Logic and Natural Language Semantics: Non Classical Logic for Natural Language Syntax

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Contents

1. Yesterday story

1. Specify semantic representations for the **lexical items** and build the representation of sentence **compositionally**: [Yesterday]

    - We have used set-theoretical interpretation, represented by λ-terms, and exploited function application and abstraction to assemble meaning representation of larger expressions compositionally.

word	**type**	**term**	**meaning**
"sara"	e	$\mathtt{s}$	sara
"walks"	$(e \to t)$	$\lambda x_e.(\mathtt{walks}(x))_t$	{sara}
"teases"	$(e \to (e \to t))$	$\lambda y_e.(\lambda x_e.(\mathtt{teases}(x, y))_t)$	$\{\langle$sara, ilaria$\rangle\}$

2. Specify the **syntax** for the natural language fragment of interest. [Today]

    - We use a Logical Grammar, a non-classical logic.

3. Capture the **syntax-semantic** interface. [Tomorrow]

    - We exploit the CH Correspondence between the logic and the λ calculus.

2. Syntax: Preliminary Notions

- **Syntax**: "setting out things together", in our case things are words. The main question addressed here is "*How do words compose together to form a grammatical sentence (s) (or fragments of it)?*"
- **Categories**: words are said to belong to *classes*/categories. The main categories are nouns (n), verbs (v), adjectives (adj), determiners (det) and adverbs (adv).
- **Constituents**: Groups of categories may form a single *unit or phrase* called constituents. The main phrases are noun phrases (np), verb phrases (vp), prepositional phrases (pp). Noun phrases for instance are: "she"; "Michael"; "Rajeev Goré"; "the house"; "a young two-year child".

  Tests like substitution help decide whether words form constituents.
- **Dependency**: Categories are interdependent, for example

Ryanair services [Pescara]$_{np}$	Ryanair flies [to Pescara]$_{pp}$
*Ryanair services [to Pescara]$_{pp}$	*Ryanair flies [Pescara]$_{np}$

  the verbs services and flies determine which category can/must be juxtaposed. If their constraints are not satisfied the structure is ungrammatical.

2.1. Long-distance Dependencies

Interdependent constituents need not be juxtaposed, but may form long-distance dependencies, manifested by gaps

- What cities does Ryanair service [. . .]?

The constituent what cities depends on the verb service, but is at the front of the sentence rather than at the object position.

Such distance can be large,

- Which flight do you want me to book [. . .]?
- Which flight do you want me to have the travel agent book [. . .]?
- Which flight do you want me to have the travel agent nearby my office book [. . .]?

2.2. Relative Pronouns and Coordination

- **Relative Pronoun** (eg. who, which): they function as e.g. the subject or object of the verb embedded in the relative clause (rc),
  - [[the [student [who [...] knows Sara]$_{rc}$]$_{n}$]$_{np}$ [left]$_{v}$]$_{s}$.
  - [[the [book [which Sara wrote [...]]$_{rc}$]$_{n}$]$_{np}$ [is interesting]$_{v}$]$_{s}$.
- **Coordination**: Expressions of the same syntactic category can be coordinated via "and", "or", "but" to form more complex phrases of the same category. E.g., a coordinated verb phrase can consist of two other verb phrases separated by a conjunction:
  - There are no flights [[leaving Denver]$_{vp}$ and [arriving in San Francisco]$_{vp}$]$_{vp}$

  The conjuncted expressions belong to vp. However, we could also have
  - I [[[want to try to write [...]] and [hope to see produced [...]]] [the movie]$_{np}$]$_{vp}$"

  Again, the interdependent constituents are disconnected from each other.

Long-distance dependencies are **challenging phenomena** for formal approaches to natural language analysis.

2.3. Scope ambiguity

- [Every student] [passed [an exam]]

  a $\forall y.\texttt{Student}(y) \rightarrow \exists x.\texttt{Exam}(x) \wedge \texttt{Passed}(y,x)$ [EVERY > A]

  b $\exists x.\texttt{Exam}(x) \wedge \forall y \texttt{Student}(y) \rightarrow \texttt{Passed}(y,x)$ [A > EVERY]

- Alice [thinks [someone left]]

  a $\texttt{think}(\text{a},\exists x.left(x))$ [THINK > LEFT]

  b $\exists x.\texttt{think}(a, left(x))$ [SOME > THINK]

The (a) readings are called "local scope reading" and the (b) are the non-local scope reading.

