# Real Time Operating Systems and Middleware

## *Real-Time Scheduling*

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# Definitions

- Algorithm $\rightarrow$ logical procedure used to solve a problem
- Program $\rightarrow$ formal description of an algorithm, using a *programming language*
- Process $\rightarrow$ instance of a program (program in execution)
  - Program: static entity
  - Process: dynamic entity
- The term *task* is used to indicate a schedulable entity (either a process or a thread)
  - Thread $\rightarrow$ flow of execution
  - Process $\rightarrow$ flow of execution + private resources (address space, file table, etc...)

# Scheduling

- Tasks do not run on bare hardware...
  - The OS kernel creates the illusion of *virtual CPU*
  - One virtual CPU per task
  - Tasks have the illusion of executing concurrently
- Concurrency is implemented by multiplexing tasks on the same CPU...
  - Tasks are alternated on a real CPU...
  - ...And the *task scheduler* decides which task executes at a given instant in time
- Tasks are associated temporal constraints (deadlines)
  - The scheduler must allocate the CPU to tasks so that their deadlines are respected

# Scheduler - 1

- The scheduler is responsible for generating a *schedule* starting from a set of tasks ready to execute
- Mathematical model
  - Let's start considering an UP system
    - A schedule $\sigma(t)$ is a function mapping time $t$ into an executing task

$$\sigma : t \rightarrow \mathcal{T} \cup \tau_{idle}$$

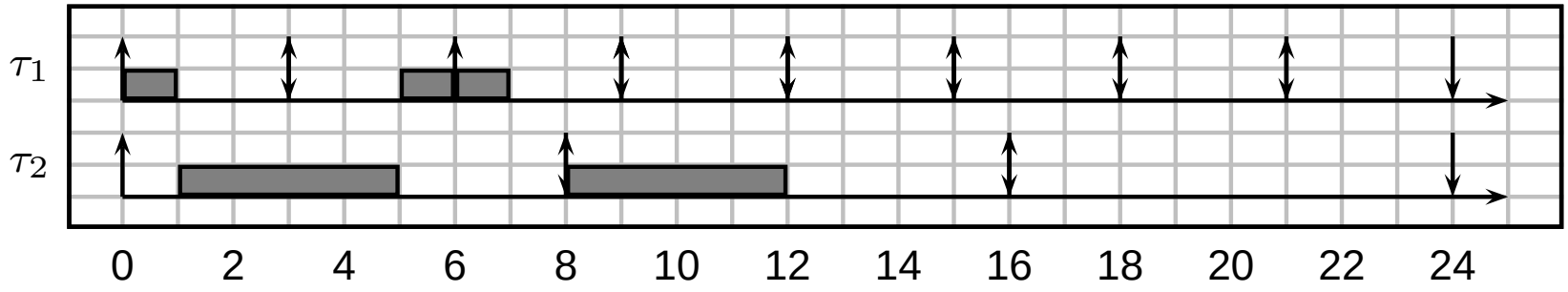
      where $\mathcal{T}$ is the set of tasks running in the system
    - $\tau_{idle}$ is the *idle task*: when it is scheduled, the CPU becomes idle
  - For an SMP system ($m$ CPUs), $\sigma(t)$ can be extended to map $t$ in vectors $\tau \in (\mathcal{T} \cup \tau_{idle})^m$

# Scheduler - 2

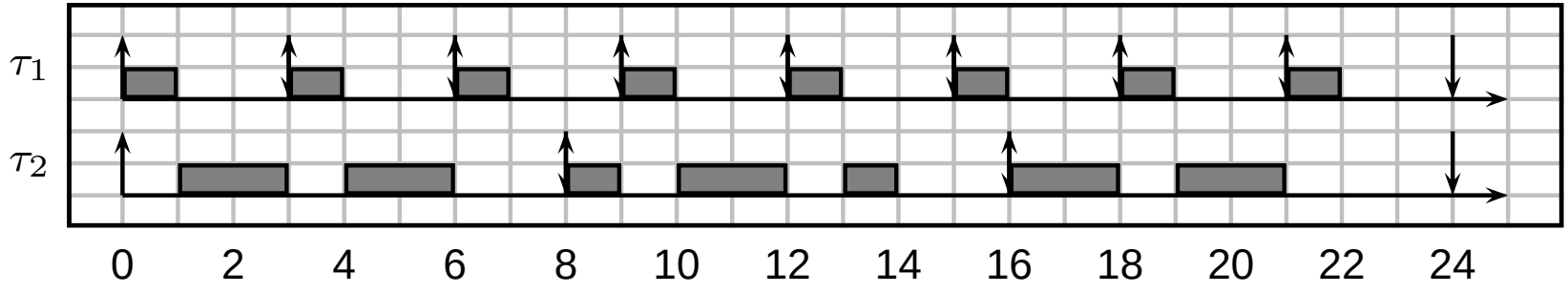
- The scheduler is responsible for selecting the task to execute
- From an algorithmic point of view
  - Scheduling algorithm $\rightarrow$ Algorithm used to select for each time instant $t$ a task to be executed on a CPU among the ready task
  - Given a task set $\mathcal{T}$, a scheduling algorithm $\mathcal{A}$ generates the schedule $\sigma_{\mathcal{A}}(t)$
- A task set is schedulable by an algorithm $\mathcal{A}$ if $\sigma_{\mathcal{A}}$ does not contain missed deadlines
- Schedulability test $\rightarrow$ check if $\mathcal{T}$ is schedulable by $\mathcal{A}$

# RT Scheduling: Why?

- The task set $\mathcal{T} = \{(1,3),(4,8)\}$ is not schedulable by FCFS




- $\mathcal{T} = \{(1,3),(4,8)\}$ is schedulable with other algorithms



# Cyclic Executive Scheduling

- Very popular in military and avionics systems
- Also called timeline scheduling or cyclic scheduling
- Originally used for periodic tasks
- Examples:
  - Air traffic control
  - Space Shuttle
  - Boeing 777

# The idea

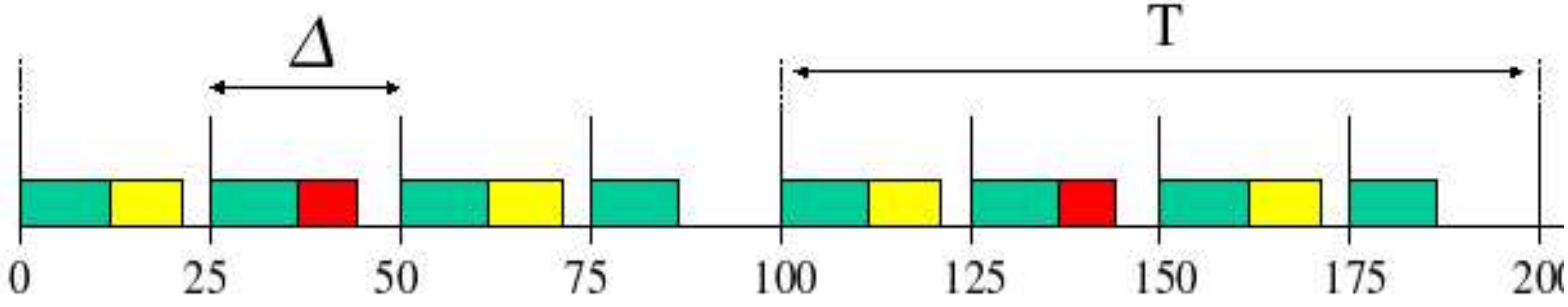
- Static scheduling algorithm
- Jobs are not preemptable
  - A scheduled job executes until termination
- The time axis is divided in time slots
- Slots are statically allocated to the tasks (scheduling table)
- A periodic timer activates execution (allocation of a slot)
  - Major Cycle: least common multiple (lcm) of all the tasks' periods (also called *hyperperiod*)
  - Minor Cycle: greatest common divisor (gcd) of all the tasks' periods
  - A timer fires every Minor Cycle $\Delta$

# Example

| task | f | T |
|---|---|---|
| A | 40 Hz | 25 ms |
| B | 20 Hz | 50 ms |
| C | 10 Hz | 100 ms |

$\Delta$ = gcd (minor cycle)

T = lcm (major cycle)




guarantee: $\begin{cases} C_A + C_B \leq \Delta \\ C_A + C_C \leq \Delta \end{cases}$

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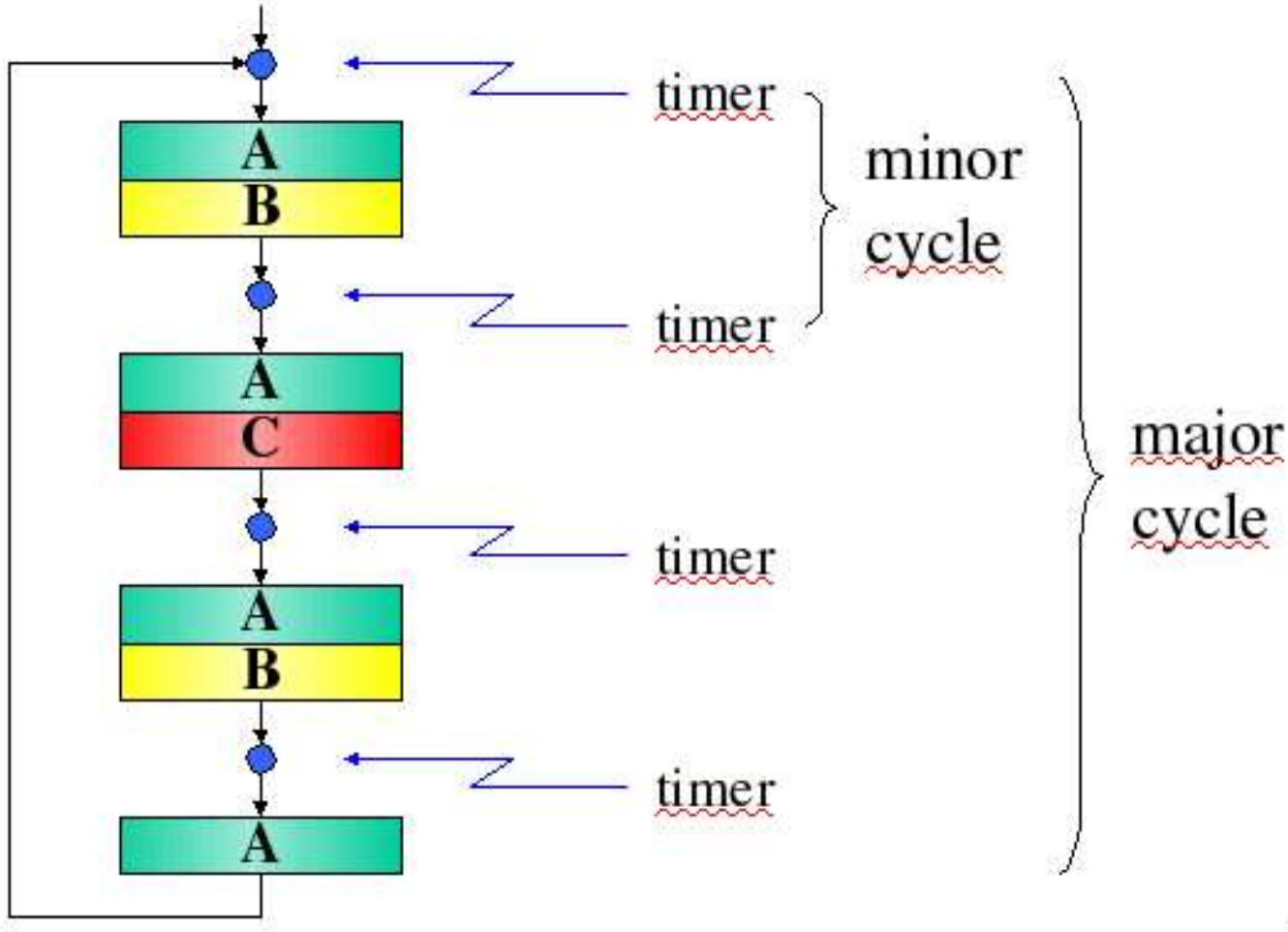
# Implementation

