# Computational Linguistics: Semantics

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## 1. CL: what have we achieved?

Main challenge of CL: to deal with ambiguity at the different levels. Which kind of ambiguities are the following ones?

- ▶ La vecchia porta sbatte.
- ▶ La vecchia porta la sbarra.
- ▶ John saw the man with the telescope.
- ▶ John saw the woman in the park with the telescope. He was at home.

We have seen how to recognize/parse these sentences so to obtain different parse trees whenever necessary.

## 2. Semantics

**Semantics**: it's the study of the meaning of words and the reference of linguistic inputs. The former is studied in **Lexical Semantics** and the latter in **Formal Semantics**.

Lexical Semantics Words are seen as having a systematic structure that governs what they mean, how they relate to actual entities and how they can be used. Studies on this topic result into e.g. Dictionary or Ontologies like WordNET.

whereas we will look at Formal Semantics.

## 3. Formal Semantics: Main questions

The main questions are:

- 1. What does a given sentence mean?
- 2. How is its meaning built?
- 3. How do we infer some piece of information out of another?

The first and last question are closely connected.

In fact, since we are ultimately interested in understanding, explaining and accounting for the entailment relation holding among sentences, we can think of **the meaning of a sentence as its truth value**.

## 4. Logical Approach

To tackle these questions we will use Logic, since using Logic helps us answering the above questions at once.

- 1. Logics have a precise semantics in terms of **models** —so if we can translate/represent a natural language sentence S into a logical formula  $\phi$ , then we have a precise grasp on at least part of the meaning of S.
- 2. Important **inference problems** have been studied for the best known logics, and often good **computational implementations** exists. So translating into a logic gives us a handle on inference.

When we look at these problems from a computational perspective, i.e. we bring in the implementation aspect too, we move from Formal Semantics to **Computational Semantics**.

### 4.1. Model: individual constants

The interpretation of a formal language has to include a specification of what constants in the language refers to. This is done by means of the **interpretation function**  $\mathcal{I}$  which assigns an appropriate **denotation** in the model  $\mathcal{M}$  to each individual and *n*-place predicate constant.

If  $\alpha$  is an individual constant,  $\mathcal{I}$  maps  $\alpha$  onto one of the entities of the universe of discourse  $\mathcal{U}$  of the model  $\mathcal{M} : \mathcal{I}(\alpha) \in \mathcal{U}$ .

### 4.2. Model: one-place predicates

One-place properties are seen as sets of individuals: the property of being orange describes the **set of individuals** that are orange. Formally, for P a one-place predicate, the interpretation function  $\mathcal{I}$  maps P onto a subset of the universe of discourse  $\mathcal{U}: \mathcal{I}(P) \subseteq \mathcal{U}$ . For instance,

### 4.3. Model: two-place predicates

Two-place predicates such as "love", "eat", "mother-of" do not denote sets of individuals, but **sets of ordered pairs of individuals**, namely all those pairs which stand in the "loving", "eating", "mother-of" relations. We form ordered pairs from two sets A and B by taking an element of A as first member of the pair and an element of B as the second member. Given the relation R, the interpretation function  $\mathcal{I}$  maps R onto a set of ordered pairs of elements of  $\mathcal{U}: \mathcal{I}(R) \subseteq \mathcal{U} \times \mathcal{U}$ 

### 4.4. Example

Let our model be based on the set of entities  $E = \{ lori, ale, sara, pim \}$  which represent Lori, Ale, Sara and Pim, respectively. Assume that they all know themselves, plus Ale and Lori know each other, but they do not know Sara or Pim; Sara does know Lori but not Ale or Pim. The first three are students whereas Pim is a professor, and both Lori and Pim are tall. This is easily expressed set theoretically. Let [w] indicate the interpretation of w:

[sara]	=	sara;
[[pim]]	=	pim;
[lori]	=	lori;
[know]	=	$\{\langle \text{lori, ale} \rangle, \langle \text{ale, lori} \rangle, \langle \text{sara, lori} \rangle, \rangle$
		$\langle \text{lori, lori} \rangle, \langle \text{ale, ale} \rangle, \langle \text{sara, sara} \rangle, \langle \text{pim, pim} \rangle \};$
[student]	=	{lori, ale, sara};
[professor]	=	$\{\text{pim}\};$
[tall]	=	${\rm [lori, pim]}.$

which is nothing else to say that, for example, the relation **know** is the **set of pairs**  $\langle \alpha, \beta \rangle$  where  $\alpha$  knows  $\beta$ ; or that 'student' is the set of all those elements which are a student.

