Logical Structures in Natural Language: Exercises
(to be done in class on Thursday 6th)
Propositional Logic

Università di Trento

1   Formalization problems

Check whether the following arguments are valid

(i) Represent the arguments formally, starting from the atomic formulae; (ii) Use truth tables to show the argument validity. Build a counter-example if the argument is not valid.

1. If all the students of the Logic Lab will pass the assignment, then Logic is an easy topic or they are very smart. All the students of the Logic Lab are very smart. Therefore, all the students of the Logic Lab will pass the exam.

2. If Spain reached the World Cup finals, then either Ireland didn't slip up or Denmark played very well. If Spain reached the World Cup finals then Ireland slipped up. Denmark did not play very well. Therefore, Spain reached the World Cup finals if and only if Ireland slipped up.

3. Smith cannot both be a running star and smoke cigarettes. Smith is not a running star. Therefore, Smith smokes cigarettes.

4. If Jones drove the car, Smith is innocent. If Brown fired the gun, then Smith is not innocent. Hence, if Brown fired the gun, then Joes did not drive the car.

2 Solutions

If all the students of the Logic Lab will pass the assignment, then Logic is an easy topic or they are very smart. All the students of the Logic Lab are very smart. Therefore, all the students of the Logic Lab will pass the exam.

- $p=$ all the students of the Logic Lab will pass the assignment
- $e=$ Logic is an easy topic
- $s=$ all the students of the Logic Lab are very smart.

$$\{p \to (e \vee s), s\} \models p$$

	p	e	s	$p \to$	$(e \vee s)$	s	p	$\models$
$\mathcal{I}_1$	T	T	T	T	T	T	T	T
$\mathcal{I}_2$	T	T	F	T	T	F	T	T
$\mathcal{I}_3$	T	F	T	T	T	T	T	T
$\mathcal{I}_4$	T	F	F	F	F	F	T	T
$\mathcal{I}_5$	F	T	T	T	T	T	F	F
$\mathcal{I}_6$	F	T	F	T	T	F	F	T
$\mathcal{I}_7$	F	F	T	T	T	T	F	F
$\mathcal{I}_8$	F	F	F	T	F	F	F	T

$\mathcal{I}_5$ and $\mathcal{I}_7$ give a counter-example of the entailment.

If Spain reached the World Cup finals, then either Ireland didn't slip up or Denmark played very well. If Spain reached the World Cup finals then Ireland slipped up. Denmark did not play very well. Therefore, Spain reached the World Cup finals if and only if Ireland slipped up.

- $s=$ Spain reached the World Cup finals
- $i=$ Ireland slipped up
- $d=$ Denmark played very well

$$\{s \to (\neg i \vee d), s \to i, \neg d\} \models s \leftrightarrow i$$

	s	i	d	$s \to$	$(\neg i$	$\vee d)$	$s \to i$	$\neg d$	$s \leftrightarrow i$	$\models$
$\mathcal{I}_1$	T	T	T	T	F	T	T	F	T	T
$\mathcal{I}_2$	T	T	F	F	F	F	T	T	T	T
$\mathcal{I}_3$	T	F	T	T	T	T	F	F	F	T
$\mathcal{I}_4$	T	F	F	F	T	F	F	T	F	T
$\mathcal{I}_5$	F	T	T	T	T	T	T	F	F	T
$\mathcal{I}_6$	F	T	F	T	F	F	T	T	F	F
$\mathcal{I}_7$	F	F	F	T	T	T	T	F	T	T
$\mathcal{I}_8$	F	F	T	T	T	T	T	T	T	T

$\mathcal{I}_6$ gives a counter-example of the entailment.

Smith cannot both be a running star and smoke cigarettes. Smith is not a running star. Therefore, Smith smokes cigarettes.

- $r=$ Smith is a running star

- $s=$ Smith smokes cigarettes

$$\{\neg(r \wedge s), \neg r\} \models s$$

	r	s	$\neg$	$(r \wedge s)$	$\neg r$	s	$\models$
$\mathcal{I}_1$	T	T	F	T	F	T	T
$\mathcal{I}_2$	T	F	T	F	F	F	T
$\mathcal{I}_3$	F	T	T	F	T	T	T
$\mathcal{I}_4^*$	F	F	T	F	T	F	F

The argument is not valid. The counter-example is given by $\mathcal{I}_4$, viz., $\mathcal{I}_4(r) = \mathcal{I}_4(s) = F$

If Jones drove the car, Smith is innocent. If Brown fired the gun, then Smith is not innocent. Hence, if Brown fired the gun, then Joes did not drive the car.

- $j =$ Jones drove the car

- $s =$ Smith is innocent

- $b =$ Brown fired the gun

$$\{j \rightarrow s, b \rightarrow \neg s\} \models b \rightarrow \neg j$$

	j	s	b	$j \rightarrow s$	$b \rightarrow$	$\neg s$	$b \rightarrow$	$\neg j$	$\models$
$\mathcal{I}_1$	T	T	T	T	F	F	F	F	T
$\mathcal{I}_2$	T	T	F	T	T	F	T	F	T
$\mathcal{I}_3$	T	F	T	F	T	T	F	F	T
$\mathcal{I}_4$	T	F	F	F	T	T	T	F	T
$\mathcal{I}_5$	F	T	T	T	F	F	T	T	T
$\mathcal{I}_6$	F	T	F	T	T	F	T	T	T
$\mathcal{I}_7$	F	F	T	T	T	T	T	T	T
$\mathcal{I}_8$	F	F	F	T	T	T	T	T	T

The entailment is valid.