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# Advantages

- Simple implementation (no real-time operating system is required)
  - No real task exist: just function calls
    - One single stack for all the “tasks”
- Non-preemptable scheduling $\Rightarrow$ no need to protect data
  - No need for semaphores, pipes, mutexes, mailboxes, etc.
- Low run-time overhead
- Jitter can be explicitly controlled

# Drawbacks

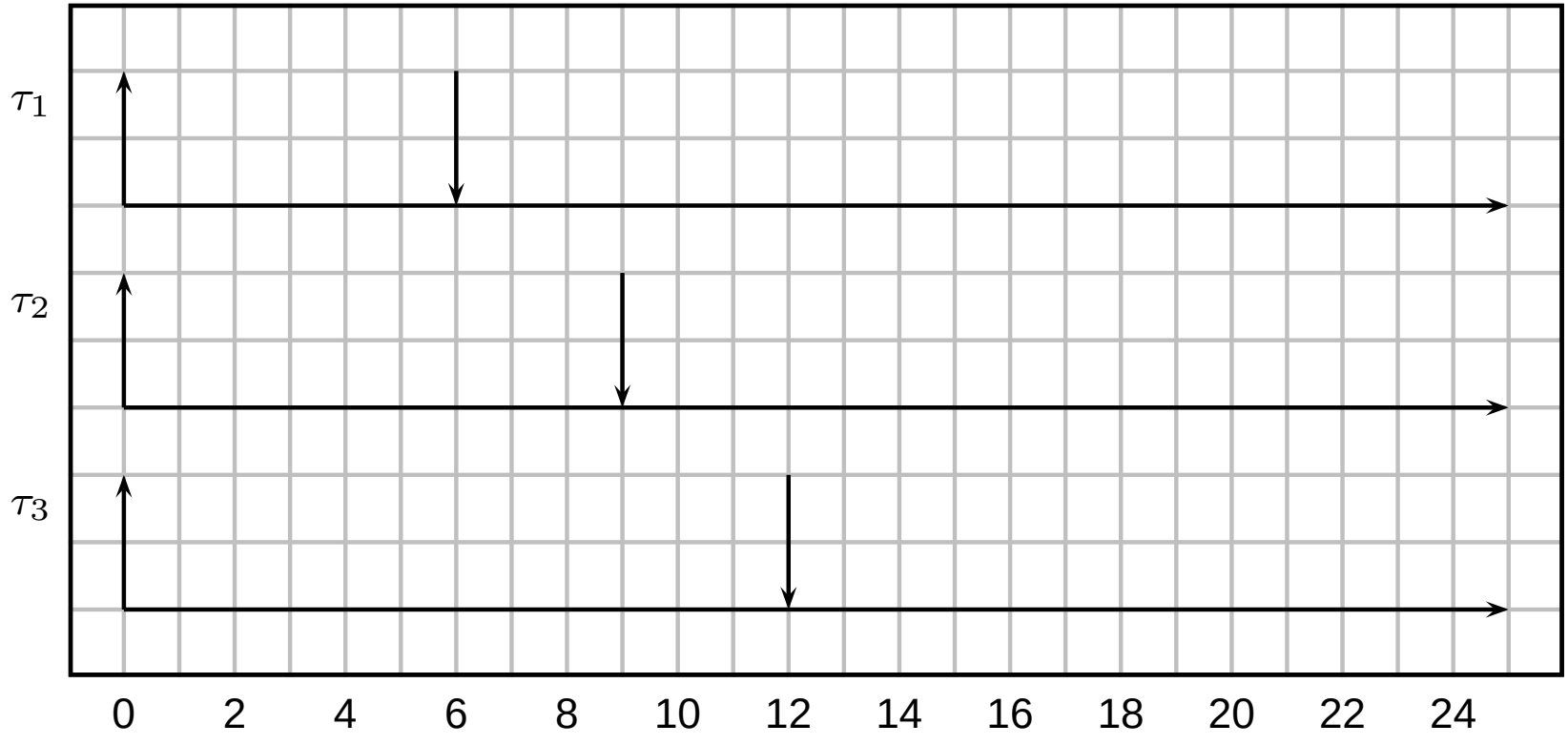
- Not robust during overloads
- Difficult to expand the schedule (static schedule)
  - New task $\Rightarrow$ the whole schedule must be recomputed
- Not easy to handle aperiodic/sporadic tasks
- All task periods must be a multiple of the minor cycle time
- Difficult to incorporate processes with long periods (big tables)
- Variable computation time $\Rightarrow$ it might be necessary to split tasks into a fixed number of fixed size procedures

# Fixed Priority Scheduling

- Very simple *preemptive* scheduling algorithm
  - Every task $\tau_i$ is assigned a fixed priority $p_i$
  - The active task with the highest priority is scheduled
- Priorities are integer numbers: the higher the number, the higher the priority
  - In the research literature, sometimes authors use the opposite convention: the lowest the number, the highest the priority
- In the following we show some examples, considering periodic tasks, constant execution times, and deadlines equal to the period

# Example of Schedule

- Consider the following task set: $\tau_1 = (2, 6, 6)$, $\tau_2 = (2, 9, 9)$, $\tau_3 = (3, 12, 12)$. Task $\tau_1$ has priority $p_1 = 3$ (highest), task $\tau_2$ has priority $p_2 = 2$, task $\tau_3$ has priority $p_3 = 1$ (lowest)



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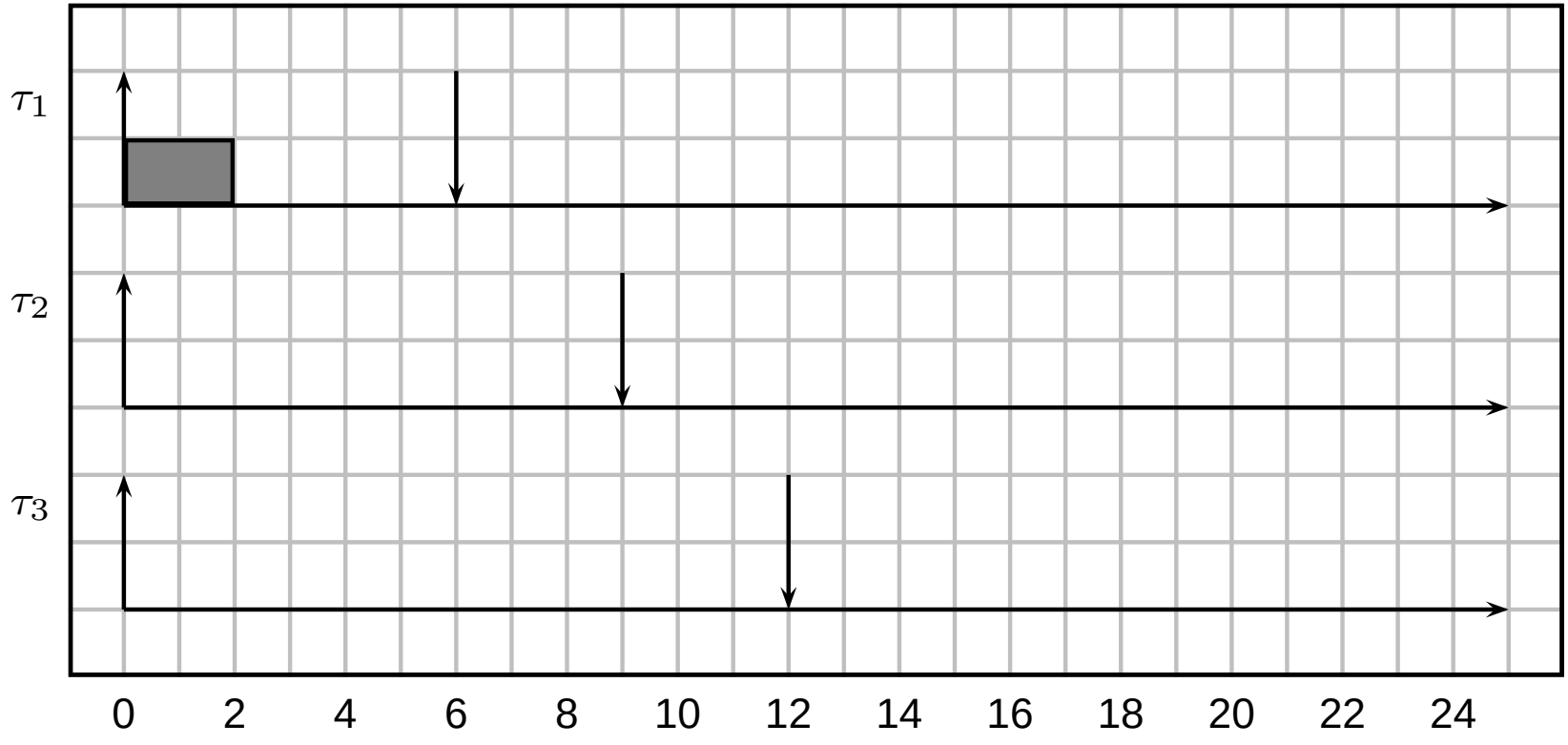

