Computational Linguistics: Semantics

Raffaella Bernardi

KRDB, Free University of Bozen-Bolzano

P.zza Domenicani, Room: 2.28, e-mail: bernardi@inf.unibz.it

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We have seen how to recognize/parse these sentences so to obtain different parse trees whenever necessary.

2. Semantics

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The first and last question are closely connected.

In fact, since we are ultimately interested in understanding, explaining and accounting for the entailment relation holding among sentences, we can think of **the meaning of a sentence as its truth value**.

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2. Important **inference problems** have been studied for the best known logics, and often good **computational implementations** exists. So translating into a logic gives us a handle on inference.

When we look at these problems from a computational perspective, i.e. we bring in the implementation aspect too, we move from Formal Semantics to **Computational Semantics**.

4.1. Model: individual constants

The interpretation of a formal language has to include a specification of what constants in the language refers to. This is done by means of the **interpretation function** $\mathcal{I}$ which assigns an appropriate **denotation** in the model $\mathcal{M}$ to each individual and n-place predicate constant .

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4.2. Model: one-place predicates

One-place properties are seen as sets of individuals: the property of being orange describes the **set of individuals** that are orange. Formally, for P a one-place predicate, the interpretation function $\mathcal{I}$ maps P onto a subset of the universe of discourse $\mathcal{U} : \mathcal{I}(P) \subseteq \mathcal{U}$. For instance,

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4.3. Model: two-place predicates

Two-place predicates such as “love”, “eat”, “mother-of” do not denote sets of individuals, but **sets of ordered pairs of individuals**, namely all those pairs which stand in the “loving”, “eating”, “mother-of” relations. We form ordered pairs from two sets A and B by taking an element of A as first member of the pair and an element of B as the second member. Given the relation R, the interpretation function $\mathcal{I}$ maps R onto a set of ordered pairs of elements of $\mathcal{U} : \mathcal{I}(R) \subseteq \mathcal{U} \times \mathcal{U}$

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4.4. Example

Let our model be based on the set of entities $E = \{\texttt{lori}, \texttt{ale}, \texttt{sara}, \texttt{pim}\}$ which represent **Lori, Ale, Sara** and **Pim**, respectively. Assume that they all know themselves, plus **Ale** and **Lori** know each other, but they do not know **Sara** or **Pim**; **Sara** does know **Lori** but not **Ale** or **Pim**. The first three are students whereas **Pim** is a professor, and both **Lori** and **Pim** are tall. This is easily expressed set theoretically. Let $[\![\texttt{w}]\!]$ indicate the interpretation of **w**:

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[\![\texttt{lori}]\!] & = & \text{lori;} \\
[\![\texttt{know}]\!] & = & \{\langle \text{lori, ale} \rangle, \langle \text{ale,lori} \rangle, \langle \text{sara, lori} \rangle, \\
 & & \langle \text{lori, lori} \rangle, \langle \text{ale, ale} \rangle, \langle \text{sara, sara} \rangle, \langle \text{pim, pim} \rangle\}; \\
[\![\texttt{student}]\!] & = & \{\text{lori, ale, sara}\}; \\
[\![\texttt{professor}]\!] & = & \{\text{pim}\}; \\
[\![\texttt{tall}]\!] & = & \{\text{lori, pim}\}.
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which is nothing else to say that, for example, the relation **know** is the **set of pairs** $\langle \alpha, \beta \rangle$ where α knows β; or that 'student' is the set of all those elements which are a student.

4.5. Relational and Functional Perspectives

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4.6. Exercises: Relations vs. Functions

Think of which function you can assign to the words in the model considered before and repeated here:

Sara, Pim, Lori, know, student, professor, tall, every man, every Mexican student, no Mexican student, some man.

4.7. Summing up

Summarizing, when trying to formalize natural language semantics, at least two sorts of objects are needed to start with: the set of **truth values** t, and the one of **entities** e.

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Furthermore, we have illustrated how one can move back and forwards between a **set/relational and a functional perspective**. The former can be more handy and intuitive when reasoning about entailment relations among expressions; the latter is more useful when looking for lexicon assignments.

References: Keenen 85.

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  $[\![\exists x.\texttt{student}(x) \wedge \texttt{left}(x)]\!] = 1$ iff standard FOL (First Order Logic) definitions are satisfied.

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We will come back to 2. next time.

