Real-Time Scheduling Analysis

Real Time Operating Systems and Middleware

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Response Time Analysis

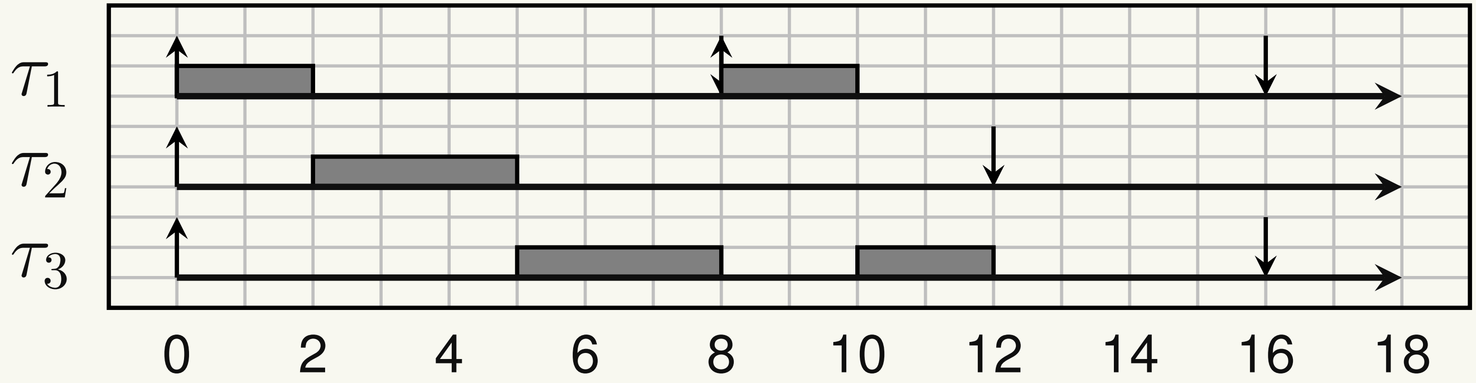
- Necessary and sufficient test: compute the *worst-case response time* for every task
- For every task τ_i:
  - Compute worst case response time R_i for τ_i
    - Remember? $R_i = \max_j\{\rho_{i,j}\}$; $\rho_{i,j} = f_{i,j} - r_{i,j}$
  - If $R_i \leq D_i$, then the task is schedulable
  - otherwise, the task is not schedulable
- No assumption on the priority assignment
  - Algorithm valid for arbitrary priority assignments
  - Not only RM / DM...
- Periodic tasks with no offsets, or sporadic tasks

The Critical Instant

- Tasks ordered by decreasing priority ($i < j \rightarrow p_i > p_j$)
- No assumptions about tasks offsets
  - $\Rightarrow$ Consider the *worst possible offsets combination*
  - A job $J_{i,j}$ released at the *critical instant* experiences the maximum response time for τ_i: $\forall k, \rho_{i,j} \geq \rho_{i,k}$
    - Simplified definition (jobs deadlines should be considered...)
  - **Theorem:** The critical instant for task τ_i occurs when job $J_{i,j}$ is released at the same time with a job in every high priority task
- **If all the offsets are 0, the first job of every task is released at the critical instant!!!**

Worst Case Response Time

- Worst case response time R_i for task τ_i depends on:
  - Its execution time...
  - ...And the execution time of higher priority tasks
    - Higher priority tasks can *preempt* task τ_i, and increase its response time