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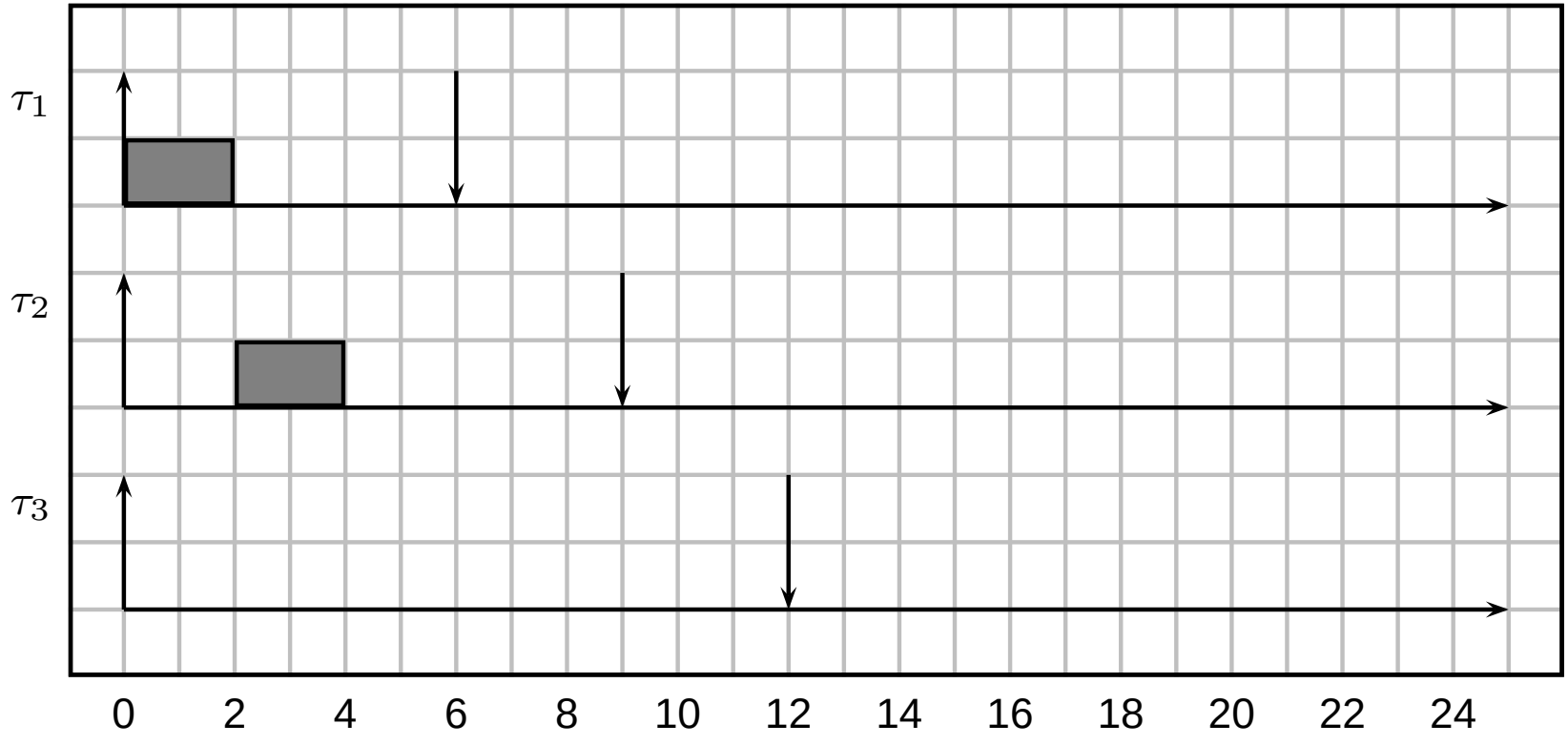

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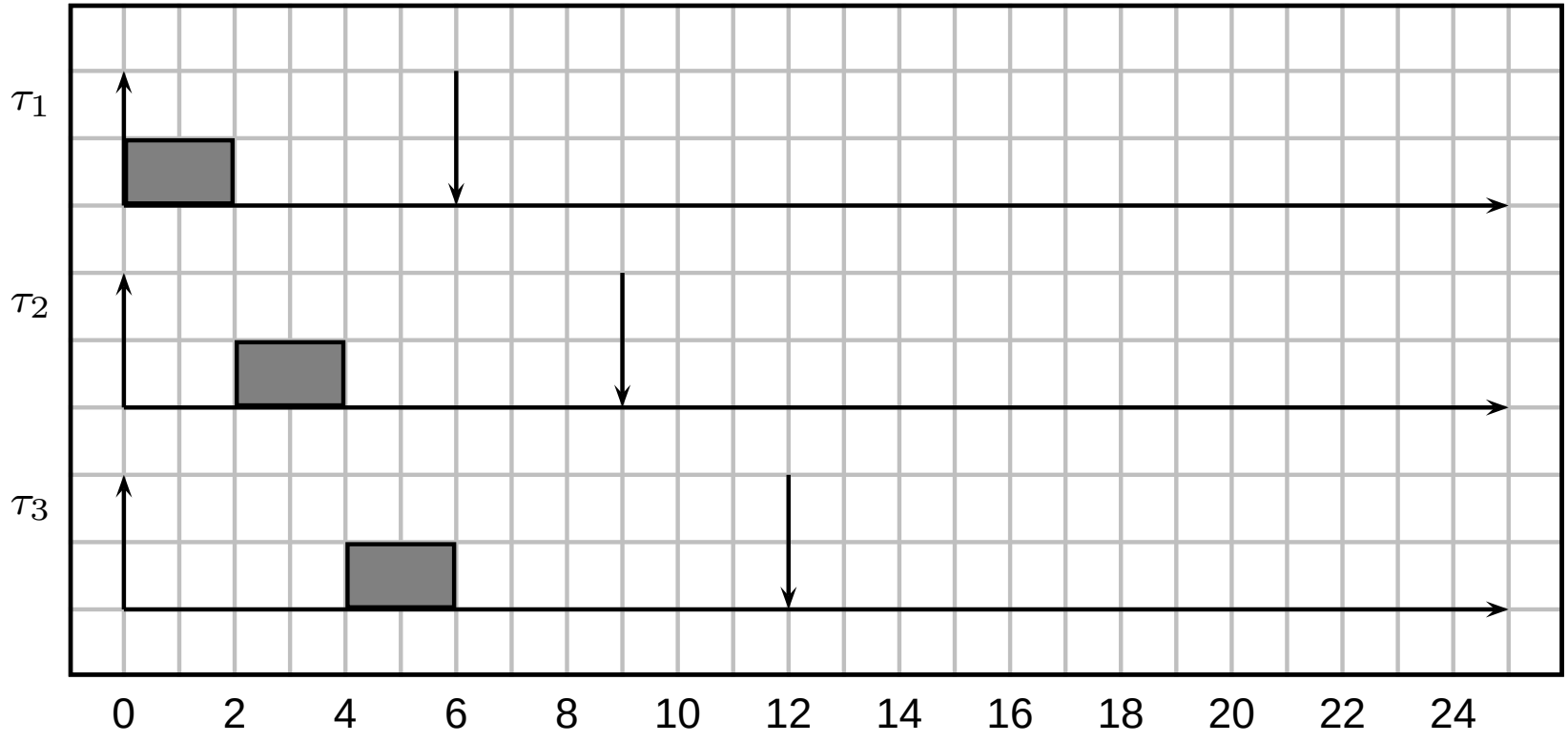

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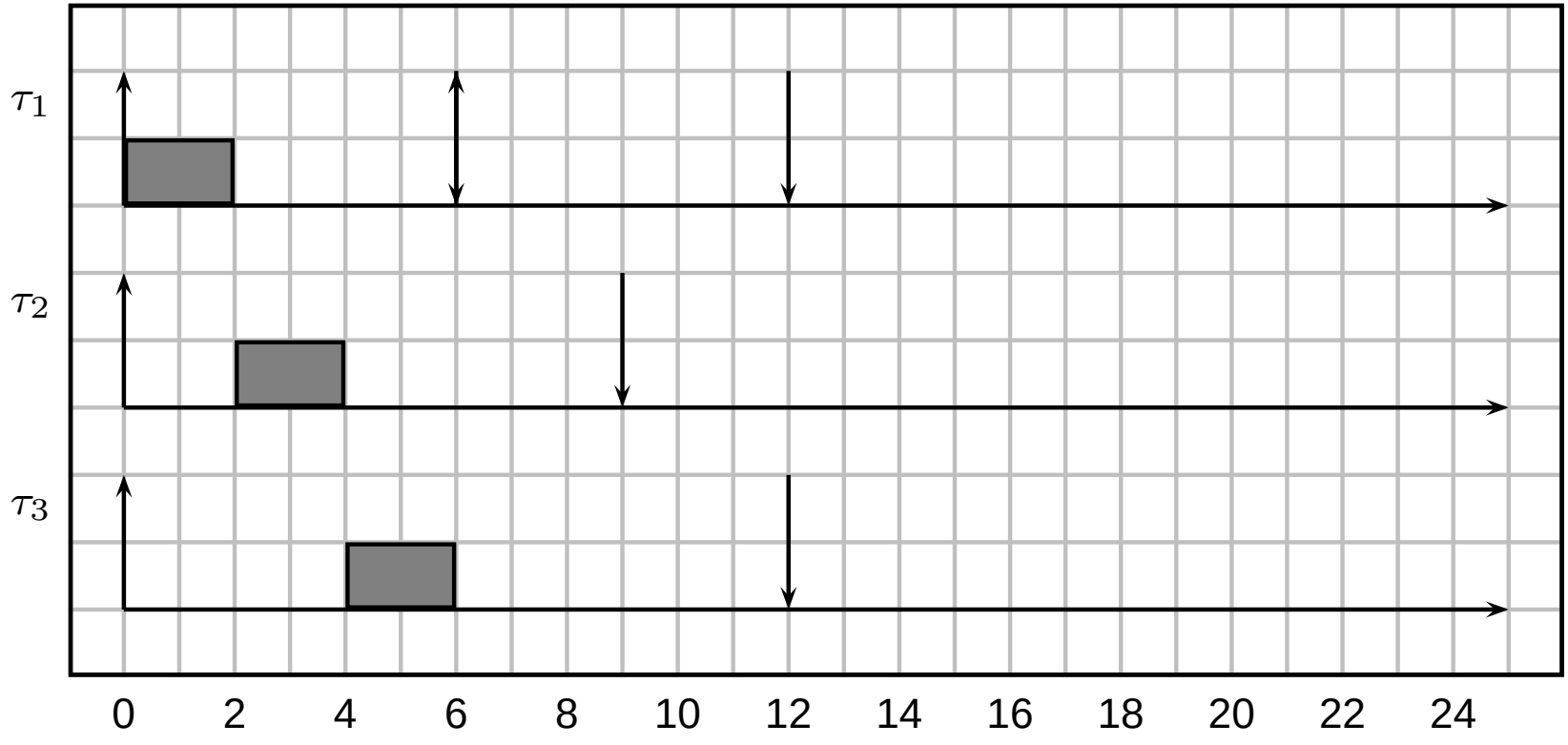