Non-local scope readings are challenging phenomena for formal approaches to natural language.

3. Formal linguistics Background

To examine how the syntax of a sentence can be computed, one must consider two things:

1. **The grammar**: A formal specification of the structures allowable in the language. [Data structures]
2. **The parsing technique**: The method of analyzing a sentence to determine its structure according to the grammar. [Algorithm]

3.1. Example of Grammar

```
S --> NP VP
VP --> IV
VP --> TV NP
NP --> DET N

NP --> john
DET --> the
N --> student
IV --> left
TV --> loves
```

The grammar recognizes that the following sentences belong to the language:

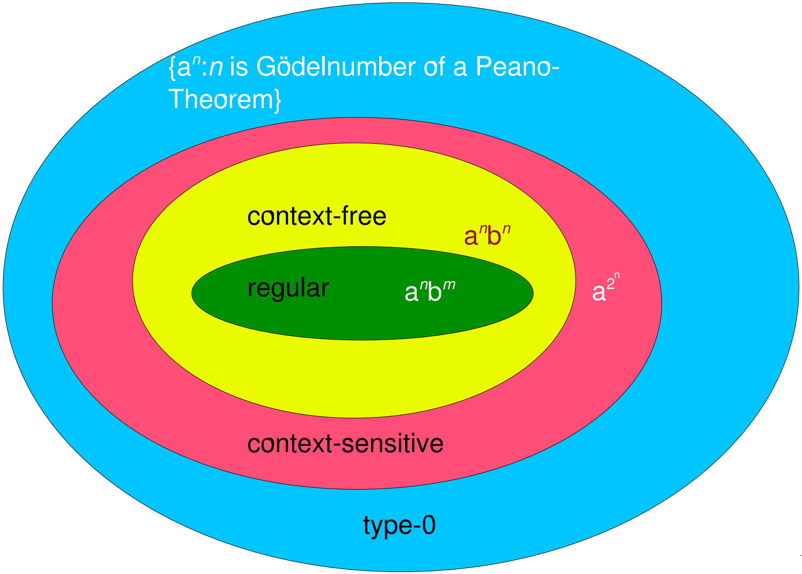
john left, the student left, john loves john, john loves the student, the student loves john, the student loves the student.

3.2. Hierarchy of Grammars and Languages

Every (formal) grammar generates a unique language. However, one language can be generated by several different (formal) grammars.

Based on this observation it's possible to construct a hierarchy of grammars, where the set of languages describable by grammars of grater power subsumes the set of language describable by grammars of less power. The most commonly used hierarchy is the **Chomsky Hierarchy of Languages** introduced in 1959.

The Chomsky Hierarchy



3.3. Where do Natural Languages fit?

Hence, the two questions to ask are:

- Where does Natural Language fit in the Hierarchy?
- Which is the generative power of the different Formal Grammars?

If we know where NL fits, we would know

- which formal language can represent NL;
- which rules to use in writing formal grammars for NL.

Moreover we would know the complexity of the parsing problem, e.g.

- For Context Free Language the problem is polynomial.
- whereas, for Context Sensitive Languages the problem is PSPACE-complete

3.4. NLs are not Regular Languages (RLs)

It is universally agreed that NLs are not regular, Chomsky (1956, 1957).

NL shows lexical dependency between one part of each structure and another: e.g. "If" must be followed by "then" "either" must be followed by "or". Moreover, these sentences can be embedded in English one in another.

Example Roughly, E.g. If either the man who said S5 is arriving today or the man who said S5 is arriving tomorrow, then the man who said S6 is arriving the day after.

$a \rightarrow$ if
$b \rightarrow$ then
$c \rightarrow$ either
$d \rightarrow$ or
$\epsilon \rightarrow$ "other words"

If we consider just the "if ... then" construction, we have $a^n b^n$ which is not a RL.

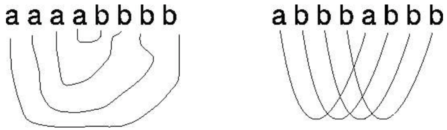
3.5. Are NL Context Free (CF)?