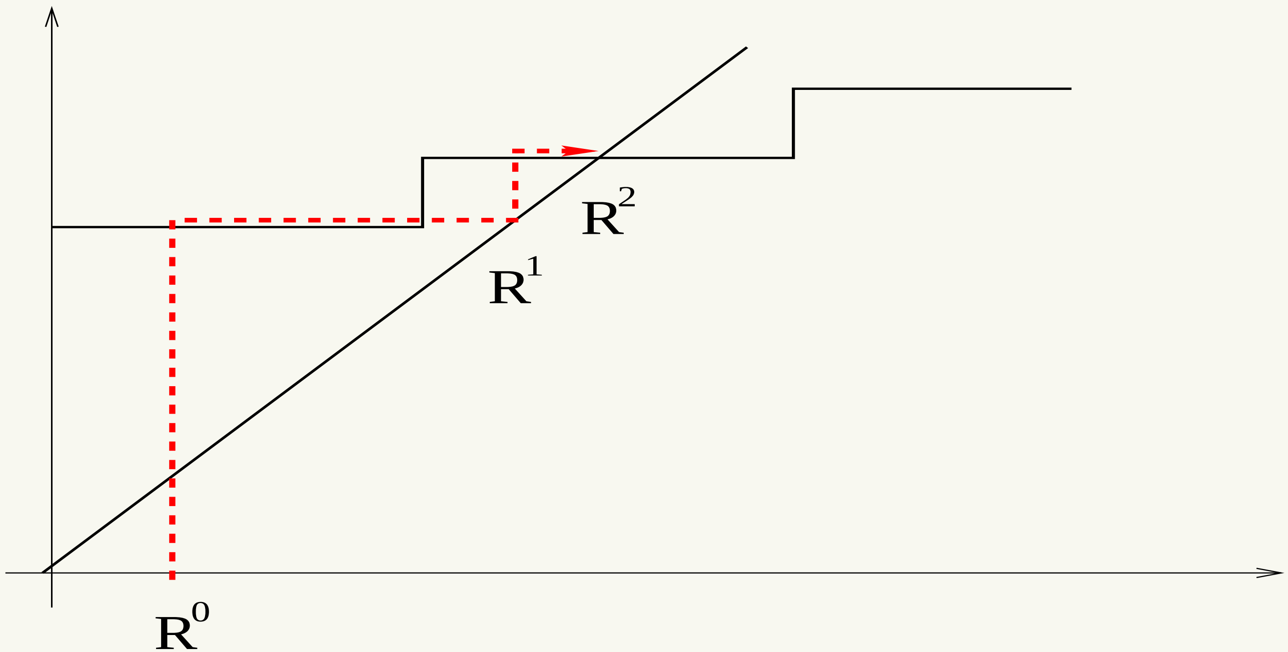


- $R_i = C_i + \sum_{h=1}^{i-1} \left\lceil \frac{R_i}{T_h} \right\rceil C_h$

Computing the Response Time - I

$$R_i = C_i + \sum_{h=1}^{i-1} \left\lceil \frac{R_i}{T_h} \right\rceil C_h$$

- Urk!!! $R_i = f(R_i)$... How can we solve it?
- There is no closed-form expression for computing the worst case response time R_i
- We need an iterative method to solve the equation



Computing the Response Time - II

- Iterative solution
  - $R_i = \lim_{k\to\infty} R_i^{(k)}$
  - $R_i^{(k)}$: worst case response time for τ_i, at step k
- $R_i^{(0)}$: first estimation of the response time
  - We can start with $R_i^{(0)} = C_i$
  - $R_i^{(0)} = C_i + \sum_{h=1}^{i-1} C_h$ saves 1 step

$$\begin{aligned} R_i^{(0)} &= C_i(+\sum_{h=1}^{i-1} C_h) \\ R_i^{(k)} &= C_i + \sum_{h=1}^{i-1} \left\lceil \frac{R_i^{(k-1)}}{T_h} \right\rceil C_h \end{aligned}$$

Computing the Response Time - III

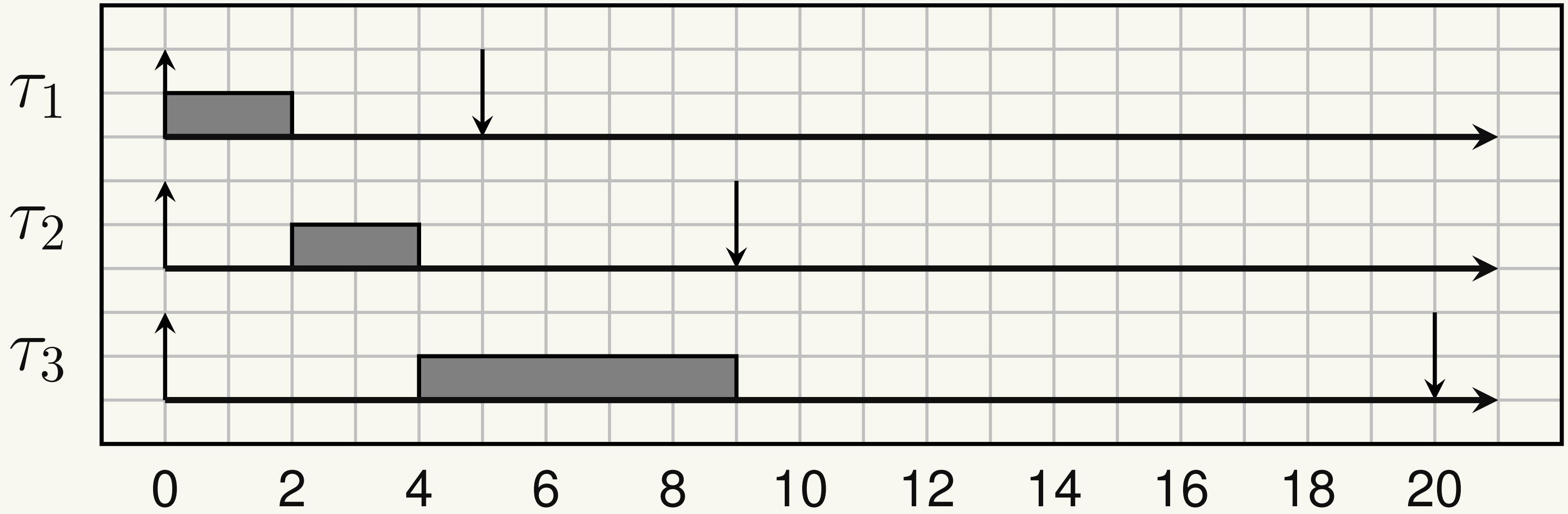
- Problem: are we sure that we find a valid solution?
- The iteration stops when:
  - $R_i^{(k+1)} = R_i^{(k)}$ *or*
  - $R_i^{(k)} > D_i$ (non schedulable);
- This is a standard method to solve non-linear equations in an iterative way
- If a solution exists (the system is not overloaded), $R_i^{(k)}$ converges to it
- Otherwise, the "$R_i^{(k)} > D_i$" condition avoids infinite iterations

