

# Logical Structures in Natural Language: Exercises

## Propositional Logic

Università di Trento

### 1 Truth Tables (in class)

Build the truth tables for the following formulas and decide whether they are satisfiable, or a tautology or a contradiction.

- $(\neg A \rightarrow B) \wedge (\neg A \vee B)$
- $P \rightarrow (Q \vee \neg R)$

### 2 Truth Tables & Reasoning (in class)

Build the truth tables for the following entailments and decide whether they are valid

1.  $P \vee Q \models Q$
2.  $P \rightarrow Q, Q \rightarrow R \models P \rightarrow R$
3.  $P \rightarrow Q, Q \models P$
4.  $P \rightarrow Q \models \neg(Q \rightarrow P)$

### 3 Formalization and Reasoning (in class)

Verify whether the following arguments are valid: (i) Represent the arguments formally by starting from atomic formulae; (ii) Use the truth tables to show the validity of the argument; (iii) Build a counter-example if the argument is not valid (build a model in which the premises are true and the conclusion is false.)

1. If Paul lives in Dublin, he lives in Ireland. Paul lives in Ireland. Therefore Paul lives in Dublin.
2. If the temperature and air pressure remained constant, there was no rain. The temperature did remain constant. Therefore, if there was rain then the air pressure did not remain constant.

# Solutions

## 3.1 Exercise: Truth Tables

$$(\neg A \rightarrow B) \wedge (\neg A \vee B)$$

	$A$	$B$	$\neg A$	$\rightarrow B$	$\neg A$	$\vee B$	$(\neg A \rightarrow B) \wedge (\neg A \vee B)$
$\mathcal{I}_1$	T	T	F	T	F	T	T
$\mathcal{I}_2$	T	F	F	T	F	F	F
$\mathcal{I}_3$	F	T	T	T	T	T	T
$\mathcal{I}_4$	F	F	T	F	F	T	F

The formula is satisfied by  $\mathcal{I}_1$  and  $\mathcal{I}_3$  and falsified by  $\mathcal{I}_2$  and  $\mathcal{I}_4$ .

$$P \rightarrow (Q \vee \neg R)$$

	$P$	$Q$	$R$	$P \rightarrow (Q \vee \neg R)$
$\mathcal{I}_1$	T	T	T	T
$\mathcal{I}_2$	T	T	F	T
$\mathcal{I}_3$	T	F	T	F
$\mathcal{I}_4$	T	F	F	T
$\mathcal{I}_5$	F	T	T	T
$\mathcal{I}_6$	F	T	F	T
$\mathcal{I}_7$	F	F	T	T
$\mathcal{I}_8$	F	F	F	T

The formula is falsified by  $\mathcal{I}_3$  and satisfied by all the other interpretations.

## 3.2 Exercise: Truth Tables and Reasoning

$$P \vee Q \models Q$$

	$P$	$Q$	$P \vee Q$	$Q$	$\models$
$\mathcal{I}_1$	T	T	T	T	T
$\mathcal{I}_2^*$	T	F	T	F	F
$\mathcal{I}_3$	F	T	T	T	T
$\mathcal{I}_4$	F	F	F	F	T

The entailment is not valid:  $\{\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3\} \not\subseteq \{\mathcal{I}_1, \mathcal{I}_3\}$ , it is falsified by  $\mathcal{I}_2$ .

$$\{P \rightarrow Q, Q \rightarrow R\} \models P \rightarrow R$$

	$P$	$Q$	$R$	$P \rightarrow Q$	$Q \rightarrow R$	$P \rightarrow R$	$\models$
$\mathcal{I}_1$	T	T	T	T	T	T	T
$\mathcal{I}_2$	T	T	F	T	F	F	T
$\mathcal{I}_3$	T	F	T	F	T	T	T
$\mathcal{I}_4$	T	F	F	F	T	F	T
$\mathcal{I}_5$	F	T	T	T	T	T	T
$\mathcal{I}_6$	F	T	F	T	F	T	T
$\mathcal{I}_7$	F	F	T	T	T	T	T
$\mathcal{I}_8$	F	F	F	T	T	T	T

The entailment is valid.

$$\{P \rightarrow Q, Q\} \models P$$

	$P$	$Q$	$P \rightarrow Q$	$Q$	$P$	$\models$
$\mathcal{I}_1$	T	T	T	T	T	T
$\mathcal{I}_2$	T	F	F	F	T	T
$\mathcal{I}_3^*$	F	T	T	T	F	F
$\mathcal{I}_4$	F	F	T	F	F	T

The entailment is not valid:  $\{\mathcal{I}_1, \mathcal{I}_3\} \not\subseteq \{\mathcal{I}_1, \mathcal{I}_2\}$ , it is falsified by  $\mathcal{I}_3$ .

$$P \rightarrow Q \models \neg(Q \rightarrow P)$$

	$P$	$Q$	$P \rightarrow Q$	$\neg$	$(Q \rightarrow P)$	$\models$
$\mathcal{I}_1^*$	T	T	T	F	T	F
$\mathcal{I}_2$	T	F	F	F	T	T
$\mathcal{I}_3$	F	T	T	T	F	T
$\mathcal{I}_4^*$	F	F	T	F	T	T

The entailment is not valid:  $\{\mathcal{I}_1, \mathcal{I}_3, \mathcal{I}_4\} \not\subseteq \{\mathcal{I}_3\}$ , it is falsified by  $\mathcal{I}_1$  and  $\mathcal{I}_4$ .

### Exercise: Formalization and Reasoning

1.

- $d$ : Paul lives in Dublin,
- $i$ : Paul lives in Ireland

$$d \rightarrow i, i \models d$$

	$d$	$i$	$d \rightarrow i$	$i$	$d$	$\models$
$\mathcal{I}_1$	T	T	T	T	T	T
$\mathcal{I}_2$	T	F	F	F	T	T
$\mathcal{I}_3^*$	F	T	T	T	F	F
$\mathcal{I}_4$	F	F	T	F	F	T

The entailment is not valid  $\{\mathcal{I}_1, \mathcal{I}_3\} \not\subseteq \{\mathcal{I}_1, \mathcal{I}_2\}$ , it is falsified by  $\mathcal{I}_3$ .

2.

- $t$ : The temperature remained constant
- $a$ : The air pressure remained constant
- $r$ : there is rain

$$(t \wedge a) \rightarrow \neg r, t \models r \rightarrow \neg a$$

	t	a	r	$(t \wedge a) \rightarrow \neg r$	t	$r \rightarrow \neg a$	$\models$
$\mathcal{I}_1$	T	T	T	F	T	F	T
$\mathcal{I}_2$	T	T	F	T	T	T	T
$\mathcal{I}_3$	T	F	T	T	T	T	T
$\mathcal{I}_4$	T	F	F	T	T	T	T
$\mathcal{I}_5$	F	T	T	T	F	F	T
$\mathcal{I}_6$	F	T	F	T	F	T	T
$\mathcal{I}_7$	F	F	T	T	F	T	T
$\mathcal{I}_8$	F	F	F	T	F	T	T

The entailment is valid  $\{\mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4\} \subseteq \{\mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4, \mathcal{I}_6, \mathcal{I}_7, \mathcal{I}_8\}$