History of the problem:

1. Chomsky 1957: conjecture that natural languages are not CF
2. sixties, seventies: many attempts to prove this conjecture
3. Pullum and Gazdar 1982:
   - all these attempts have failed
   - for all we know, natural languages (conceived as string sets) might be context-free
4. Huybregts 1984, Shieber 1985: shown that Swiss German is not context-free
5. Culy 1985: proof that Bambara is not context-free

Nested and crossing dependencies

- CFLs—unlike regular languages—can have unbounded dependencies
- however, these dependencies can only be **nested**, not **crossing**
- example:
  - $a^n b^n$ has unlimited nested dependencies → context-free
  - the copy language has unlimited crossing dependencies → not context-free



Cross-serial dependencies Swiss German Today many theorists believe natural languages are not context-free. (Huybregts 1984, Shieber 1985).

Evidences are given by **cross-serial dependencies** found in Swiss German where verbs and their argument can be ordered cross-serially.

A sentence can have a string of

dative nouns followed by a string of accusative nouns, followed by a string of dative-taking verbs, followed by a string of accusative-taking verbs. E.g.

> mer em Hans es huus halfed aastriiche
> we Hans/Dat the house/Acc helped paint.
>
> tr. we helped Hans paint the house.

The number of verbs requaring dative objects must equal the number of dative NPs and similarly for accusatives, and this number can be arbitrary. Hence, the language representing this phenomenon is $wa^nb^mxc^nd^my$ which is not Context Free (CF).

However, notice that those construction types used to prove that NLs is not CF appear to be hard to understand for humans too.

3.7. Beyond CFG

However, how large NL are continues to be a less simple matter. There are two main non-compatible views:

1. Natural Language forms a class of languages that **includes the CF** family, but is larger than it.

2. Natural Language occupies a position eccentric with respect to that hierarchy, in such a way that it does not contain any whole family in the hierarchy but is **spread along all of them**

Mildly Context-Sensitive Languages The first view gave rise to a new family of languages which is of clear linguistic interest, Mildly Context-Sensitive Languages.

According to Joshi (1985, p. 225): MCS **only slightly more powerful than CFGs** but the parsing problem for L is solvable in **polynomial time**.

3.8. Parsing as deduction

We want to find the Logic that properly models natural language syntax-semantics interface.

- We consider syntactic categories to be logical formulas
- As such, they can be atomic or complex (not just plain A, B, a, b etc.).
- They are related by means of the derivability relation ($\vdash$)
- To recognize that a string/structure is of a certain category (e.g. john loves mary $\in s$) reduces to prove the formulas corresponding to the structure and the category are in a derivability relation $\Gamma \vdash A$ ($NP\ TV\ NP \vdash s$)

The slogan is:

> "Parsing as deduction"

The question is:

> which logic do we need?

4. Recall: Classical Logic: Logical Rules

$$A, \Gamma \vdash A, \Delta$$

$$\frac{A, B, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash \Delta, A \quad \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \wedge B}\ (\wedge R)$$

$$\frac{A, \Gamma \vdash \Delta \quad B, \Gamma \vdash \Delta}{A \vee B, \Gamma \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash \Delta, A, B}{\Gamma \vdash \Delta, A \vee B}\ (\vee R)$$

$$\frac{\Gamma \vdash \Delta, A \quad B, \Gamma \vdash \Delta}{A \rightarrow B, \Gamma \vdash \Delta}\ (\rightarrow L) \qquad \frac{A, \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \rightarrow B}\ (\rightarrow R)$$

$$\frac{\Gamma \vdash \Delta, A}{\neg A, \Gamma \vdash \Delta}\ (\neg L) \qquad \frac{A, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg A}\ (\neg R)$$

A, B stand for logical formulas. Γ, Δ, Σ stand for **sets** of formulas. Hence the **order** and **quantity** of formulas occurrences don't count.

The , stand for "**and**" when occurring on the right of $\vdash$ and for "**or**" on its left

4.1. Recall: Classical Logic: Structural Rules

An important discovery in Gentzen's thesis [1935] is that in logic there are rules of inference that don't involve any logical constant. Gentzen called such rules "structural".