### 4.5. Relational and Functional Perspectives

Alternatively, one can assume a functional perspective and interpret, for example, **student** as a function from individual (entities) to truth values, student(enrico) = 1, student(raffaella) = 0.

The shift from the relational to the functional perspective is made possible by the fact that the sets and their characteristic functions amount to the same thing:

if  $f_X$  is a function from Y to  $\{0, 1\}$ , then  $X = \{y \mid f_X(y) = 1\}$ . In other words, the assertion  $y \in X$  and  $f_X(y) = 1$  are equivalent.

Therefore, the two notations y(z)(u) and y(u, z) are equivalent.

### 4.6. Exercises: Relations vs. Functions

Think of which function you can assign to the words in the model considered before and repeated here:

Sara, Pim, Lori, know, student, professor, tall,

## 4.7. Summing up

Summarizing, when trying to formalize natural language semantics, at least two sorts of objects are needed to start with: the set of truth values t, and the one of entities e.

Moreover, we spoke of more complex objects as well, namely functions. More specifically, we saw that the kind of functions we need are **truth-valued functions** (or boolean functions).

Furthermore, we have illustrated how one can move back and forwards between a **set/relational and a functional perspective**. The former can be more handy and intuitive when reasoning about entailment relations among expressions; the latter is more useful when looking for lexicon assignments.

References: Keenen 85.

## 5. Formal Semantics: What

#### What does a given sentence mean?

The meaning of a sentence is its truth value. Hence, this question can be rephrased in "Which is the meaning representation of a given sentence to be evaluated as true or false?"

- ▶ Meaning Representations: Predicate-Argument Structures are a suitable meaning representation for natural language sentences. E.g. the meaning representation of "Vincent loves Mia" is loves(vicent,mia) whereas the meaning representation of "A student loves Mia" is ∃x.student(x)∧ loves(x,mia).
- Interpretation: a sentence is taken to be a proposition and its meaning is the truth value of its meaning representations. E.g.

 $[\exists x.\mathtt{student}(x) \land \mathtt{left}(x)] = 1$  iff standard FOL (First Order Logic) definitions are satisfied.

## 6. Formal Semantics: How

#### How is the meaning of a sentence built?

To answer this question, we can look back at the example of "Vincent loves Mia". We see that:

- "Vincent" contributes the constant vincent
- ▶ "Mia" contributes the constant mia
- ▶ "loves" contributes the relation symbol loves

This observation can bring us to conclude that the **words** making up a sentence contribute all the bits and pieces needed to build the sentence's meaning representation.

In brief, meaning flows from the lexicon.

### 6.1. Formal Semantics: How (cont'd)

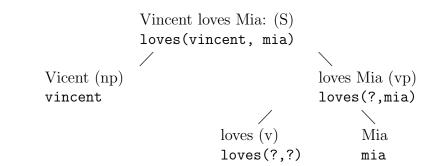
But,

- 1. Why the meaning representation of "Vincent loves Mia" is not love(mia, vincent)?
- 2. What does "a" contribute to in "A student loves Mia"?

As for 1., the missing ingredient is the **syntactic structure**! [Vincent [loves<sub>v</sub>  $Mia_{np}]_{vp}]_s$ .

We will come back to 2. next time.

#### 6.2. Formal Semantics: How (Cont'd)



Briefly, syntactic structure guiding gluing.

### 6.3. Compositionality

The question to answer is: "How can we specify in which way the bit and pieces combine?"

- 1. Meaning (representation) ultimately flows from the lexicon.
- 2. Meaning (representation) are combined by making use of syntactic information.
- 3. The meaning of the whole is function of the meaning of its parts, where "parts" refer to substructures given us by the syntax.

## 6.4. Ambiguity

A single linguistic sentence can legitimately have different meaning representations assigned to it.