Example

Task set: $\tau_1 = (2, 5)$, $\tau_2 = (2, 9)$, $\tau_3 = (5, 20)$; $U = 0.872$

$$R_i^{(k)} = C_i + \sum_{h=1}^{i-1} \left\lceil \frac{R_i^{(k-1)}}{T_h} \right\rceil C_h$$

$$R_3^{(0)} = C_3 + 1 \cdot C_1 + 1 \cdot C_2 = 9$$



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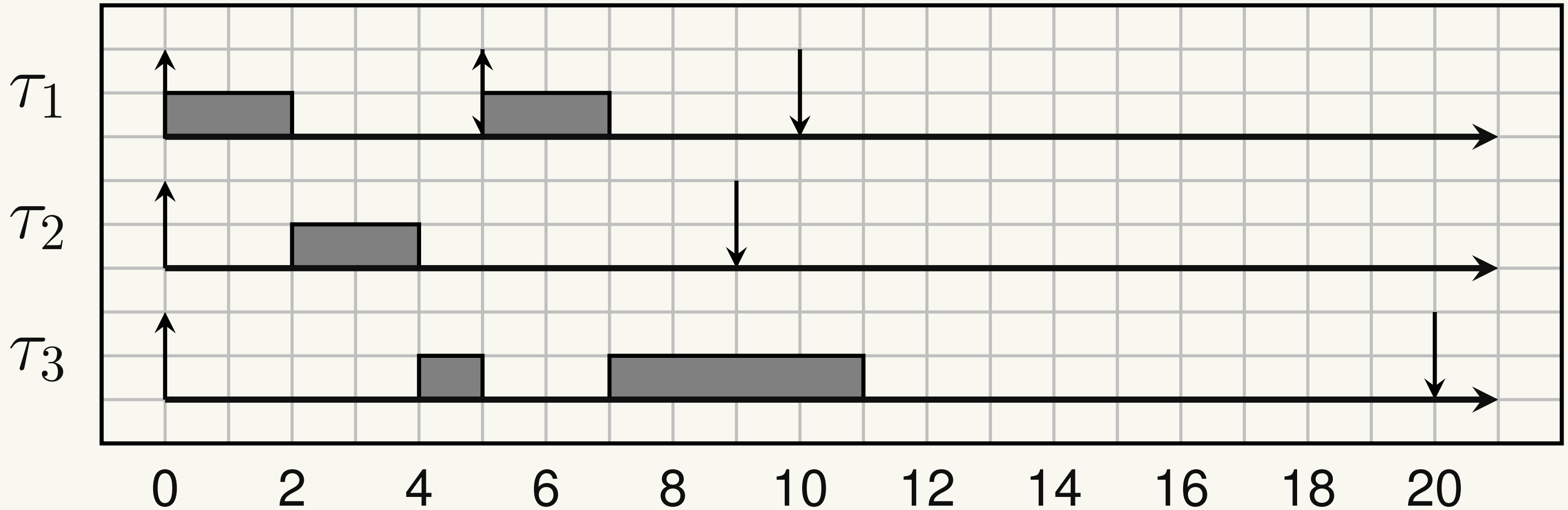

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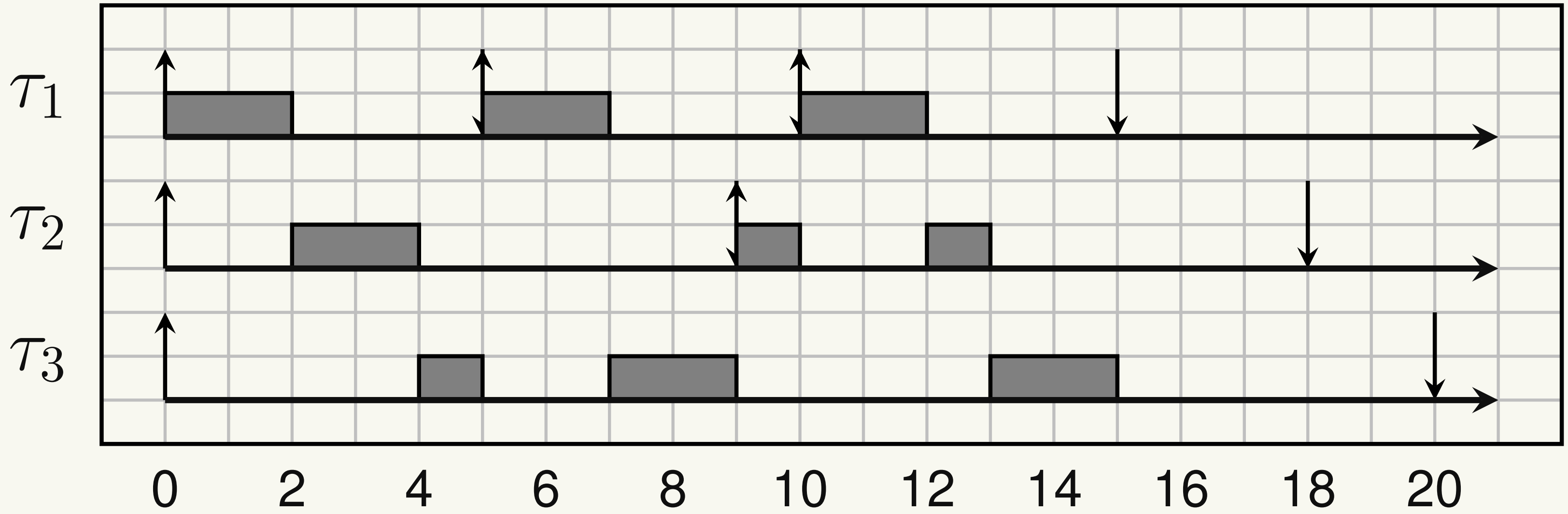