Hidden in Classical Logic there are the following structural rules.

Weakening	$\dfrac{\Lambda \vdash \Sigma}{B, \Lambda \vdash \Sigma}$	$\dfrac{\Lambda \vdash \Sigma}{\Lambda \vdash B, \Sigma}$	Axiom: $\Gamma, A \vdash \Delta, A$
Contraction	$\dfrac{A, A, \Gamma \vdash \Sigma}{A, \Gamma \vdash \Sigma}$	$\dfrac{\Gamma \vdash A, A, \Sigma}{\Gamma \vdash A, \Sigma}$	structures are sets: **nr. does not count**
Permutation	$\dfrac{A, B, \Gamma \vdash \Sigma}{B, A, \Gamma \vdash \Sigma}$	$\dfrac{\Gamma \vdash A, B, \Sigma}{\Gamma \vdash B, A, \Sigma}$	structures are sets: **order doesn't count**

Furthermore, the comma is associative: $(A, (B, C)) = ((A, B), C)$.

4.2. The Core of Logic

The distinction between logical and structural rules helped

(i) capturing the core of the family of substructural logics

(ii) fine tune logics on the base of the object of investigation.

In particular, the separation of structural rules from logical rules helped highlighting the crucial role played by conditional and the residuation condition that is at the core of Logic.

4.3. The "comma"

The distinction between logical vs. structural rules shed lights on the role and properties of the "comma", that indicates the combination of premises.

- It showed that the behavior of premises combination is also important in determining the behavior of the conditional. As the comma's behavior varies, so does the conditional.
- Hence, there can exist logics that share logical rules while differ with respect to the structures of the premises.
- Intuitionistic logic is obtained by taking the structure on the right of the $\vdash$ to stand for single formulas $\Delta \vdash A$, so to reject the classical tautologies $(p \vee \neg p)$.
- this leads to substructural logics, i.e. those non-classical logics that lack one or all structural rules, while share the core of logic (residuation condition.)

5. Research Goal

We are interested in using a Logic as a Grammar, hence we want:

(a) tailoring the logic of natural language syntactic composition

(b) capturing its correlation with the semantic representation.

We want to grasp the **fragment of logic** that suits our needs, and use a logic as a grammar.

5.1. Logical Grammar requirements

- We know we need to speak about complete vs. incomplete expressions
- We know that we want both compose and decompose linguistic structures.
- We know the core logical grammar basic requirements; linguistic structures are:
  - not commutative: mary walks $\vdash s$ but walks mary $\not\vdash s$.
  - not associativity [the student]$_{np}$ walks $\vdash s$ but [the [student walks]$_?$ $\not\vdash s$.
  - sensitive to the occurrence of words (we cannot freely reduce or add them): mary walks $\vdash s$; mary mary walks $\not\vdash s$, and mary walks $\vdash s$ but walks $\not\vdash s$.
  - natural languages differ w.r.t word order.
- We know that syntax and semantics representations are tiddly connected. And there can be non local scope phenomena.

5.2. Non associative Lambek Calculus (**NL**)

No contraction, no weakening, no associativity, no commutativity. Hence,

- right ($A \backslash B$ if A then B $A \rightarrow B$) and left (B/A B if A, $B \leftarrow A$) implications

$$\dfrac{\dfrac{\dfrac{B \vdash B \quad A \vdash A}{A/B, B \vdash A}\ (/L)}{B, A/B \vdash A}\ (\text{Com})}{A/B \vdash B \backslash A}\ (\backslash R)$$

- conjunction is seen as fusion
- structures are lists
- And no negation.

