Logic: Introduction

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1. What is Logic?

Lewis Carroll "Through the Looking Glass":

> "Contrariwise", continued Tweedledee, "if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic."

Question What's your answer?

Moshe Vardi's students

- the ability to determine correct answers through a standardized process
- the study of formal inference
- a sequence of verified statements
- reasoning, as opposed to intuition
- the deduction of statements from a set of statements

Wikipedia

> Logic [...] is most often said to be the study of criteria for the evaluation of arguments [..], the task of the logician is: to advance an account of *valid and fallacious inference* to allow one to distinguish logical from flawed arguments.

2. History

- Traditionally, logic is studied as a branch of *philosophy*.
- Since the mid-nineteenth century logic has been commonly studied in *mathematics* and law.
- More recently logic is applied in *computer science* and artificial intelligence.

Question What do you know of Logic & Philosophy, and Logic & Computer Science?

2.1. The First Age of Logic: Symbolic Logic (500 B.C. - 19th Century)

- logic was originally studied by the Sophists, who engaged in *formal debates*
- eventually, they sought to devise an objective *system of rules* to determine beyond any doubt who had won a debate
- so originally logic dealt with arguments in natural language used by humans
- natural language is *very ambiguous*
- natural language lead also to *paradoxes*

  “This sentence is a lie”

2.2. The First Age of Logic: Symbolic Logic (500 B.C.)

It was used to demonstrate the correctness of arguments like the following:

All men are mortal

Socrates is a man

Therefore, Socrates is mortal

If we change *all* to *some* the argument doesn't hold. Expressions like these two play a crucial role in reasoning.

All N are B

C is a N

Therefore, C is B

2.3. Ambiguity

There can be some which are *ambiguous* like *any*

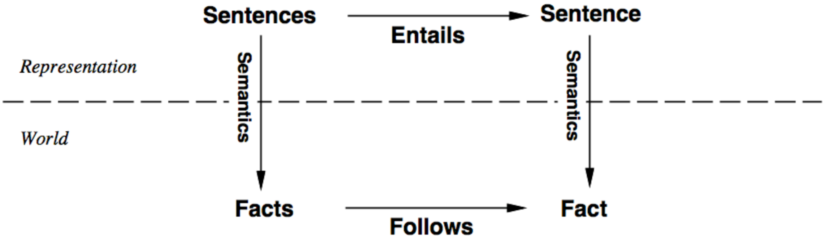
> "Eric does not believe that Mary can pass *any* test"

it could be taken to mean either *all* or *one*.

Paradoxes and the ambiguities of natural language led to the effort to formulate logic in a *symbolic language*.

For more information on the History of Logic read Moshe Vardi, "A Brief History of Logic".

3. Logic



A logic allows the axiomatization of the domain information, and the drawing of conclusions from that information.

- Syntax
- Semantics
- Logical inference = *reasoning*

3.1. Our main concern

Modern Logic teaches us that one claim is a *logical consequence* of another if

> there is no way the latter could be *true* without the former also being true.

It is also used to *disconfirm* a theory

> if a particular claim is a logical consequence of a theory, and we discover that the claim is *false*, then we know the theory itself must be incorrect in some way or another.

Examples of theories: physical theory; economic theory, etc.

Our main concern in this course how to recognize when a specific claim follows logically from others, and conversely, when it does not. Better,

we will look at methods to *prove* that one claim is a logical consequence of others, or that it is not.

3.2. Important Questions in Logic

- **Expressive Power** of representation language

⇝ able to *represent* the problem

- **Correctness** of entailment procedure

⇝ *no false* conclusions are drawn

- **Completeness** of entailment procedure

⇝ *all correct* conclusions are drawn

- **Decidability** of entailment problem

⇝ there exists a (terminating) algorithm to compute entailment

- **Complexity**

⇝ resources needed for computing the solution

3.3. What is a Logic

Clearly distinguish the definitions of:

- the *formal language*
  - Syntax
  - Semantics
  - Expressive Power
- the *reasoning problem* (e.g., entailment)
  - Decidability
  - Computational Complexity
- the *problem solving procedure*
  - Soundness and Completeness
  - (Asymptotic) Complexity

3.4. The ideal Logic

- Expressive
- With decidable reasoning problems
- With sound and complete reasoning procedures
- With efficient reasoning procedures – possibly sub-optimal

3.5. Many Logics

- Propositional Logic
- First Order Logic
- Modal Logic
- Temporal Logic
- Relevant Logic
- . . .

3.6. Types of Logics

- Logics are characterized by what they commit to as “primitives”
- Ontological commitment: what exists—facts? objects? time? beliefs?
- Epistemological commitment: what states of knowledge?

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief 0…1
Fuzzy logic	degree of truth	degree of belief 0…1

4. Logic and Computer Science

Computer scientists design and study systems through the use of formal languages that can themselves be interpreted by a formal system.

Formal languages of modern logic serve as a working tool for computer science. Some of the most basic applications of this tool are:

- Boolean circuits: The design of hardware built out of gates that implement Boolean logic primitives.
- Some problems seem to be so hard that computers cannot solve them no matter how fast they are. The reason for the difficulty is a combinatorial explosion that seem to be inherent in this problem. Logic plays a crucial role in the development of the theory of NP-completeness, which formalize the concept of combinatorial explosion.
- Semantics: To make sure that different implementation of a programming language yield the same results, programming languages need to have a formal semantics. Logic provide the tool to develop such a semantics.

- Design Validation and verification: to verify the correctness of a design with a certainty beyond that of conventional testing. It uses temporal logic .
- AI: mechanized reasoning and expert systems.
- Security: With increasing use of network, security has become a big issue. Hence, the concept of “proof-carrying code” (proof of safety and correctness)
- Semantic Web: Description Logics have been used as web ontology languages.

Logic is sometime described as the “calculus of computer science”.

5. Outline of the Course

Lectures' topics. Theory plus exercises on

1. Propositional Logic:
   - Truth Tables (13.10, 20.10)
   - Tableuax (27.10, 03.11)
2. First Order Logic
   - Truth Tables (10.11, 17.11)
   - Tableuax (24.11, 01.12)
3. Applications (15.12) [not part of the exam]
4. Proof Theory (22.12) [not part of the exam]
5. Summary (12.01.2007)

6. Learning Outcomes

You will learn how to:

- Use reasoning tools (Truth Tables and Tableaux) to prove that a claim is a logical consequence of others (or is not).
- How to formalize a problem in PL and FOL so to solve it and check logical consequences.

7. Administrativia

- Lectures (2hrs per week – Fridays 08:30-10:30)
- Lab instructor: Rosella Gennari (1hr per week – Fridays 10:30-11:30)
- Recommended texts:
  - Main text book: “The Essence of Logic”, by John Kelly. Prentice Hall, 1997. Ch. 1, 2, 6, 8 (don’t miss a word of them!)
  - A Mathematical Introduction to Logic, by H. Enderton.
    Mathematical Logic, by H. D. Ebbinghaus, J. Flum, W. Thomas.
    Language, Proof and Logic, by Jon Barwise and John Etchemendy.
- Assessment:
  Mid-term: 30% (planned for the 22nd of December)
  Final written exam: 70% or 100% for those who have not done the mid-term.
- Office hours: use them!