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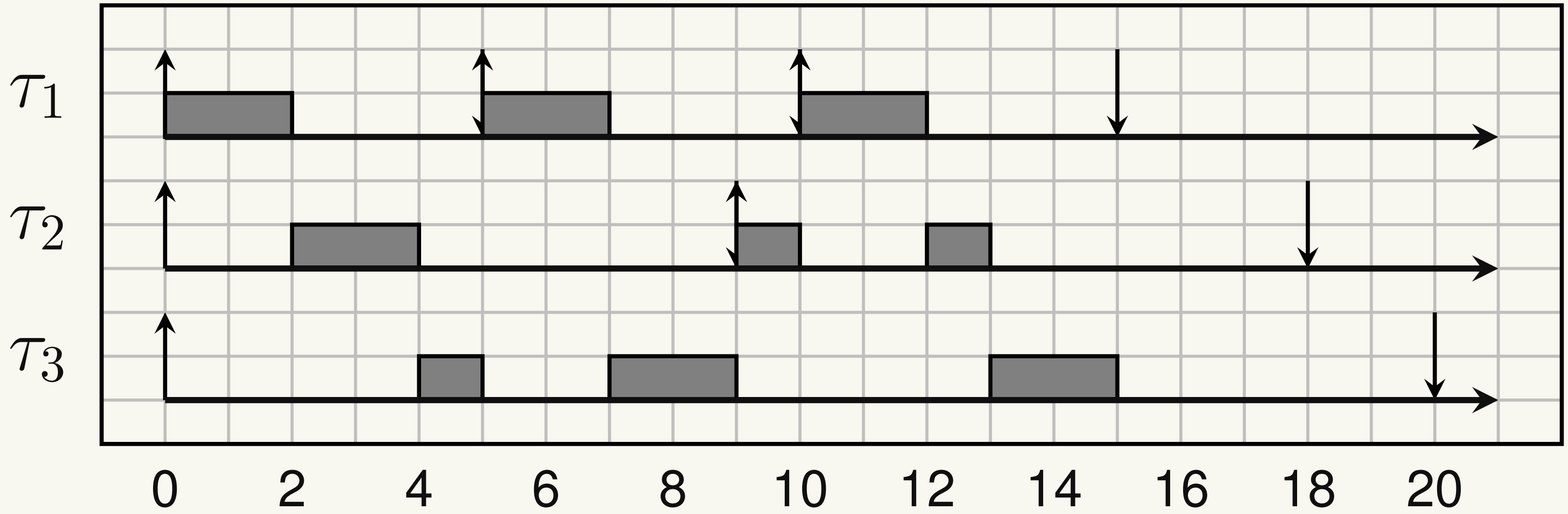


