Logic: First Order Logic

Raffaella Bernardi

bernardi@inf.unibz.it
P.zza Domenicani 3, Room 2.28

Faculty of Computer Science, Free University of Bolzano-Bozen

http://www.inf.unibz.it/~bernardi/Courses/Logic06

Contents

1. Summary of the Last Lesson

- scope of a quantifier
- free vs. bound variable
- different order of quantifiers & different meaning
- domain of interpretation (Δ)
- value assignment –for variables– (α)
- interpretation of FOL formulae ($\mathcal{I}$)
- satisfiability of FOL formulae
- entailment of FOL formulae

2. Basic exercises

2.1. Exercises: Satisfiability of a given FOL formula

a) say whether it is unsatisfiable, satisfiable, or valid, b) in the cases when an interpretation exists, show it.

1. $\forall x(P(x)) \wedge \exists y(\neg P(y))$
2. $\forall x(P(x)) \rightarrow \exists y(P(y))$
3. $\exists y(P(y) \rightarrow \forall x.P(x))$
4. $P(a) \rightarrow \neg(\exists x.P(x))$
5. $\exists x(P(x)) \rightarrow \forall y(\neg P(y))$
6. $\forall x(P(x)) \rightarrow \exists x(\neg P(x))$
7. $\exists x(\neg P(x)) \rightarrow \forall x(P(x))$

2.2. Exercises: Entailment among given FOL formulae

Prove whether the following entailments are valid and give a counter-example (a domain and an interpretation) if they are not.

- $\forall x(F(x) \rightarrow G(x)), \neg\exists x(G(x)) \models \neg F(a)$
- $\neg\exists x(F(x) \wedge G(x)) \models \neg F(a)$
- $\forall x(F(x)) \rightarrow \forall x(G(x)), \neg\exists x G(x) \models \exists x \neg F(x)$

2.3. Exercise: Validity of an argument

a) Check by means of tableaux method whether the argument below is valid.

$$\forall y(\texttt{Suspect}(y)), \exists x(\texttt{Murder}(x)) \models \neg\exists x(\forall y(\texttt{Suspect}(x) \rightarrow \texttt{Murder}(y))).$$

b) Build a counterexample if the argumentation is not valid.

3. More complex problems

4. Administrativia

Change in the schedule: again!! :

- 1st of December: two hours (09:30-11:30) with Rosella Gennari (as announced last time!).
- 6th of December: one hour (17:00-18:00) with Raffaella Bernardi (last time I made a mistake!).
- 15th of December: one hour (08:30-09:30) to ask all your last minute doubts. (09:30-11:30): mid term (= 30% of the exam). (change asked by some of you!)
- 22nd of December: three hours (08:30-11:30) with Raffaella Bernardi

Topics for the mid-term will be listed on the course web site by the 6th of December.