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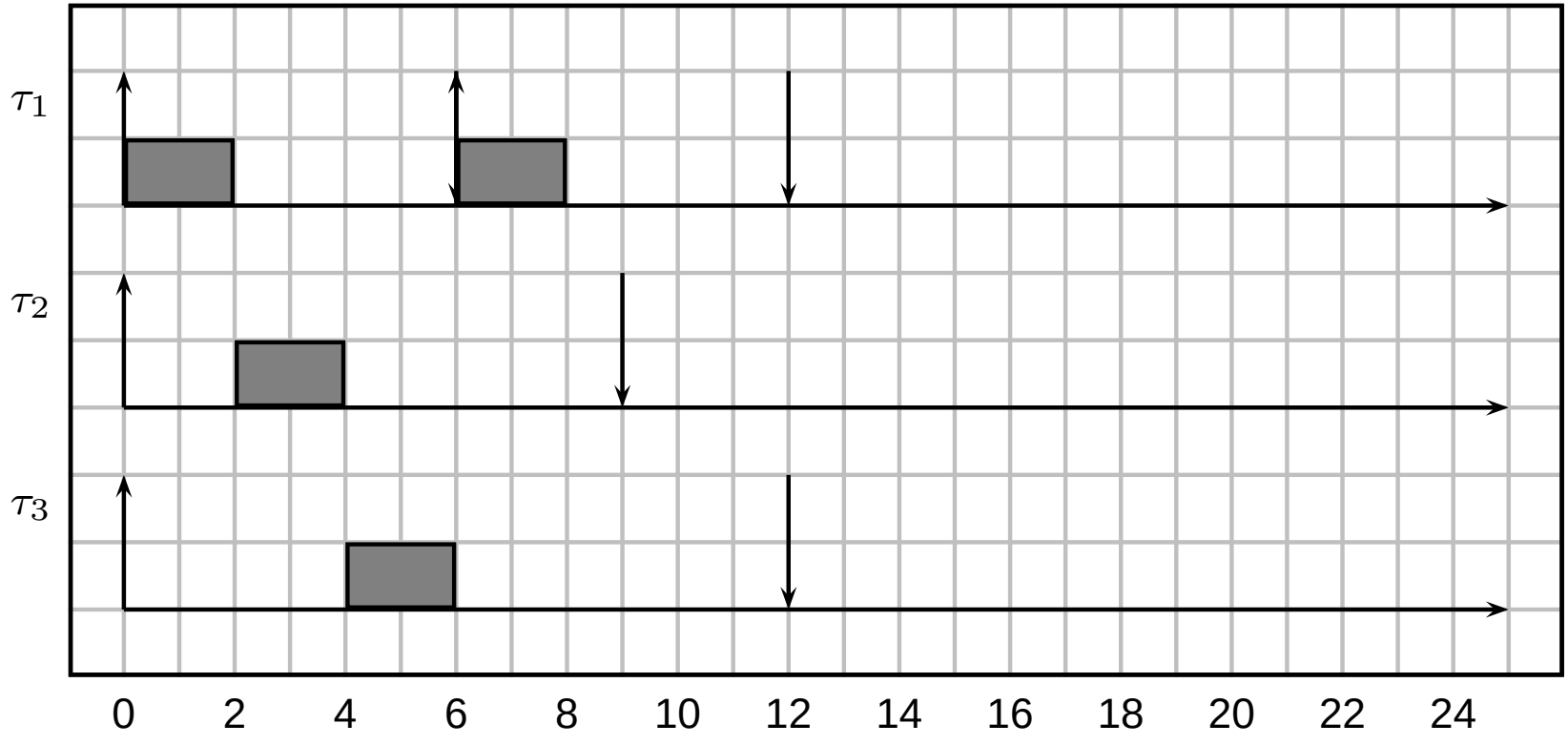

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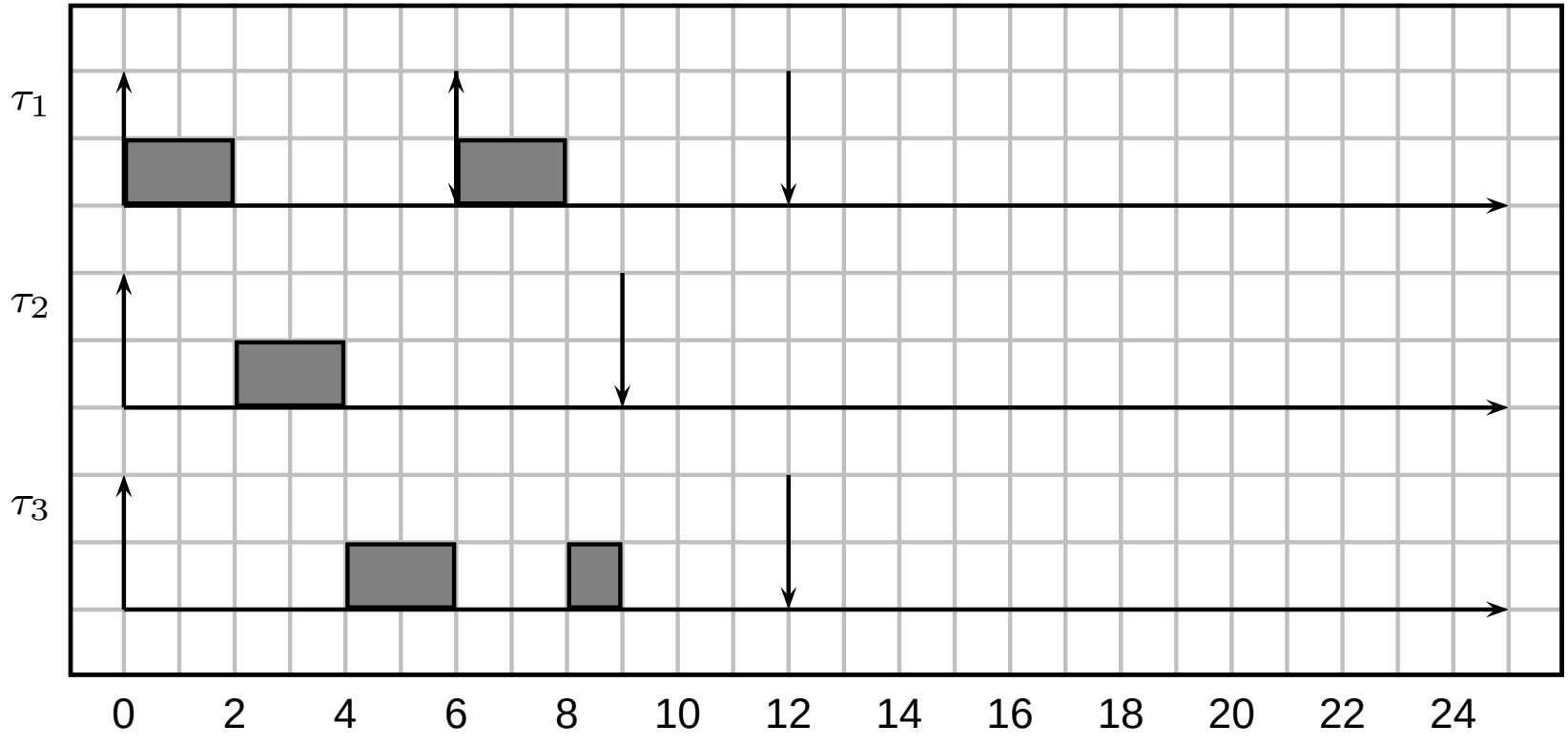

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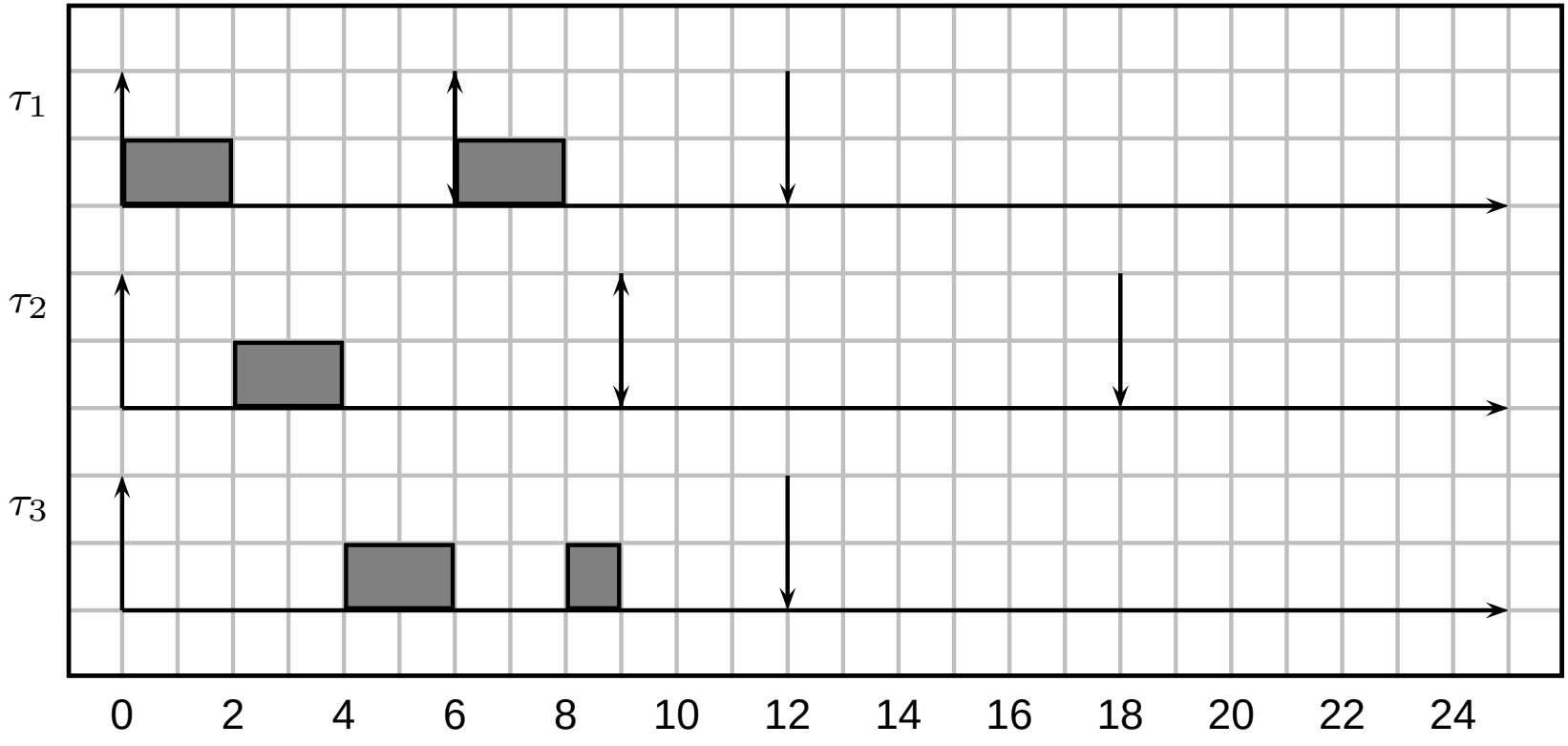