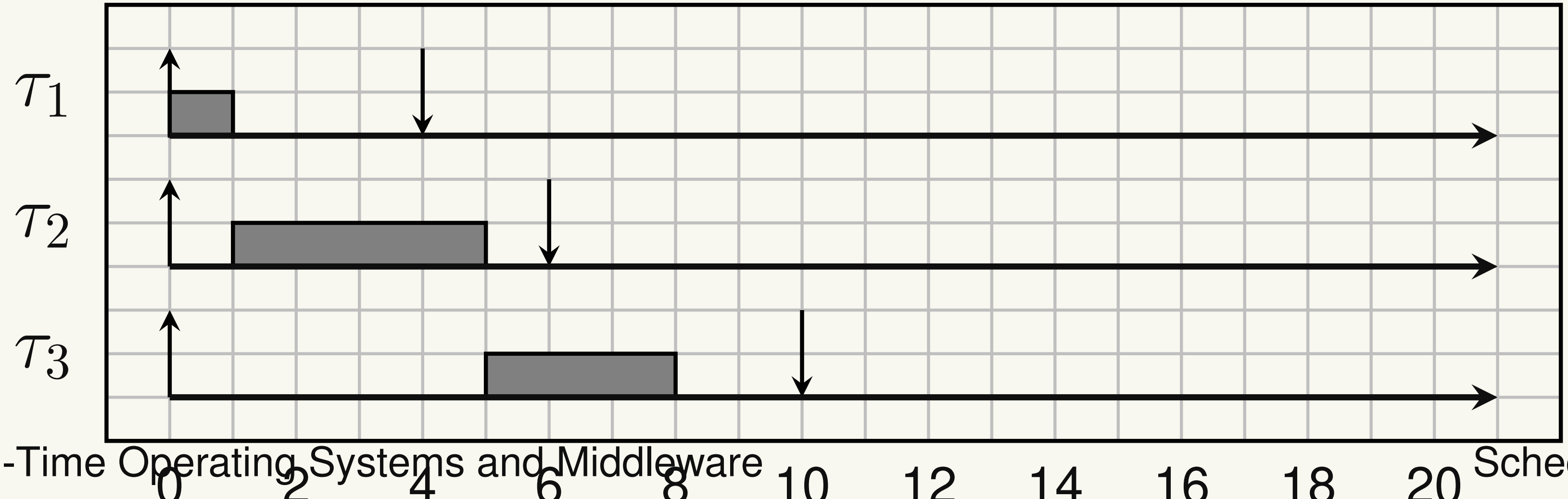
Another Example with DM

What about different priority assignments and deadlines different from periods?

$\tau_1 = (1, 4, 4), p_1 = 3$, $\tau_2 = (4, 6, 15), p_2 = 2$,
$\tau_3 = (3, 10, 10), p_3 = 1; U = 0.72$

$$R_i^{(k)} = C_i + \sum_{h=1}^{i-1} \left\lceil \frac{R_i^{(k-1)}}{T_h} \right\rceil C_h$$

$R_3^{(0)} = C_3 + 1 \cdot C_1 + 1 \cdot C_2 = 8$



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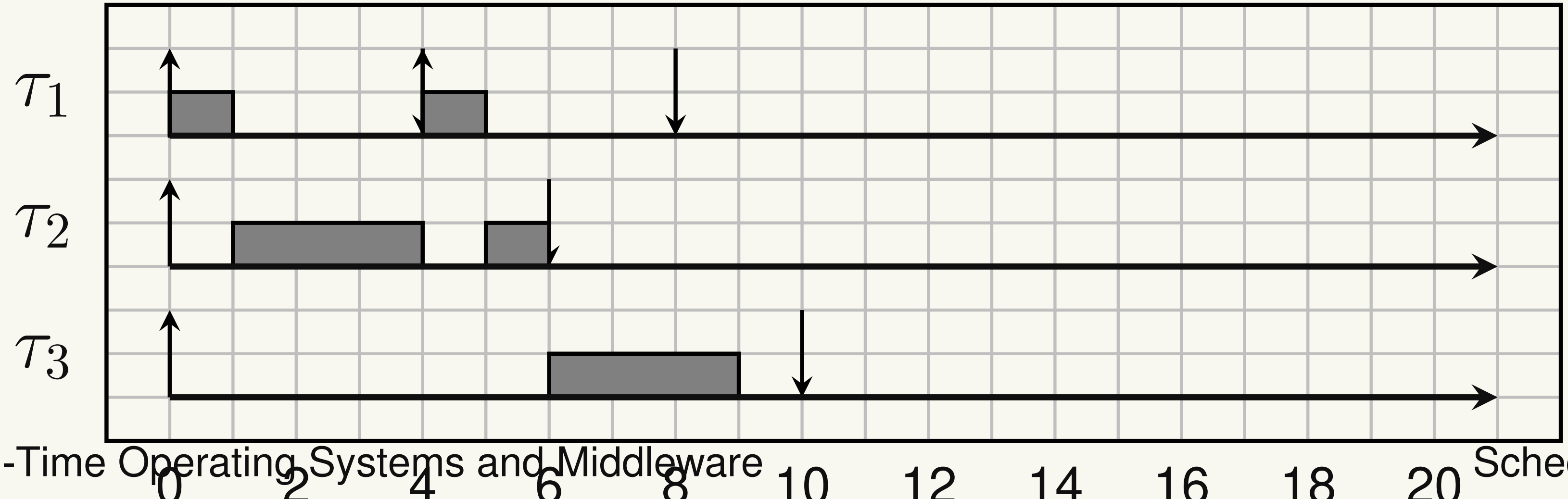

