FREIE UNIVERSITÄT BOZEN
LIBERA UNIVERSITÀ DI BOLZANO
FREE UNIVERSITY OF BOZEN - BOLZANO

FACULTY OF COMPUTER SCIENCE, PIAZZA DOMENICANI 3, I-39100 BOZEN-BOLZANO, ITALY

Logic: 2006-2007

Written Examination

15th of December 2006

First name		Last name	
STUDENT NUMBER		SIGNATURE	

Instructions:

- Write clearly First Name, Last Name, Student Number and Signature where indicated. In absence of this information, the exam paper will not be marked.
- Do not cheat or speak to any other student during the examination. If you cheat or speak to another student, your exam will be not valid.
- Use a pen, not a pencil.
- Write neatly and clearly.

Specific instructions related to this exam:

- This assignment will constitute the 30% of the overall coursework marking. Positive marks are rentained for the next three exam sessions.
- Note, it is required to write down the answers in a very precise way, and in all formal details. Part of the assessment will be based on the ability to correctly use the logical notation. Please, attach this cover sheet to your answers.

1) Propositional Logic. [4 points]

Check by means of truth tables whether the following entailments are valid. If an entailment is not valid, mark the interpretations that falsify it.

- $\{P \vee Q, \neg P, P \rightarrow (Q \wedge \neg P)\} \models Q$
- $\{P \rightarrow Q, P \rightarrow \neg Q\} \models \neg P$
- $\{P \rightarrow Q\} \models \neg(Q \rightarrow P)$
- $\{P, \neg P\} \models Q$

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2) Propositional logic. [4 points]

Check whether the following argument is valid: (a) formalize it in propositional logic (remember to give the keys of your formalization); (b) find the solution using the truth table or tableau procedure; (c) show an interpretation that falsifies the argument if it is not valid.

> If you are dancing on the moon, then you are alive. You are alive. Therefore, you are dancing on the moon.

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3) Propositional logic. [7 points]

Consider the following situation.

> Suppose that a village is inhabited by exactly two persons, Timoty and Sam. If Timoty likes an inhabitant then this likes Sam. But if Sam likes an inhabitant then this does not like Timoty. Can it be that Timoty likes himself?

(a) Formalise the essential facts using a suitable propositional language and *(b)* use truth tables or tableaux to establish whether the argument is satisfiable (a.k.a. consistent in the terminology of Kelly).

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4) First Order Logic. [5 points]

Let $\mathcal{L} = \langle P, S \rangle$ be a first-order language where S and P are a binary and a unary relation symbol, respectively. Given the $\mathcal{L}$-interpretation I with domain $\{0, 1\}$,

$$\begin{aligned} P^I &= \{1\}, \\ S^I &= \{(0,1), (1,1)\}, \end{aligned}$$

check whether I is a model of the following formulas and justify your answers:

$$\forall x(\exists y(S(x,y) \rightarrow P(y)))$$
$$\exists x(\forall y(S(x,y) \wedge P(y)))$$

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5) First Order Logic. [10 points]

Check by means of tableaux whether the following entailments are valid. If an entailment is not valid give a counter-example.

- $\exists x(F(x) \wedge G(x)) \models (\exists x F(x)) \wedge (\exists x G(x))$
- $\{\exists x F(x), \exists x G(x)\} \models \exists x(F(x) \wedge G(x))$