# Logical Structures in Natural Language: Exercises
# (to be done in class on Monday 10th)
# Propositional Logic

Università di Trento

## 1 Tableaux: Satisfiability of a formula

$$\begin{array}{c} A \wedge (B \wedge \neg A) \\ A \\ B \\ \neg A \\ \mathrm{X} \end{array}$$

All the branches are closed, the formula is unsatisfiable.

$$\begin{array}{ccc} & (A \to B) \to \neg B & \\ \neg(A \to B) & & \neg B \\ A & & \\ \neg B & & \end{array}$$

The formula is satisfiable.

$$\begin{array}{cccc} & A \to (B \to A) & & \\ \neg A & & B \to A & \\ & & \neg B & A \end{array}$$

The formula is satisfiable.

$$\begin{array}{ccc} & (B \to A) \to A & \\ \neg(B \to A) & & A \\ B & & \\ \neg A & & \end{array}$$

The formula is satisfiable.

$$\begin{array}{cccc} & A \to (B \vee \neg C) & & \\ \neg A & & B \vee \neg C & \\ & & B & \neg C \end{array}$$

The formula is satisfiable.

## 2 Tableaux: Satisfiability of a set of formulas

| | | | | |
|---|---|---|---|---|
| 1. | | $\neg B \to B$ | | |
| 2. | | $\neg(A \to B)$ | | |
| 3. | | $\neg A \vee \neg B$ | | |
| 4. | | $A$ | | from 2 |
| 5. | | $\neg B$ | | from 2 |
| 6. | $B$ | | $B$ | from 1 |
| 7. | X | $\neg A$ | $\neg B$ | from 3. |
| | | X | X | |

The set of formulas is unsatisfiable.

The second set of formulas is also unsatisfiable.

## 3 Tableaux: Tautology

We look at those formulas that we have shown to be satisfiable and check whether they are a tautology. We negate the formula, if its negation is unsatasifiable (all branches close) the formula is a tautology.

P.S. Notice, there is typo in the premise. I did the tablueax starting from a different premise than the one used above.There is a negation more.

$$\neg(\neg(A \to B) \to \neg B)$$
$$\neg(A \to B)$$
$$B$$
$$A$$
$$\neg B$$
$$\text{X}$$

$\neg(\neg(A \to B) \to \neg B)$ is unsatisfiable. Hence $\neg(A \to B) \to \neg B$ is a tautology.

| | | |
|---|---|---|
| | $\neg((B \to A) \to A)$ | |
| | $B \to A$ | |
| | $\neg A$ | |
| $\neg B$ | | $A$ |
| | | X |

$\neg((B \to A) \to A)$ is satisfiable, hence $(B \to A) \to A$ is falsifiable.