Logic: Fallacies of reasoning

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Contents

1. Still to practice

- Formalization of problems: make explicit implicit knowledge needed to infer the solution. (in class: today, and in January.)
- practice with CNF (on your own, and ask for feedbacks if you have doubts.)

2. Animal problem

Consider the following problem

1. The only animals in this house are cats.
2. Every animal that loves to look at the moon is suitable for a pet.
3. When I detest an animal, I avoid it.
4. All animals that don't prowl at night are carnivorous.
5. No cat fails to kill mice.
6. No animals ever take to me, except the ones in this house.
7. Kangaroos are not suitable for pets.
8. All animals that are carnivorous kill mice.
9. I detest animals that do not take to me.
10. Animals that prowl at night always love to look at the moon.

Is it true that I always avoid kangaroo?

(a) Represent the facts above as FOL sentences and formalize the problem. Remember to give the keys of your formalization and add the extra information you might need in order to answer the question. (b) Give the proof of your answer by means of tableaux.

2.1. Solution: Animal Problem

$\texttt{Animal}(x)$:	"x is an animal"
$\texttt{Cat}(x)$:	"x is a cat"
$\texttt{Inhouse}(x)$:	"x is in this house"
$\texttt{Detest}(x,y)$:	"x detests y"
$\texttt{Avoid}(x,y)$:	"x avoids y"
$\texttt{Take}(x,y)$:	"x takes to y"
$\texttt{Kan}(x)$:	"x is a kangaroo"

$$\forall x.(\texttt{Animal}(x) \wedge \texttt{Inhouse}(x)) \longrightarrow \texttt{Cat}(x) \qquad [1.]$$

$$\forall x.\texttt{Detest}(r,x) \longrightarrow \texttt{Avoid}(r,x) \qquad [3.]$$

$$\forall x.(\texttt{Animal}(x) \wedge \neg\texttt{Inhouse}(x)) \longrightarrow \neg Take(x,r) \qquad [6.]$$

$$\forall x.(\texttt{Animal}(x) \wedge \neg\texttt{Take}(x,r)) \longrightarrow \texttt{Detest}(r,x) \qquad [9.]$$

$$\forall x.\texttt{Kan}(x) \longrightarrow \texttt{Animal}(x) \qquad [\text{extra}]$$

$$\forall x.\texttt{Cat}(x) \leftrightarrow \neg\texttt{Kan}(x) \qquad [\text{extra}]$$

$$\neg(\forall x.\texttt{Kan}(x) \longrightarrow \texttt{Avoid}(r,x)) \qquad [\text{negation of Con.}]$$

3. Fallacies of reasoning

Fallacies are mistakes that occur in arguments and that superficially appear to be good arguments.

There are many kinds of fallacies, and philosophers and logicians have identified patterns of bad reasoning habits.

Here I give a few examples. See John Nolt, Dennis Rohatyn and Achille Varzi *Logic*. for a detailed description.

3.1. Fallacies of Relevances

They occur when the premises of an argument have no bearing upon its conclusion.

There are a number of these fallacies, e.g. *ad hominem abusive* (i.e. arguments that attack a person's age, character, family, etc. when there is no reason to take the person's views seriously).

Jones advocates fluoridation of the city water supply.
Jones is a convicted thief.
Therefore, we should not fluoridate the city water supply.

Remark Even if Jones is a convicted thief, this has no bearing on whether the water supply should be fluoridated.

3.2. Circular Reasoning

They occur when an argument assumes its own conclusion. Such an argument is always true, but it is useless as a means of proving its conclusion.

1. Example Eating ice cream in public is immoral because it is just plain wrong.

   Remark The premise "Eating ice cream in public is plain wrong" and the conclusion "Eating ice cream in public is immoral". These two sentences say basically the same think.

2. Example

   1. Capital punishment is justified.
   2. For our country is full of criminals who commit barbarous acts of murder. Therefore, it is perfectly legitimate to punish such inhuman people by putting them to death.

   Remark The conclusion and the first premise say the same thing.

3.3. Semantic Fallacies

They occur when the language employed to express an argument has multiple meanings or is vague. E.g.

1. It is silly to fight over mere words.
2. Discrimination is just a word

Therefore, it is silly to fight over discrimination

3.4. Inductive Fallacies

They occur when an inductive probability of an argument (i.e. the probability of its conclusion given its premises) is low.

1. The patient became violently ill immediately after eating lunch.
   There were no sign of illness prior to eating, and she was in good spirits during the meal.
2. She is in good health overall, and her medical history shows no record of physical problems.
   Therefore, she was the victim of food poisoning.

3.5. Formal Fallacies

They occur when 1.) we misapply a valid rule of inference or else 2.) follow a rule which is demonstrably invalid.

If a formal fallacy is suspected, it is important to ascertain both that 1.) the rule on which the reasoning seems to be based is invalid, 2.) the argument itself is invalid (by means of giving a counter-example)

3.5.1. Formal Fallacies: example

For instance:

If it rains heavily tomorrow, the game will be postponed.
It will not rain heavily tomorrow
Therefore, the game will not be postponed.