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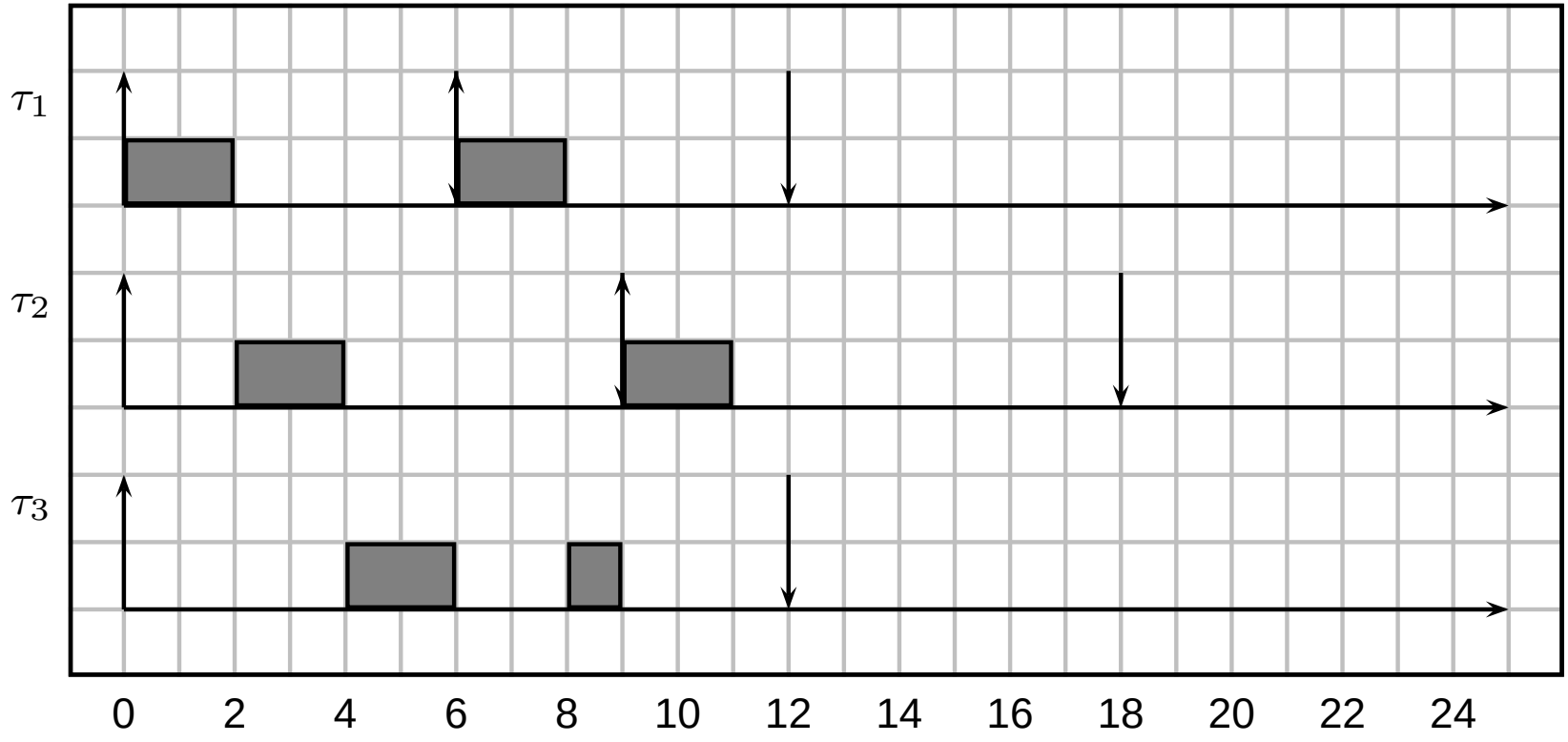

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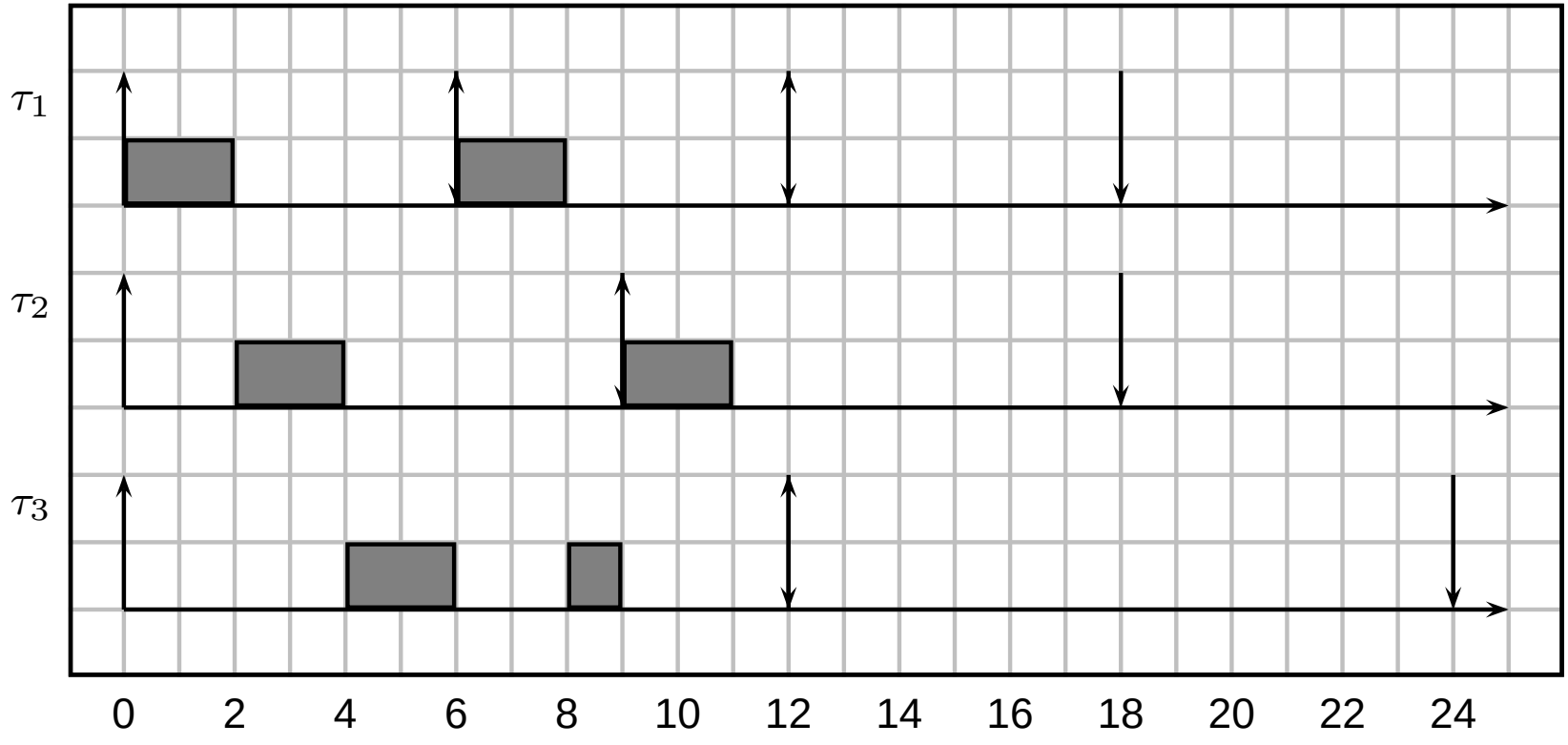

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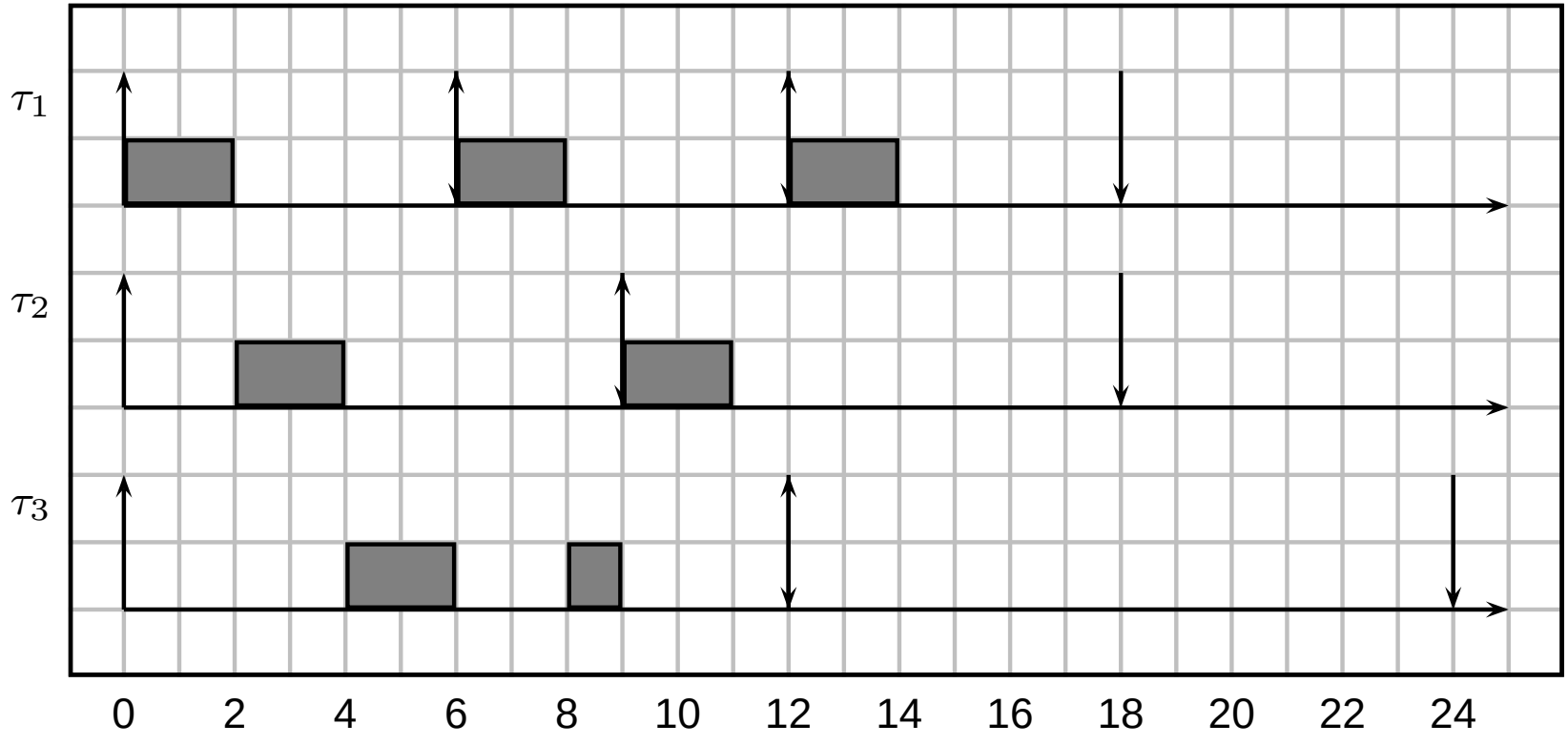