5.3. The core of logic: Residuation

$$\text{(REF)}\quad A \vdash A \quad \text{and} \quad \text{(TRANS)}\quad \text{if } A \vdash B \text{ and } B \vdash C, \text{then } A \vdash C$$

$$\text{(RES)}\quad C \vdash A\backslash B \quad \text{iff} \quad A \otimes C \vdash B \quad \text{iff} \quad A \vdash B/C$$

$$(\text{RES}_{def})\quad y \leq g(x,z) \quad \text{iff} \quad f(x,y) \leq z \quad \text{iff} \quad x \leq h(z,y)$$

$$[RES_2] \quad \begin{pmatrix} 2 \leq \frac{9}{4} \\ \text{iff} \\ 2 \times 4 \leq 9 \\ \text{iff} \\ 4 \leq \frac{9}{2} \end{pmatrix} \quad [RES_2] \quad \begin{pmatrix} np : mary \leq \frac{s}{iv:walks} \\ \text{iff} \\ np : mary \times iv : walks \leq s \\ \text{iff} \\ walks : iv \leq \frac{s}{np:mary} \end{pmatrix}$$

Similarly principle holds for unary operators and there is a dual residuation principle (math: $(+,-)$) too.

5.4. NL: logical rules

$$\frac{}{A \vdash A}\ (\text{axiom})$$

$$\frac{\Delta \vdash B \quad \Gamma[A] \vdash C}{\Gamma[(A/B \otimes \Delta)] \vdash C}\ (/L) \qquad \frac{\Gamma \otimes C \vdash B}{\Gamma \vdash B/C}\ (/R)$$

$$\frac{\Delta \vdash B \quad \Gamma[A] \vdash C}{\Gamma[(\Delta \otimes B\backslash A)] \vdash C}\ (\backslash L) \qquad \frac{C \otimes \Gamma \vdash B}{\Gamma \vdash C\backslash B}\ (\backslash R)$$

$$\frac{\Gamma[(A \otimes B)] \Rightarrow C}{\Gamma[A \bullet B] \Rightarrow C}\ (\bullet L) \qquad \frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{(\Gamma \otimes \Delta) \Rightarrow A \bullet B}\ (\bullet R)$$

Look at (/ L) has Modus Ponens: $A/B\ B \Rightarrow A$ (similarly for (\ L))

Look at (/ R) has hypothetical reasoning: assuming I've a C I arrive to prove $A \otimes C \vdash B$, then $A \vdash B/C$

It's called the Logic of Residuation (the core logic). See web site for reference on the connection between Gentzen Sequent presentation and the residuation principle.

5.5. Compose expressions: (\ L) and (/L)

Complete exp: e.g. "sara" $\in np$ vs. incomplete exp: eg. left $\in np\backslash s$.

sara left $\in s$? $\qquad np \otimes (np\backslash s) \vdash s$? $\qquad \dfrac{np \vdash np \quad s \vdash s}{np \otimes (np\backslash s) \vdash s}\,(\backslash L)$

Syntactic categories are seen as sets of expressions: those expressions that belong to a given category.

$$\underbrace{\{\text{john, sara}\}}_{np} \underbrace{\times}_{\otimes} \underbrace{\{\text{left, teases mary}\}}_{(np\backslash s)} \underbrace{\subseteq}_{\vdash} \underbrace{\{\text{john left, sara left, john teases mary, sara teases mary}\}}_{s}$$

$\otimes$ merges expressions.

$$\dfrac{np \vdash np \qquad \dfrac{np \vdash np \quad s \vdash s}{np \otimes (np\backslash s) \vdash s}\,(\backslash L)}{np \otimes ((np\backslash s)/np \otimes np) \vdash s}\,(/L)$$

5.6. Extraction: Left-branch

$\underbrace{\text{The student who } [[\ldots]\text{teases Lori}]_s}_{np}\ \underbrace{\text{left}}_{np\backslash s}$

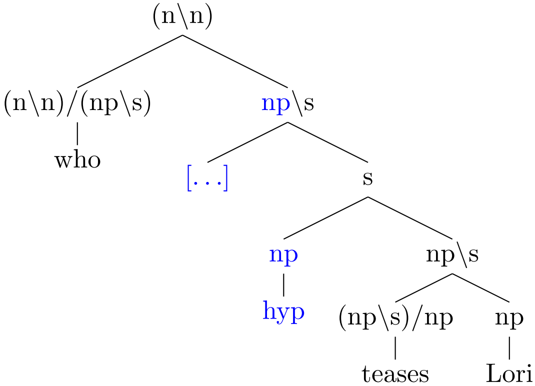
Lexicon

Lori	np	left	$np\backslash s$
student	n	teases	$(np\backslash s)/np$
the	np/n	who	$(n\backslash n)/(np\backslash s)$