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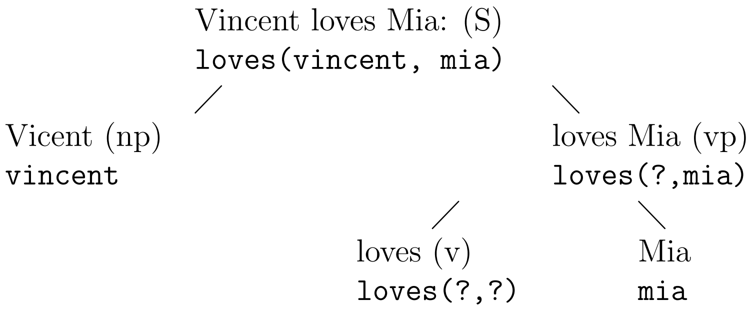

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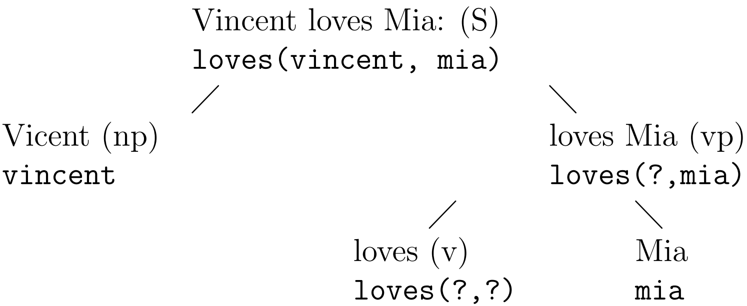

Briefly, **syntactic structure guiding gluing**.

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Different parse trees result into different meaning representations!

7. How far can we go with FOL?

FOL can capture the **what** (partially) and cannot capture the **how**, i.e.

Problems with the "what":

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- There was a red book on the table. $\exists x(Book(x) \wedge Red(x) \wedge On\_the\_table(x))$.
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Moreover, when interested in Computational Semantics, all three tasks need to be carried out in a way that leads to computational implementation naturally.

We have looked at Task 1 in lecture 2 and 3 (formal grammars) and at their computational side (Implementation in Prolog, Recognition and Parsing) during the Labs.

Today we will start looking at the other two tasks.

9. Lambda Calculus

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- It has a **variable binding operators** λ. Occurrences of variables bound by λ should be thought of as place-holders for missing information: they explicitly mark where we should substitute the various bits and pieces obtained in the course of semantic construction.
- An **operation** called β-conversion performs the required substitutions.

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Such expressions are called **functional applications**, the left-hand expression is called the **functor** and the right-hand expression is called the **argument**. The functor is applied to the argument. Intuitively it says: fill all the the placeholders in the functor by occurrences of the term `vincent`.

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The substitution is performed by means of β-conversion, obtaining `left(vincent)`.

9.2. Functional Application

Summing up:

- FA has the form: Functor(Argument). E.g. $(\lambda x.love(x, mary))(john)$
- FA triggers a very simple operation: Replace the λ-bound variable by the argument. E.g. $(\lambda x.love(x, mary))(john) \Rightarrow love(john, mary)$

9.3. β-conversion

Summing up:

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9.4. Exercise

Give the lambda term representing a transitive verb.

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(a) Build the meaning representation of "John saw Mary" starting from:

- John: $\mathtt{j}$
- Mary: $\mathtt{m}$
- saw: $\lambda x.\lambda y.\mathtt{saw}(y, x)$

(b) Build the parse tree of the sentence by means the bottom-up method.

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(b) Build the parse tree of the sentence by means the bottom-up method.

(c) Compare what you have done to assembly the meaning representation with the way you have built the tree.

9.5. α-conversion

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$\lambda x.\lambda y.\texttt{Love}(y,x)$ from $\lambda z.\lambda y.\texttt{Love}(y,z)$.

When working with lambda calculus we always α-covert before carrying out β-conversion. In particular, we always rename all the bound variables in the functor so they are distinct from all the variables in the argument. This prevents accidental binding.

10. Lambda-Terms Interpretations

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- λ-terms speak of functions and we've used also **variables standing for functions**. Therefore, we need a more complex concept of interpretation, or better a more **complex concept of domain** to provide the fine-grained distinction among the objects we are interested in: truth values, entities and functions.
- For this reason, the λ-calculus is of Higher Order.

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  - is of type $(e \rightarrow t)$,
  - is represented by the term $\lambda x_e(\texttt{walk}(x))_t$

10.2. Lambda-calculus: some remarks

The pure lambda calculus is a theory of functions as rules invented around 1930 by Church. It has more recently been applied in Computer Science for instance in “Semantics of Programming Languages”.

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- e and t are types.
- If a and b are types, then $(a \rightarrow b)$ is a type.

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In the Lab we have seen that the solution proposed for task 1 leads itself to computational implementation naturally. Next week we will see that this holds for task 2 and 3 too (though we won't go into the detail of it. If you are interested in it and you know Prolog already: good topic for a project!)