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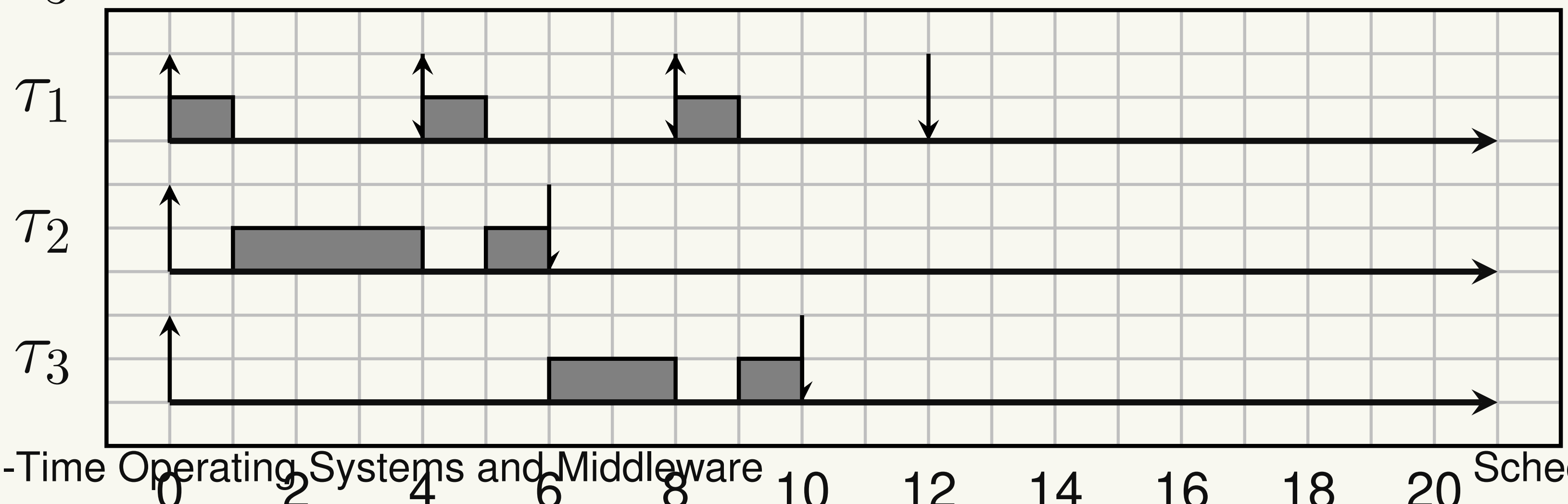
$R_3^{(2)} = C_3 + 3 \cdot C_1 + 1 \cdot C_2 = 10$



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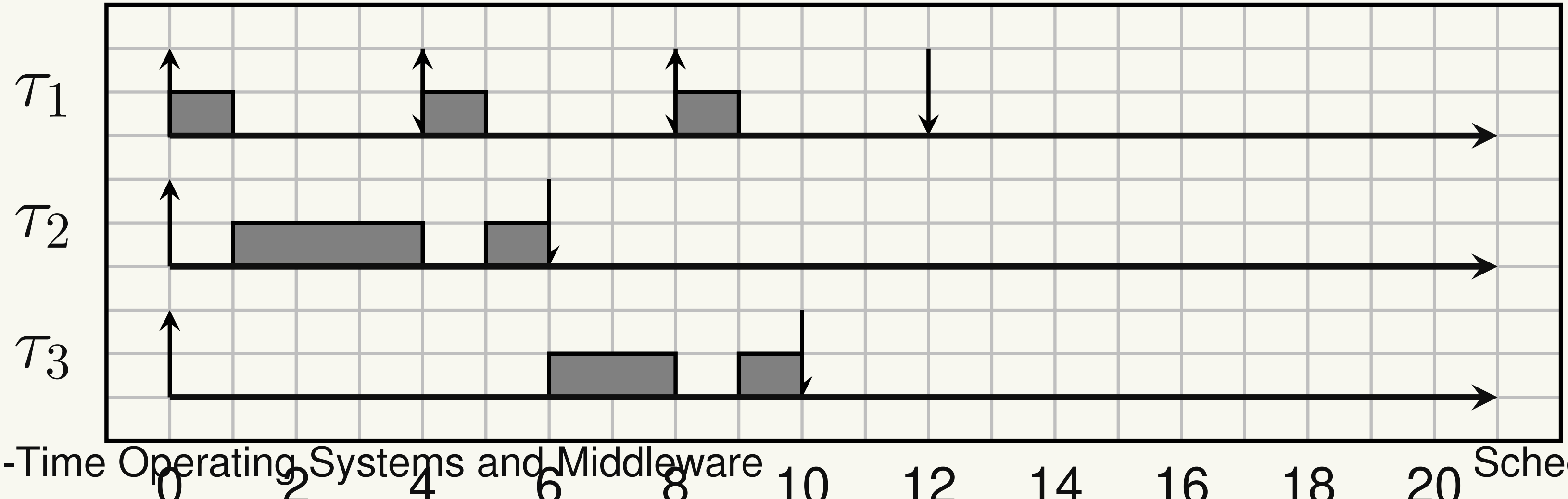

Considerations

- The response time analysis is an efficient algorithm
  - In the worst case, the number of steps N for the algorithm to converge is exponential
    - Depends on the total number of jobs of higher priority tasks in the interval $[0, D_i]$:

$$N \propto \sum_{h=1}^{i-1} \left\lceil \frac{D_h}{T_h} \right\rceil$$

    - If s is the minimum granularity of the time, then in the worst case $N = \frac{D_i}{s}$;
  - However, such worst case is very rare: usually, the number of steps is low.