$$\underbrace{\text{The student who } [[\ldots]\text{teases Lori}]_s}_{np}\ \underbrace{\text{left}}_{np\backslash s}$$

$$\dfrac{\dfrac{np \vdash np \qquad \dfrac{np \vdash np \quad s \vdash s}{np \otimes np\backslash s \vdash s}\,(\backslash L)}{np \otimes ((np\backslash s)/np \otimes np) \vdash s}\,(/L)}{\dfrac{(np\backslash s)/np \otimes np \vdash np\backslash s}{}\,(\backslash R)} $$

$$\dfrac{(np\backslash s)/np \otimes np \vdash np\backslash s \qquad n\backslash n \vdash n\backslash n}{\underbrace{(n\backslash n)/(np\backslash s)}_{who} \otimes \underbrace{(np\backslash s)/np}_{teases} \otimes \underbrace{np}_{Lori} \vdash n\backslash n}\,(/L)$$



Note, how (\R) decomposes the built structure by removing/abstracting the np.

5.7. Extraction: Right-branch

"which Sara wrote [...]" requires (some form of) associativity.

$$
\dfrac{
  \dfrac{
    \dfrac{
      np \vdash np \qquad \dfrac{np \vdash np \quad s \vdash s}{np \otimes np\backslash s \vdash s}\,(\backslash L)
    }{
      np \otimes ((np\backslash s)/np \otimes np) \vdash s
    }\,(/L)
  }{
    \dfrac{(np \otimes (np\backslash s)/np) \otimes np \vdash s}{np \otimes (np\backslash s)/np \vdash s/np}\,(/R)
  }\,(Ass)
  \qquad n\backslash n \vdash n\backslash n
}{
  \underbrace{(n\backslash n)/(s/np)}_{which} \otimes (\underbrace{np}_{Sara} \otimes \underbrace{(np\backslash s)/np}_{wrote}) \vdash n\backslash n
}\,(/L)
$$

But

- global structural rules are "unsound" when reasoning with natural language.
- The logical grammar will overgenerate proving as grammatical also ungrammatical sentence.

5.8. Local structural control: Modalities

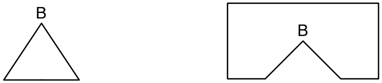
As in other domains, in Linguistic as well, there is the need of locally controlling structural reasoning and account for the different compositional relations linguistic phenomena may exhibit. For instance,

- E.g. Latin: "cum magna laude" vs. "cum laude magna" Need some sort of commutativity.
- E.g. Coordination: "[[I want to try to write [...]] and [hope to see produced [...]] the movie]" need of some form of associativity.

But differently from linear logic, the control is expressed either by means of **modes** of composition or by means of **unary** or the **dual** logical operators living within the same algebraic structure (residuation) of the binary ones.

5.9. Non local scope: Expressions and Contexts

Recall: natural language exhibit non local scope. In the Logical Grammar approach, this fact has been accounted for by considering a logic language that can represent expressions (viz. structures) and contexts (viz. holes):



Composition Expressions are composed in a bottom-up fashion, whereas contexts are composed in a "top-down" one.

Interaction Expressions and contexts need to interact. E.g. the QP has to act locally as an expression and scope on the structure as a context.

Duality Expressions and contexts are dual.

5.10. Lambek Calculi: From '57 till now – the Utrecht school

- Jim Lambek: Lambek Calculus (L) Non-associative Lambek Calculus (NL) (59, 61)
- Michael Moortgat and Dick Oehrle: **modes** of operators (1986) to control structural rules.
- Michael Moortgat and Natasha Kurtonina: **unary** of operators (1995)
- Richard Moot: Lambek Calculi and **Proof Nets** (2001)
- Dirk Helyen, Willmeijn Vermaat, Raffaella Bernardi: unary operators for feature propagation, long distance phenomena, licensing relations and quantifiers scope distribution. (1999-2003)
- Michael Moortgat and Raffaella Bernardi: Lambek **Grishin** Calculus (or Symmetric Categorial Grammar) (2010)
- Arno Bastenhof currently carrying out is PhD thesis on Lambek Grishin: many results on the proof-theoretical part.
- Michael Moortgat & Richard Moot (in preparation) Proof nets for the Lambek-Grishin calculus. In Heunen, Sadrzadeh, Grefenstette (eds.) Compositional methods in Physics and Linguistics, OUP.