For instance, "John saw a man with the telescope"

- a. John [saw [a man [with the telescope]<sub>pp</sub>]<sub>np</sub>]<sub>vp</sub>  $\exists x.Man(x) \land Saw(j,x) \land Has(x,t)$
- b. John [[saw [a man]<sub>np</sub>]<sub>vp</sub> [with the telescope]<sub>pp</sub>]<sub>vp</sub>  $\exists x.Man(x) \land Saw(j,x) \land Has(j,t)$

Different parse trees result into different meaning representations!

## 7. How far can we go with FOL?

FOL can capture the **what** (partially) and cannot capture the **how**, i.e. Problems with the "what":

- order ► Swimming is healthy. *Healthy(Swim)*: wrong! (property of property)
  - ▶ John has all the properties of Santa Clause  $\forall P(P(s) \rightarrow P(j))$ : wrong! (quantification over properties)
  - ▶ Red has something in common with green.  $\exists P(P(red) \land P(green))$ : wrong! (quant. over properties of properties)
  - adj. ► There was a red book on the table. ∃x(Book(x) ∧ Red(x) ∧ On\_the\_table(x)).
    ► There was a small elephant in the zoo. ∃x(Elephant(x) ∧ Small(x) ∧ In\_the\_zoo(x)).: wrong!
  - adv. ► Milly swam slowly. (modifier of the verb rather than of individuals!)
    ► Milly swam terribly slow (modifier of a modifier).

### 7.1. FOL: How?

Problems with the how:

**Constituents:** it cannot capture the meanings of constituents. **Assembly:** it cannot account for meaning representation assembly.

## 8. Building Meaning Representations

To build a meaning representation we need to fulfill three tasks:

- Task 1 Specify a reasonable syntax for the natural language fragment of interest.
- Task 2 Specify semantic representations for the lexical items.
- **Task 3** Specify the **translation** of constituents **compositionally**. That is, we need to specify the translation of such expressions in terms of the translation of their parts, parts here referring to the substructure given to us by the syntax.

Moreover, when interested in Computational Semantics, all three tasks need to be carried out in a way that leads to computational implementation naturally.

We have looked at Task 1 in lecture 2 and 3 (formal grammars) and at their computational side (Implementation in Prolog, Recognition and Parsing) during the Labs.

Today we will start looking at the other two tasks.

## 9. Lambda Calculus

FOL augmented with Lambda calculus can capture the "how" and accomplish tasks 2 and 3.

▶ It has a variable binding operators  $\lambda$ . Occurrences of variables bound by  $\lambda$  should be thought of as place-holders for missing information: they explicitly mark where we should substitute the various bits and pieces obtained in the course of semantic construction.

• An operation called  $\beta$ -conversion performs the required substitutions.

### 9.1. Lambda-terms: Examples

Here is an example of lambda terms:

 $\lambda x.\texttt{left}(x)$ 

The prefix  $\lambda x$ . binds the occurrence of x in student(x). We say it **abstracts** over the variable x. The purpose of abstracting over variables is to mark the slots where we want the substitutions to be made.

To glue vincent with "left" we need to apply the lambda-term representing "left" to the one representing "Vincent":

 $\lambda x.\texttt{left}(x)(\texttt{vincent})$ 

Such expressions are called **functional applications**, the left-hand expression is called the **functor** and the right-hand expression is called the **argument**. The functor is applied to the argument. Intuitively it says: fill all the the placeholders in the functor by occurrences of the term **vincent**.

The substitution is performed by means of  $\beta$ -conversion, obtaining left(vincent).

### 9.2. Functional Application

Summing up:

- ► FA has the form: Functor(Argument). E.g.  $(\lambda x.love(x, mary))(john)$
- ► FA triggers a very simple operation: Replace the  $\lambda$ -bound variable by the argument. E.g.  $(\lambda x.love(x, mary))(john) \Rightarrow love(john, mary)$

### 9.4. Exercise

Give the lambda term representing a transitive verb.

(a) Build the meaning representation of "John saw Mary" starting from:

- ▶ John: j
- ▶ Mary: m
- ▶ saw:  $\lambda x.\lambda y.\texttt{saw}(y,x)$

(b) Build the parse tree of the sentence by means the bottom-up method.

(c) Compare what you have done to assembly the meaning representation with the way you have built the tree.

### 9.5. $\alpha$ -conversion

Warning: Accidental bounds, e.g.  $\lambda x \cdot \lambda y \cdot Love(y, x)(y)$  gives  $\lambda y \cdot Love(y, y)$ . We need to rename variables before performing  $\beta$ -conversion.