Remark

1. The argument has an invalid form:

   $R \rightarrow P$
   $\neg R$
   $\neg P.$

   the invalidity of the rule can be verified by e.g. truth tables.

2. The counter-example (premises both true and conclusion false.) to show the invalidity of the argument, could be, e.g., The game will be postponed because it will snow heavily, or the visiting team misses the flight etc.

3.5.2. Formal Fallacies: example

For instance:

If Smith inherited a fortune, then she is rich.
She is reach.
Therefore, she inherited a fortune.

Remark This is a fallacy of *affirming the consequent.*

1. The argument has an invalid form:

   $P \to Q$
   Q
   P.

   The invalidity of the rule can be verified by e.g. truth tables.

2. The counter-example (premises both true and conclusion false.) E.g., Smith made the fortune by creating a software corporation.

3.5.3. Formal Fallacies: example

For instance:

Every sentence in this slide is well written.
Therefore, this slide is well written

Remark This is a typical fallacy of *composition*.

$p_1, \ldots, p_n$ are parts of w.
$p_1, \ldots, p_n$ have property F.
Therefore, w has property F

3.5.4. Formal Fallacies: example

For instance:

This slide is written in English.
Therefore, every sentence in this book is in English.

Remark This is a typical fallacy of *division*.

w has property F
$p_1, \ldots, p_n$ are parts of w.
Therefore, $p_1, \ldots, p_n$ have property F.

4. Additions: Language

Consider the assertion of “Jo Ann’s father likes music”. How would you represented it in FOL?

$$\exists x.(F(x, j) \wedge L(x, m))$$

Is this ok?

This reads as “Jo Ann has at least one father and he likes music”. This is not what the sentence above meant.

We can use *function symbols* to refer to specific objects, so that $f(a)$ is “Paul”. We have:

$$L(f(a), m)$$

4.1. Functions: Exercise 1

Consider a first order language where g is a two-placed function symbol and R is a binary predicate symbol. Consider an interpretation I with domain $\{a, b\}$, where:

$g^I = \{(a, a) \mapsto a, (a, b) \mapsto b, (b, a) \mapsto a, (b, b) \mapsto a\}$

$R^I = \{(a, a), (a, b), (b, a), (b, b)\}$

Check whether I is a model of the formula

$\exists x.(\exists y.R(y, g(x, x)) \wedge \exists y.R(g(x, y), x)).$

4.2. Equality: Example

- Equality is a special predicate.
- $t_1 = t_2$ is true under a given interpretation $(\mathcal{I}, \alpha \models t_1 = t_2)$ if and only if t_1 and t_2 refer to the same object: ${t_1}^{\mathcal{I},\alpha} = {t_2}^{\mathcal{I},\alpha}$

E.g., $\forall x.\ (\times(sqrt(x), sqrt(x)) = x)$ is satisfiable
$2 = 2$ is valid

4.3. Equality: Exercise

Try to define (full) *Sibling* in terms of *Parent*:

$$
\begin{aligned}
&\forall x, y\,\boldsymbol{.}\; Sibling(x, y) \leftrightarrow \\
&\qquad ((x \neq y) \wedge \\
&\qquad\; \exists z, v\,\boldsymbol{.}\; (z \neq v) \wedge Parent(z, x) \wedge Parent(v, x) \wedge \\
&\qquad\qquad\quad Parent(z, y) \wedge Parent(v, y)\ )
\end{aligned}
$$

4.4. Functions: Exercise

Consider a first order language where f is a one-placed function symbol and P is a unary predicate symbol. Let Φ be the following sentence:

$\exists x.(\exists y.(Q(x,y) \wedge f(x) \neq f(y)) \rightarrow P(x))$

Define two interpretations I_1 and I_2, each with underlying domain $\{1,2,3\}$, such that $I_1 \models \Phi$ and $I_2 \not\models \Phi$.

5. Exercise

Housing lotteries are often used by university housing administrators to determine which students get first choice of dormitory rooms.

Consider the following problem:

1. Bob is ranked immediately ahead of Jim.
2. Jim is ranked immediately ahead of a woman who is a biology major.
3. Lisa is not near to Bob in the ranking.
4. Mary or Lisa is ranked first.

Is it true that Jim is immediately ahead of Lisa?

6. Administrativa

- Note: change of schedule. The 3 hours of the 12th of January have been moved to the Friday 19th of January (08:30-10:30 plus 10:30-11:30).
- Exam: 8th of February.

Next time:

- Decidability and Complexity of FOL.
- Where is Logic used?
- More exercises on formalization.

Before the exam, I will be away: 10.01.07-12.01.07, 23.01.07-06.02.07. During these days, I will be reading urgent e-mails only (i.e. not yours :)). Hence, if you have questions after the 23rd of Jan., please contact Rosella Gennari.