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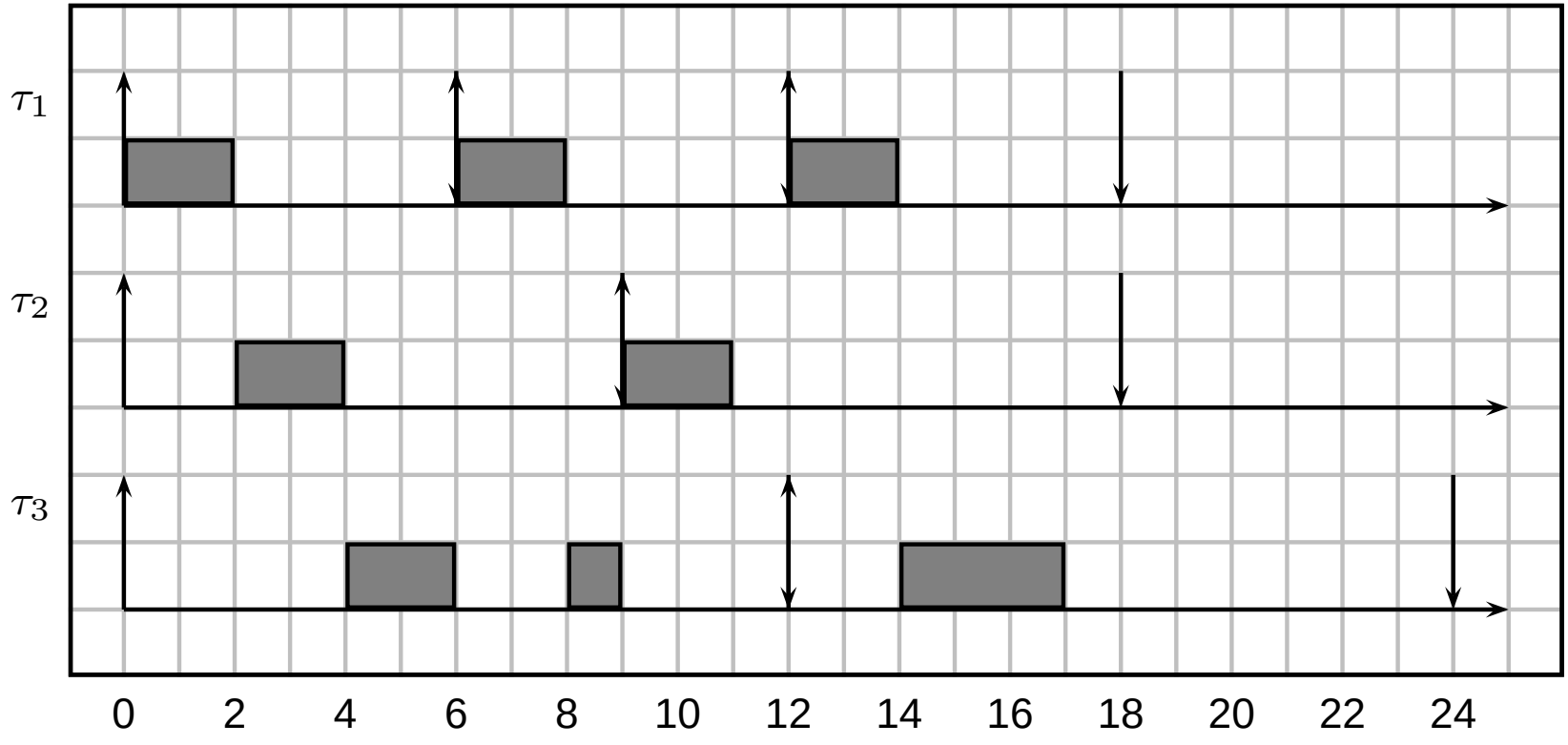

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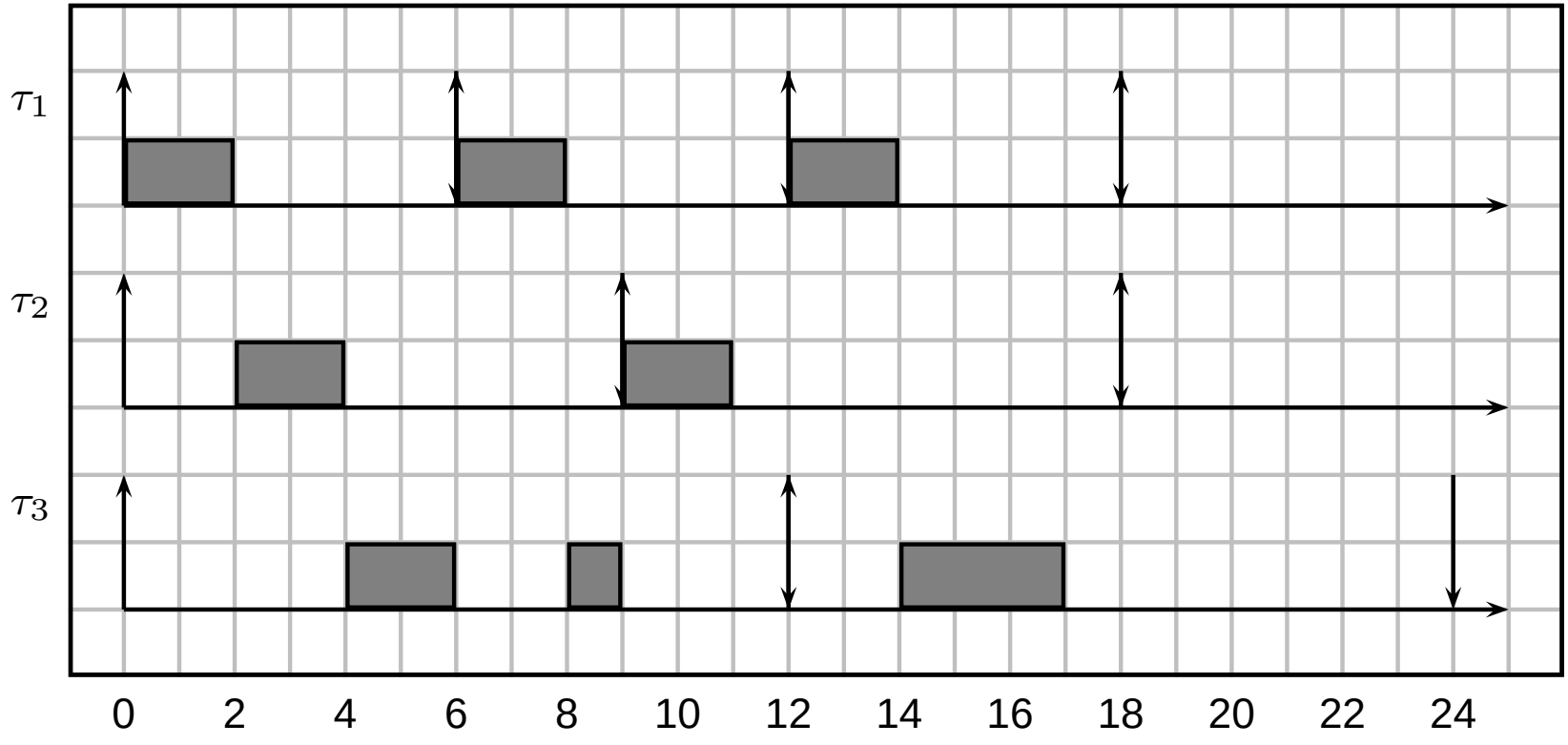

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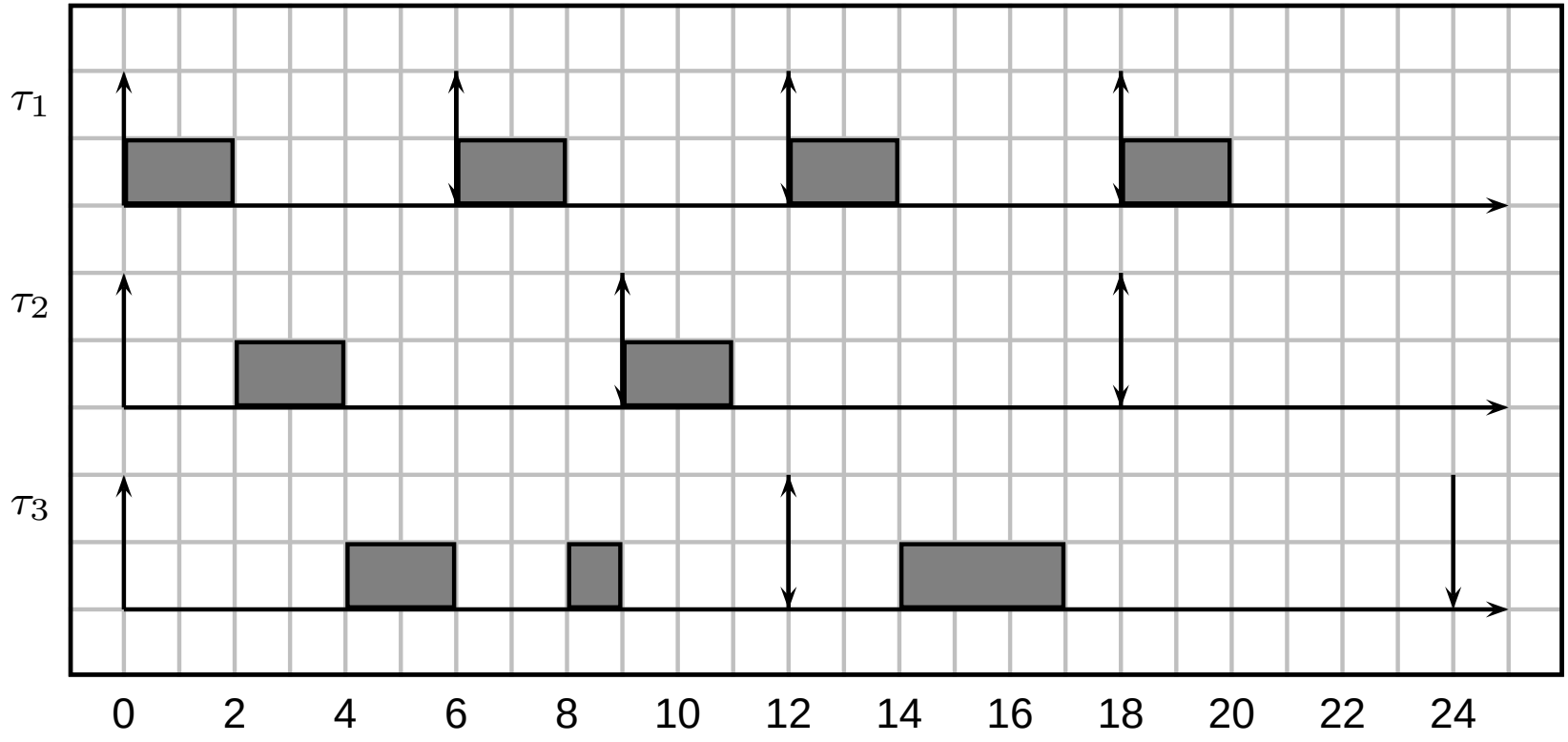