Time Demand Analysis: the Idea

- Processor Demand approach has been proposed by Lehoczky and others in 1989
- Refined by Audsley and others in 1993 and by Baruah in 1990
- The basic idea is very simple: *in any interval, the computation demanded by all tasks in the set must never exceed the available time*
- The problem is: how to compute the time demanded by a task set $\mathcal{T}$?
  - Remember: we have to look only at jobs released at the critical instant
  - Offsets $= 0 \Rightarrow$ only consider the first job of each task...

The Processor Demand

- Given an interval $[t_1, t_2]$,
- let $\mathcal{J}_{t_1,t_2}$ be the set of jobs started after t_1 and with deadline lower than or equal to t_2:

$$\mathcal{J}_{t_1,t_2} = \{J_{i,j} : r_{i,j} \geq t_1 \wedge d_{i,j} \leq t_2\}$$

- the processor demand in $[t_1, t_2]$ is defined as:

$$W(t_1, t_2) = \sum_{J_{i,j} \in \mathcal{J}_{t_1,t_2}} c_{i,j}$$

- Worst case: use C_i instead of $c_{i,j}$

Computing the Processor Demand - 1

- Guaranteeing a task set $\mathcal{T}$ based on $W(t_1, t_2)$ can take a loooong time
  - $\forall (t_1, t_2), W(t_1, t_2) \leq t_2 - t_1$
  - Need to check all the (t_1, t_2) combinations in a hyperperiod?
- But...
  - We only need to check the first job of every task τ_i!
- $W_i(t_1, t_2)$: time demanded in $[t_1, t_2]$ by all tasks τ_j with $p_j \geq p_i$ ($\Rightarrow j \leq i$)

Computing the Processor Demand - 2

- We can consider only $W_i(0, t)$
- For task τ_i, only check $W_i(0, t)$ for $0 \leq t \leq D_i$
- Change $\forall$ into $\exists$: consider worst case for $W_i()$
  - Number of jobs in $[0, t]$: $\left\lfloor \frac{t}{T_i} \right\rfloor$
  - Use $\lceil \rceil$ instead!
- We already have hints about computing an upper bound for $W_i(0, t)$...

$$W_i(0, t) = C_i + \sum_{h=1}^{i-1} \left\lceil \frac{t}{T_h} \right\rceil C_h$$

TDA: The Guarantee

- Task τ_i is schedulable **iff** $\exists t : 0 \leq t \leq D_i \wedge W_i(0,t) \leq t$
- A task set $\mathcal{T}$ is schedulable **iff**

$$\forall \tau_i \in \mathcal{T}, \exists t : 0 \leq t \leq D_i \wedge W_i(0,t) \leq t$$

- Sometimes, different notations in literature:
  - $W_i(0,t) \rightarrow W_i(t) = \sum_{h=1}^{i} \left\lceil \frac{t}{T_h} \right\rceil C_h$ (this is equivalent, because $0 \leq t \leq T_i$)

TDA: Alternative Formulations

- Someone defines

$$L_i(t_1, t_2) = \frac{W_i(t_1, t_2)}{t_2 - t_1}$$

$$L_i = \min_{0 \leq t \leq D_i} L_i(0, t); L = \max_{\tau_i \in \mathcal{T}} L_i$$

- The guarantee tests then becomes

Task τ_i is schedulable **iff** $L_i \leq 1$
$\mathcal{T}$ is schedulable **iff** $L \leq 1$

TDA Simplifications

- The test might still be long (need to check many values of $L(0,t)$ to find the minimum)...
- The number of points to check for computing W_i / L_i can be reduced
  - *Scheduling points*: $S_i = \{kT_h | h \leq i; 1 \leq k \leq \left\lfloor \frac{T_i}{T_h} \right\rfloor\}$
    - (multiples of T_h for $h \leq i$)
    - $L_i = \min_{t \in S_i} L_i(0,t)$

Example - 1

- $\tau_1 = (20, 100)$, $\tau_2 = (40, 150)$, $\tau_3 = (100, 350)$
- τ_1 is schedulable: $20 < 100$
- What about τ_2?
  - $S_2 = \{100, 150\}$
  - $W_2(0, 100) = 40 + 20 = 60 \leq 100$: τ_2 **is schedulable**

Example - 2

- And now, τ_3:
  - $S_3 = \{100, 150, 200, 300, 350\}$
  - $W_3(0, 100) = 100 + 20 + 40 = 160 > 100$
  - $W_3(0, 150) = 100 + 2 * 20 + 40 = 180 > 150$
  - $W_3(0, 200) = 100 + 2 * 20 + 2 * 40 = 220 > 200$
  - $W_3(0, 300) = 100 + 3 * 20 + 2 * 40 = 240 \leq 300$: τ_3 **is schedulable**

Example - Continued 1

- But we already knew that the task set is schedulable: $\frac{20}{100} + \frac{40}{150} + \frac{100}{350} = 0.2 + 0.2\overline{6} + 0.2857 = 0.753 < 0.779$
- Now, let's change C_1 to 40:
  - $\tau_1 = (40, 100)$, $\tau_2 = (40, 150)$, $\tau_3 = (100, 350)$
- $U = \frac{40}{100} + \frac{40}{150} + \frac{100}{350} = 0.953$
  - $U_{lub} \leq U \leq 1$: using utilisation-based analysis, we cannot say is $\mathcal{T}$ is schedulable or not...
- τ_1 is schedulable: $40 < 100$

Example - Continued 2

- τ_1 is schedulable, what about the other tasks?
- τ_2:
  - $W_2(0, 100) = 40 + 40 \leq 100$: τ_2 is schedulable
- τ_3:
  - $W_3(0, 300) = 100 + 3 * 40 + 2 * 40 = 300 \leq 300$: τ_3 is still schedulable!!!
- Note: to simplify, I directly checked the "right" scheduling point...

