s$ stands for "left".
2. Hence, replace p and t with $\lambda X.\exists y.X(y)$ and $\lambda z.\texttt{left}(z)$, resp. by β-reduction twice: $\exists y.\texttt{left}(y)$