6. Symmetric CG – Lambek Grishin

(Bernardi & Moortgat 2007/2010, Moortgat 2007/2009) proposed LG:

- Lambek Calculus: $\backslash, \otimes, /$
- Dual Lambek Calculus: $\oslash, \oplus, \obslash$

Multiplicative composition	$A \otimes B \vdash C$	iff	$B \vdash A \backslash C$
	$x \times y \leq z$	iff	$y \leq \frac{z}{x}$
Additive composition	$C \vdash B \oplus A$	iff	$C \oslash A \vdash B$
	$z \leq y + x$	iff	$z - x \leq y$

with Linear Distributivity postulates between the two families of operators. (Grishin)

Additive composition $\oplus$ is also known as "multiplicative disjunction" (viz. the dual of "multiplicative conjunction" $\otimes$).

6.1. Modus Ponens

Modus Ponens and its dual

$$B/A \otimes A \vdash B \qquad \text{dually} \qquad B \vdash A \oplus A \oslash B$$

- Multiplicative MP: The function (B/A) turns an expression A into an expression B.
- Additive MP: The dual-function $(A \oslash B)$ turns the context of an A (the need of a A) into the context of a B (the need of a B).

$\otimes$ composes **expressions** to produce an expression, $\oplus$ composes contexts to produce a context.

Focus

- $B/A \otimes A \vdash_{\triangleright} \underline{B}$ [focus on the right of $\vdash \underline{B}$]
- $\underline{B} \vdash_{\triangleleft} A \oplus A \oslash B$ [focus on the left of $\underline{B} \vdash$ (contrapositive)]

6.2. Hypothetical Reasoning

$$\frac{\Delta \otimes A \vdash_{\triangleright} \underline{B}}{\Delta \vdash_{\triangleright} \underline{B/A}} \qquad \text{dually} \qquad \frac{\underline{B} \vdash_{\triangleleft} A \oplus \Delta}{\underline{A \oslash B} \vdash_{\triangleleft} \Delta}$$

- Multiplicative HR If Δ together with A produces a B, then Δ alone produces B provided an A is given (and vice versa.)
- Additive HR If by adding up the A and Δ debits, we obtain the B debit, than from the Δ debit, we obtain a B debit without the A debit (and vice versa.)

Lambek and Dual Lambek Calculi A sentence can be built with a bottom up or top down view by putting together either expressions (Lambek) or contexts (dual Lambek).

$$\underbrace{np}_{\text{[Lori}} \otimes \underbrace{((np\backslash s)/np \otimes (np/n \otimes n))}_{\text{[teases [the student]]}} \vdash s \ \text{ dually } \ s \vdash \underbrace{((n \oplus (n \oslash np)) \oplus ((np \oslash s) \oslash np))}_{\text{[[student the] teases]}} \oplus \underbrace{np}_{\text{Lori]}}$$

No gain.

6.3. Lambek Grishin

(a) We need the two "views" to work together and

(b) We need to express that some words require both views.

(a) Grishin Postulates (Linear Distributivity)

$$
\begin{array}{ll}
(\otimes\ \mathrm{MA}) & (B \oslash C) \otimes A \vdash B \oslash (C \otimes A) \\
(\otimes\ \mathrm{MC}) & A \otimes (B \oslash C) \vdash B \oslash (A \otimes C)
\end{array}
$$

Claim scope phenomena are not an evidence against the principle of compositionality, the meaning of the whole it's indeed a function of the meaning of its parts, but the latter are both expressions and contexts.

Scope operators are a combination of expression and context: they are syntactic expressions that act locally and are holes that can be filled in by the structure in which their twin expression is embedded.