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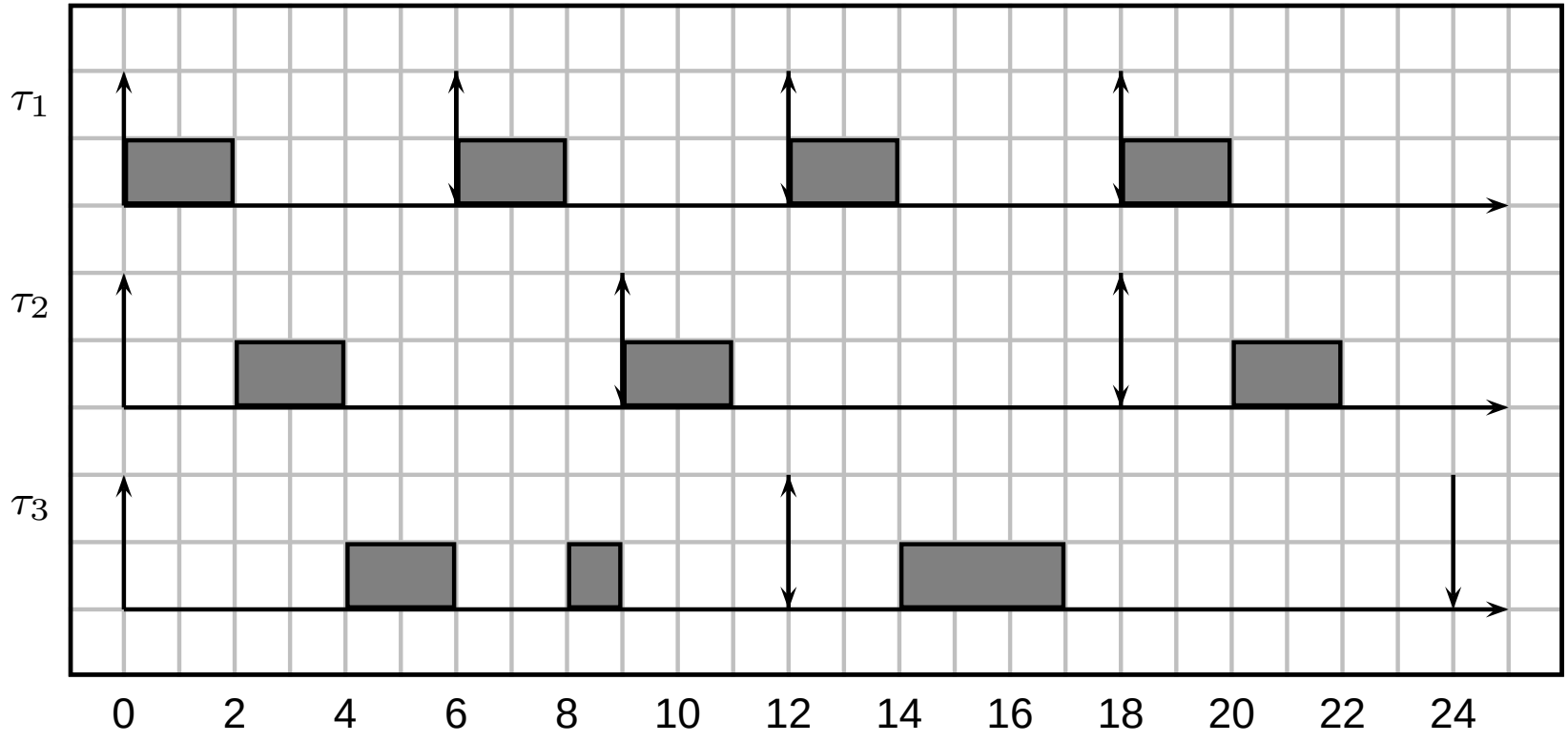
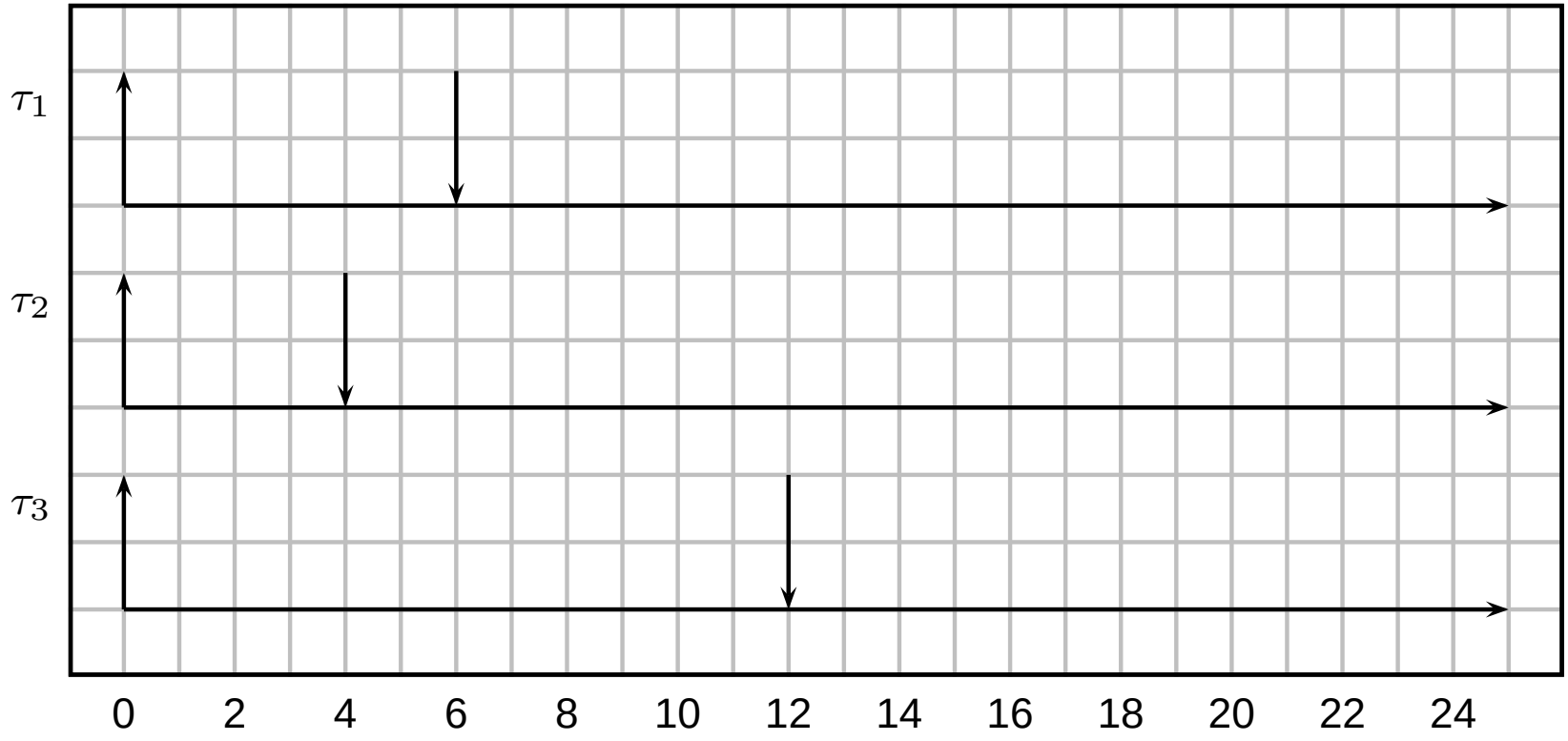


# Another Example (non-schedulable)

- Consider the following task set: $\tau_1 = (3, 6, 6)$, $p_1 = 3$, $\tau_2 = (2, 4, 8)$, $p_2 = 2$, $\tau_3 = (2, 12, 12)$, $p_3 = 1$




In this case, task $\tau_2$ misses its deadline!

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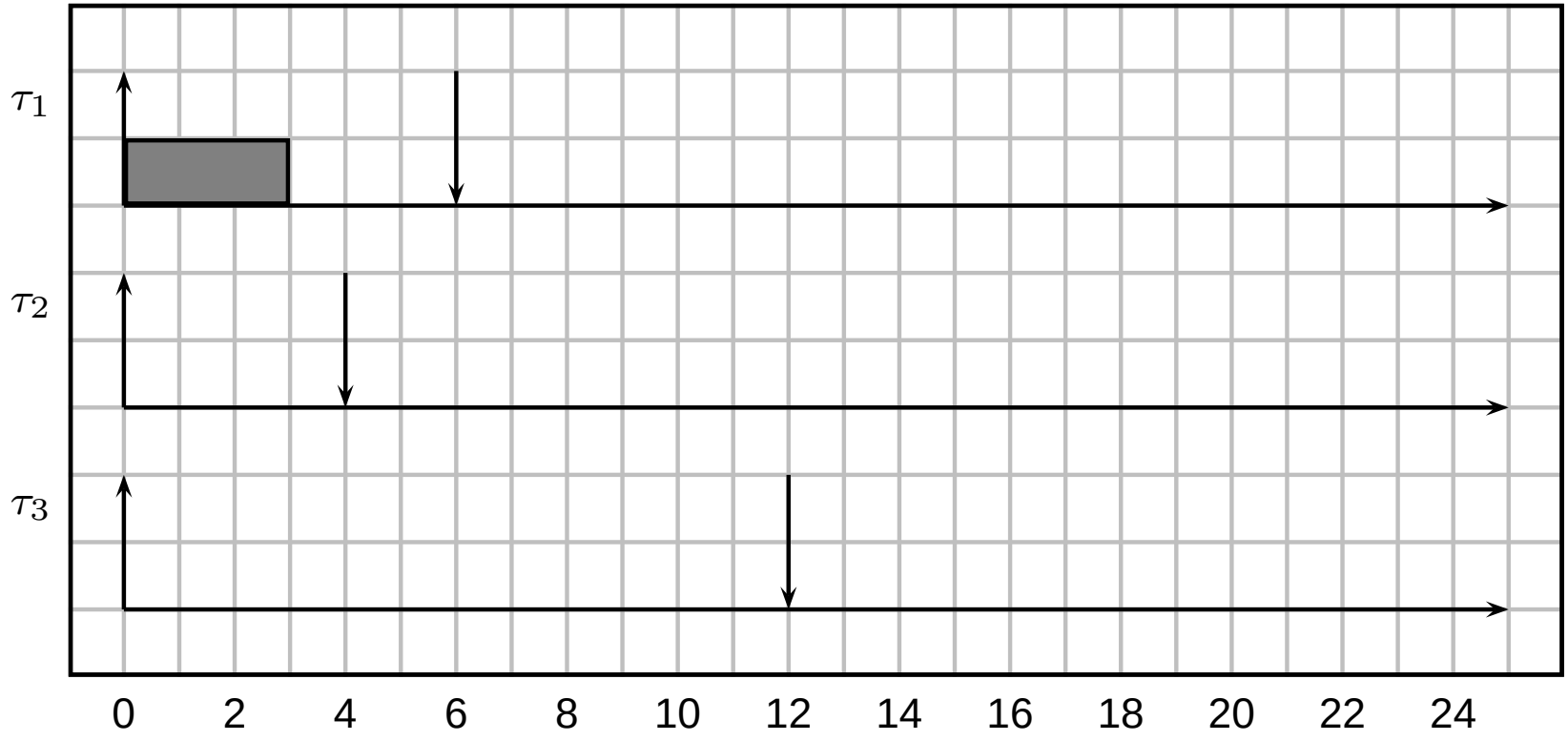


