Logica & Linguaggio: Tablaux

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Contents

1. Animal problem

Consider the following problem

1. The only animals in this house are cats.
2. Every animal that loves to look at the moon is suitable for a pet.
3. When I detest an animal, I avoid it.
4. All animals that don't prowl at night are carnivorous.
5. No cat fails to kill mice.
6. No animals ever take to me, except the ones in this house.
7. Kangaroos are not suitable for pets.
8. All animals that are carnivorous kill mice.
9. I detest animals that do not take to me.
10. Animals that prowl at night always love to look at the moon.

Is it true that I always avoid kangaroo?

(a) Represent the facts above as FOL sentences and formalize the problem. Remember to give the keys of your formalization and add the extra information you might need in order to answer the question. (b) Give the proof of your answer by means of tableaux.

1.1. Solution: Animal Problem

$\texttt{Animal}(x)$:	"x is an animal"
$\texttt{Cat}(x)$:	"x is a cat"
$\texttt{Inhouse}(x)$:	"x is in this house"
$\texttt{Detest}(x,y)$:	"x detests y"
$\texttt{Avoid}(x,y)$:	"x avoids y"
$\texttt{Take}(x,y)$:	"x takes to y"
$\texttt{Kan}(x)$:	"x is a kangaroo"

$$\forall x.(\texttt{Animal}(x) \wedge \texttt{Inhouse}(x)) \rightarrow \texttt{Cat}(x) \qquad [1.]$$

$$\forall x.\texttt{Detest}(r,x) \rightarrow \texttt{Avoid}(r,x) \qquad [3.]$$

$$\forall x.(\texttt{Animal}(x) \wedge \neg\texttt{Inhouse}(x)) \rightarrow \neg\texttt{Take}(x,r) \qquad [6.]$$

$$\forall x.(\texttt{Animal}(x) \wedge \neg\texttt{Take}(x,r)) \rightarrow \texttt{Detest}(r,x) \qquad [9.]$$

$$\forall x.\texttt{Kan}(x) \rightarrow \texttt{Animal}(x) \qquad [\text{extra}]$$

$$\forall x.\texttt{Cat}(x) \leftrightarrow \neg\texttt{Kan}(x) \qquad [\text{extra}]$$

$$\neg(\forall x.\texttt{Kan}(x) \rightarrow \texttt{Avoid}(r,x)) \qquad [\text{negation of Con.}]$$

2. Coloring Problem

Given a map of adjacent regions and a set of colors, determine whether an assignment of colors to regions is possible (exists) such that no two adjacent regions have the same color. Axiomatize this problem in first-order logic for the following map and three colors:

```
+-----------------+
|      |          |
|      |   reg4   |
| reg1 |          |
|      +----------+
|      |          |
+------+--+       |
|         | reg3  |
|  reg2   |       |
|         |       |
+---------+-------+
```

3. Solutions

3.1. Coloring Problem

Graph Coloring Problem

- **Reflexivity of edges**: $\forall x.\forall y.(Con(x,y) \leftrightarrow Con(y,x))$
- **Coloring of nodes**: $\forall x.((Blue(x) \vee Green(x)) \vee Red(x))$
- **Uniqueness of colors per node**: $\forall x.(Blue(x) \leftrightarrow (\neg Green(x) \wedge \neg Red(x)))$
- **Irriflexivity of relation**: $\forall x.\neg Con(x,x)$

Secondly, we have to add the given constraints: the coloring function has to be such that no two connected vertices have the same color:

- **Constraint**: $\forall x.\forall y.(Con(x,y) \rightarrow ((Blue(x) \leftrightarrow \neg Blue(y)) \wedge (Red(x) \leftrightarrow \neg Red(y)) \wedge (Green(x) \leftrightarrow \neg Green(y))))$

Finally, we have to describe the domain.

$$
\begin{array}{ll}
Con(reg_1, reg_2) & Con(reg_1, reg_3) \\
Con(reg_1, reg_4) & Con(reg_2, reg_3) \\
Con(reg_3, reg_4) & \neg Con(reg_2, reg_4)
\end{array}
$$

$$\forall x.\forall y.(Con(x,y) \rightarrow (((((x = reg_1) \vee (x = reg_2)) \vee (x = reg_3)) \vee (x = reg_4)) \wedge ((((y = reg_1) \vee (y = reg_2))(y = reg_3))$$

Alternatively, we could treat the colors as object using the language below:

- Object := $blue, green, red, reg_1, reg_2, reg_3, reg_4$
- Relation := $Color(x,y), Con(x,y)$

With this language, we can formalize the problem as follows:

- **Reflexivity of edges**: $\forall x.\forall y.(Con(x,y) \leftrightarrow Con(y,x))$
- **Coloring of nodes**: $\forall x.((Color(x, blue) \vee Color(x, green)) \vee Color(x, red))$
- **Uniqueness of colors**: $(blue \neq green) \wedge (blue \neq red) \wedge (green \neq red)$

- **Irriflexivity of relation**: $\forall x.\neg Con(x,x)$

The constraint is reduced to the simple formula below.

- **Constraint**: $\forall x.\forall y.(Con(x,y) \rightarrow \neg\exists z.(Color(x,z) \wedge Color(y,z)))$

Finally, we have to describe the domain as we did above.