6.4. Lambek-Grishin Calculus: full picture

$$\text{(REF)} \quad A \vdash A$$

$$\text{(TRAN)} \quad \text{if} \quad A \vdash B \text{ and } \quad B \vdash C, \text{then} \quad A \vdash C$$

$$\text{(RES)} \; C \vdash A \backslash B \quad \text{iff} \quad A \otimes C \vdash B \quad \text{iff} \quad A \vdash B/C$$

$$\text{(DRES)} \; (B \oslash A) \vdash C \quad \text{iff} \quad B \vdash C \oplus A \quad \text{iff} \quad C \obslash B \vdash A$$

$$(\otimes\ \text{MA}) \quad (B \obslash C) \otimes A \vdash B \obslash (C \otimes A) \qquad (\oplus\ \text{MA}) \quad (A \oplus C)/B \vdash A \oplus (C/B)$$

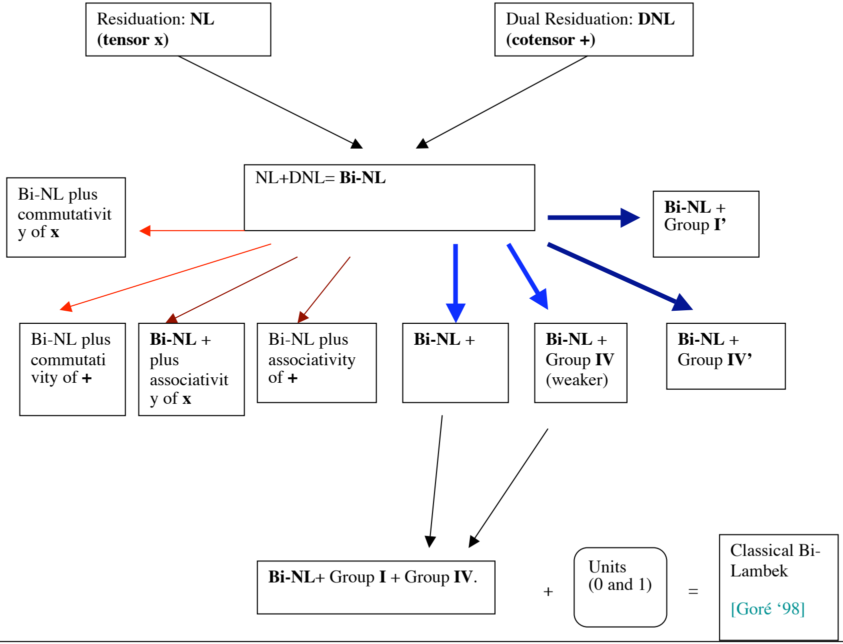
$$(\otimes\ \text{MC}) \quad A \otimes (B \obslash C) \vdash B \obslash (A \otimes C) \qquad (\oplus\ \text{MC}) \quad (C \oplus A)/B \vdash (C/B) \oplus A$$

$$(\otimes\ \text{MA}^{\sim}) \quad A \otimes (C \oslash B) \vdash (A \otimes C) \oslash B \qquad (\oplus\ \text{MA}^{\sim}) \quad B \backslash (C \oplus A) \vdash (B \backslash C) \oplus A$$

$$(\otimes\ \text{MC}^{\sim}) \quad (C \oslash B) \otimes A \vdash (C \otimes A) \oslash B \qquad (\oplus\ \text{MC}^{\sim}) \quad B \backslash (A \oplus C \vdash A \oplus (B \backslash C)$$

Plus, unary operators. (a bit about them tomorrow)

6.5. Family of Logics



7. Summing up

We have seen that

- To analyze linguistic structures we need to determine the category of any constituents.
- This means we need to compose and decompose structures.
- Composition and Decompositions are captured by Residuation.
- Substructural Logics are the logics sharing this property.
- They differ on their structural rules.
- (Global) Structural Rules do not capture the core of natural language assembly.
- Hence, we are interested in the pure calculus of residuation.
- The pure calculus of residuation is mirrored by its dual, the two calculi need to interact via MA and MC postulates (studied by Grishin) to gain expressivity.
- Lambek Grishin calculus captures also the miss-match between syntax-semantics exhibited by scope operators in natural language.

Next time we look at the syntax-semantic relation.

8. Other MCS systems

See also:

- Steedman: Combinatory Categorial Grammar (CCG)
- Joshi: Tree Adjoining Grammars (TAG)
- Morrill and Fadda: Discontinuous Lambek Calculi