Example - Checking the Results

- TDA says that the task set is schedulable, but I do not believe it!!! ($U = 0.953$ looks very high)

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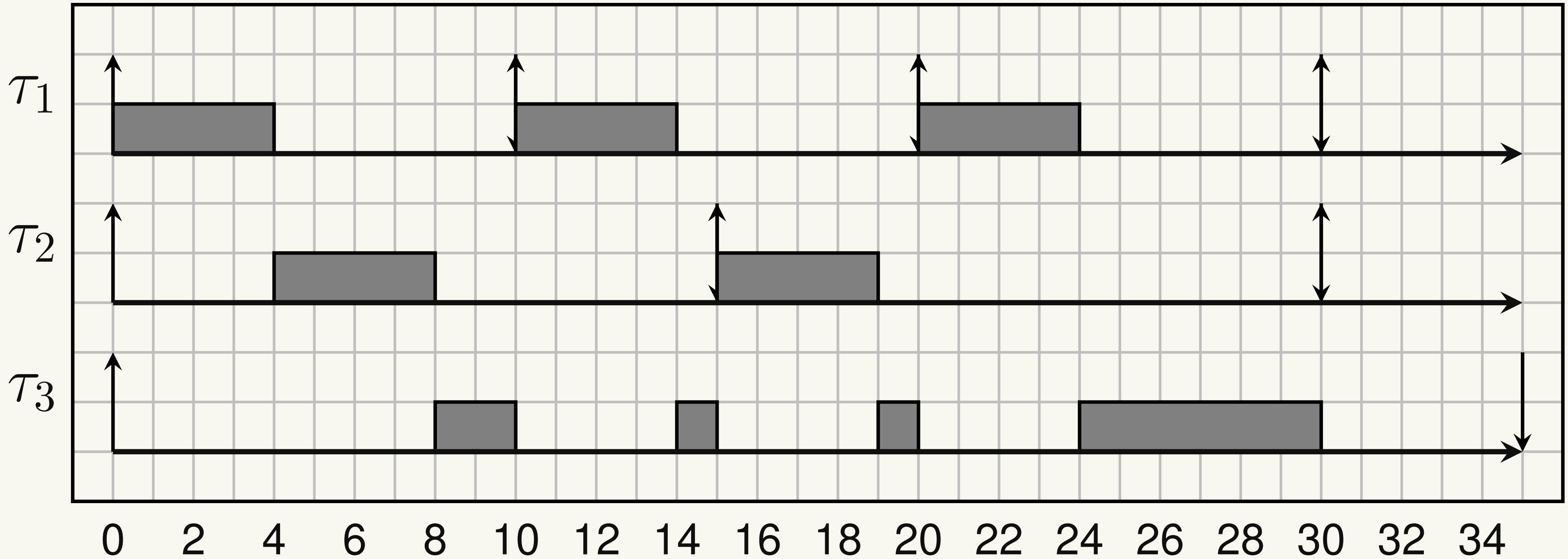
- TDA says that the task set is schedulable, but I do not believe it!!! ($U = 0.953$ looks very high)
- So, here is the schedule...



Checking the Results - Response Time

- And what about using response time analysis?
- $\mathcal{T} = \{(40, 100), (40, 150), (100, 350)\}$
  - $R_3^{(0)} = 100$
  - $R_3^{(1)} = 100 + \left\lceil \frac{100}{100} \right\rceil 40 + \left\lceil \frac{100}{150} \right\rceil 40 = 100 + 40 + 40 = 180$
  - $R_3^{(2)} = 100 + \left\lceil \frac{180}{100} \right\rceil 40 + \left\lceil \frac{180}{150} \right\rceil 40 = 100 + 80 + 80 = 260$
  - $R_3^{(3)} = 100 + \left\lceil \frac{260}{100} \right\rceil 40 + \left\lceil \frac{260}{150} \right\rceil 40 = 100 + 120 + 80 = 300$
  - $R_3^{(4)} = 100 + \left\lceil \frac{300}{100} \right\rceil 40 + \left\lceil \frac{300}{150} \right\rceil 40 = 100 + 120 + 80 = 300$
  - $R_3^{(4)} = R_3^{(3)} \Rightarrow$ stop. $R_3 = 300$
- $R_3 = 300 \leq 350 \Rightarrow$ the system is schedulable.
- The previous result is confirmed...