 $\alpha\text{-conversion}$  is the process used in the  $\lambda\text{-calculus}$  to rename bound variables. For instance, we obtain

 $\lambda x.\lambda y. \texttt{Love}(y, x) \text{ from } \lambda z.\lambda y. \texttt{Love}(y, z).$ 

When working with lambda calculus we always  $\alpha$ -covert before carrying out  $\beta$ conversion. In particular, we always rename all the bound variables in the functor so they are distinct from all the variables in the argument. This prevents accidental binding.

## **10.** Lambda-Terms Interpretations

In the Logic course you've seen that an Interpretation is a pair consisting of a domain  $(\mathcal{D})$  and an interpretation function  $(\mathcal{I})$ .

- ▶ In the case of FOL we had only one domain, namely the one of the objects/entities we were reasoning about. Similarly, we only had one type of variables. Moreover, we were only able to speak of propositions/clauses.
- λ-terms speak of functions and we've used also variables standing for functions. Therefore, we need a more complex concept of interpretation, or better a more complex concept of domain to provide the fine-grained distinction among the objects we are interested in: truth values, entities and functions.
- ▶ For this reason, the  $\lambda$ -calculus is of Higher Order.

### 10.1. Models, Domains, Interpretation

In order to interpret meaning representations expressed in FOL augmented with  $\lambda$ , the following facts are essential:

- ▶ Sentences: Sentences can be thought of as referring to their truth value, hence they denote in the the domain  $D_t = \{1, 0\}$ .
- Entities: Entities can be represented as constants denoting in the domain D<sub>e</sub>, e.g. D<sub>e</sub> = {john, vincent, mary}
- ▶ Functions: The other natural language expressions can be seen as incomplete sentences and can be interpreted as **boolean functions** (i.e. functions yielding a truth value). They denote on functional domains  $D_b^{D_a}$  and are represented by functional terms of type  $(a \rightarrow b)$ .

For instance "walks" misses the subject (of type e) to yield a sentence (t).

- $\triangleright$  denotes in  $D_t^{D_e}$
- ▷ is of type  $(e \rightarrow t)$ ,
- ▷ is represented by the term  $\lambda x_e(\texttt{walk}(x))_t$

### 10.2. Lambda-calculus: some remarks

The pure lambda calculus is a theory of functions as rules invented around 1930 by Church. It has more recently been applied in Computer Science for instance in "Semantics of Programming Languages".

In Formal Linguistics we are mostly interested in lambda conversion and abstraction. Moreover, we work only with typed-lambda calculus and even more, only with a fragment of it.

The types are the ones we have seen above labeling the domains, namely:

- $\blacktriangleright$  e and t are types.
- ▶ If a and b are types, then  $(a \rightarrow b)$  is a type.

## 11. The Three Tasks Revised

- Task 1 Specify a reasonable syntax for the natural language fragment of interest. We can do this using CFG.
- Task 2 Specify semantic representations for the lexical items. We know what this involves
- Task 3 Specify the translation of an item  $\mathcal{R}$  whose parts are  $\mathcal{F}$  and  $\mathcal{A}$  with the help of functional application. That is, we need to specify which part is to be thought of as functor (here it's  $\mathcal{F}$ ), which as argument (here it's  $\mathcal{A}$ ) and then let the resultant translation  $\mathcal{R}'$  be  $\mathcal{F}'(\mathcal{A}')$ . We know that  $\beta$ -conversion (with the help of  $\alpha$ -conversion), gives us the tools needed to actually construct the representation built by this process.

In the Lab we will seen that the solution proposed for task 1 leads itself to computational implementation naturally. Next week we will see that this holds for task 2 and 3 too (though we won't go into the detail of it. If you are interested in it and you know Prolog already: good topic for a project!)

## 12. Practical things

Next classes/labs

- ▶ Wed 28th, 18:00-19:00 (exercises on lambda calculus)
- ▶ Thu 29th, 17:00-19:00 (CFG in Prolog, bottom-up and top-down parsing).
- ▶ Tue 3rd, 08:30-09:30 Semantics II.
- ▶ Tue 3rd, 09:30-10:30 Michael Moortgat "Parsing as Deduction".