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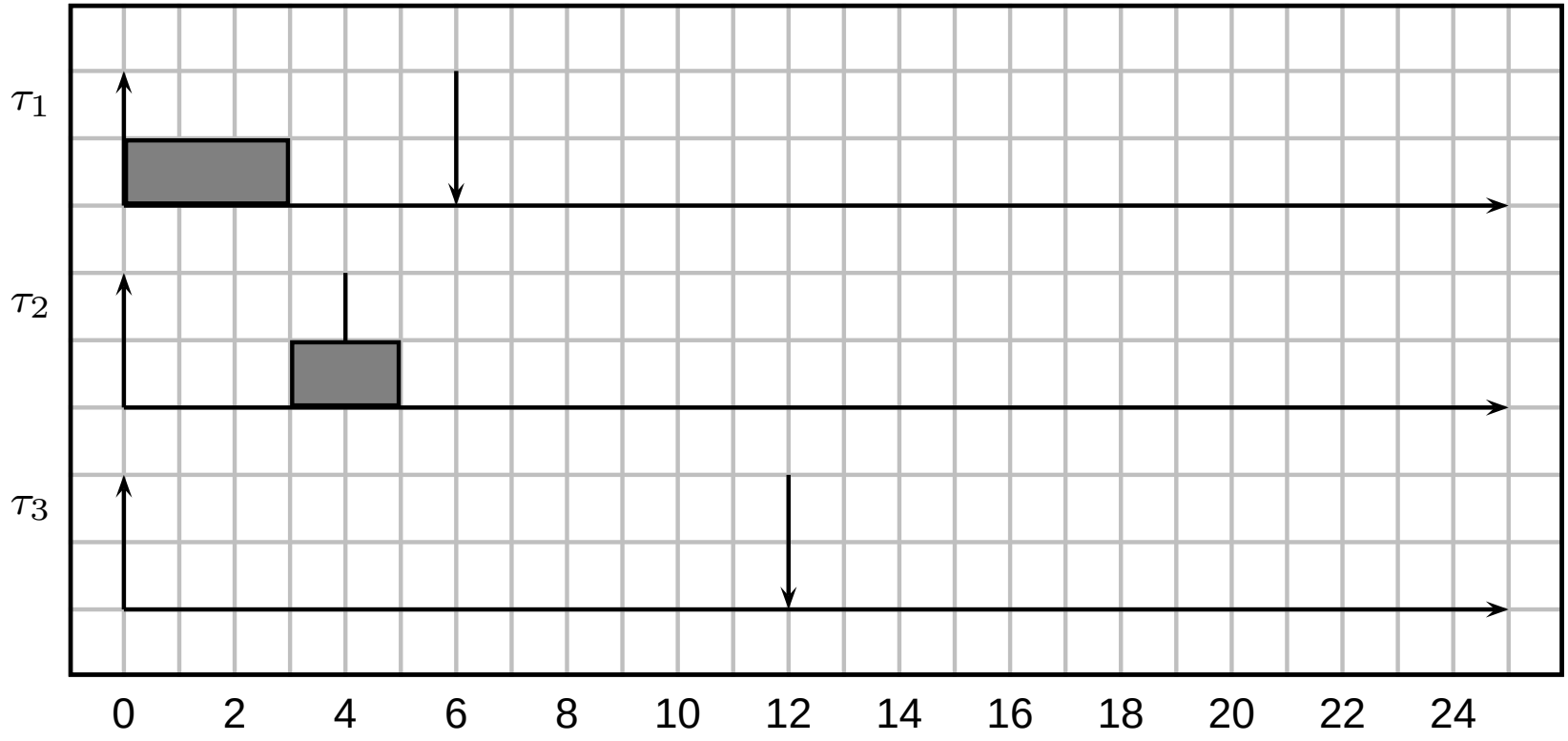


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# Notes about Priority Scheduling

- Some considerations about the schedule shown before:
  - The response time of the task with the highest priority is minimum and equal to its WCET
  - The response time of the other tasks depends on the *interference* of the higher priority tasks
  - The priority assignment may influence the schedulability of a task set
    - Problem: how to assign tasks' priorities so that a task set is schedulable?

# Priority assignment

# Priority assignment

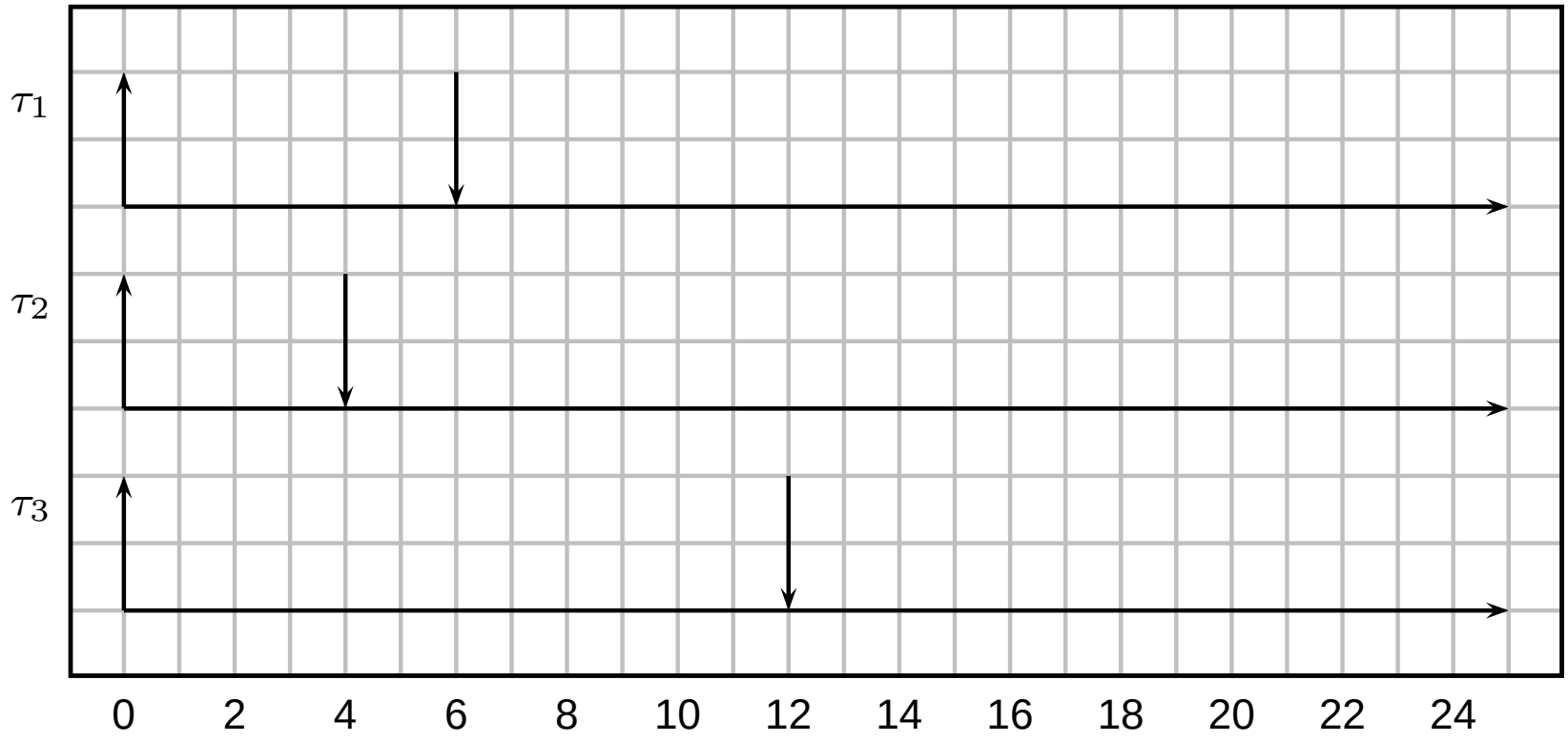
- Given a task set, how to assign priorities?
- There are two possible objectives:
  - Schedulability (i.e. find the priority assignment that makes all tasks schedulable)
  - Response time (i.e. find the priority assignment that minimise the response time of a subset of tasks)
- By now we consider the first objective only
- An *optimal* priority assignment *Opt* is such that:
  - If the task set is schedulable with another priority assignment, then it is schedulable with priority assignment *Opt*
  - If the task set is not schedulable with *Opt*, then it is not schedulable by any other assignment

# Optimal Priority Assignment

- Given a periodic task set T with all tasks having relative deadline $D_i$ equal to the period $T_i$ ($\forall i, \quad D_i = T_i$), and with all offsets equal to 0 ($\forall i, \quad r_{i,0} = 0$):
  - The best assignment is the *Rate Monotonic* (RM) assignment
  - Shorter period $\rightarrow$ higher priority
- Given a periodic task set with deadline different from periods, and with all offsets equal to 0 ($\forall i, \quad r_{i,0} = 0$):
  - The best assignment is the *Deadline Monotonic* assignment
  - Shorter relative deadline $\rightarrow$ higher priority
- For sporadic tasks, the same rules are valid as for periodic tasks with offsets equal to $0$

# Example revised

- Consider the example shown before with deadline monotonic: $\tau_1 = (3, 6, 6)$, $p_1 = 2$, $\tau_2 = (2, 4, 8)$, $p_2 = 3$, $\tau_3 = (2, 12, 12)$, $p_3 = 1$



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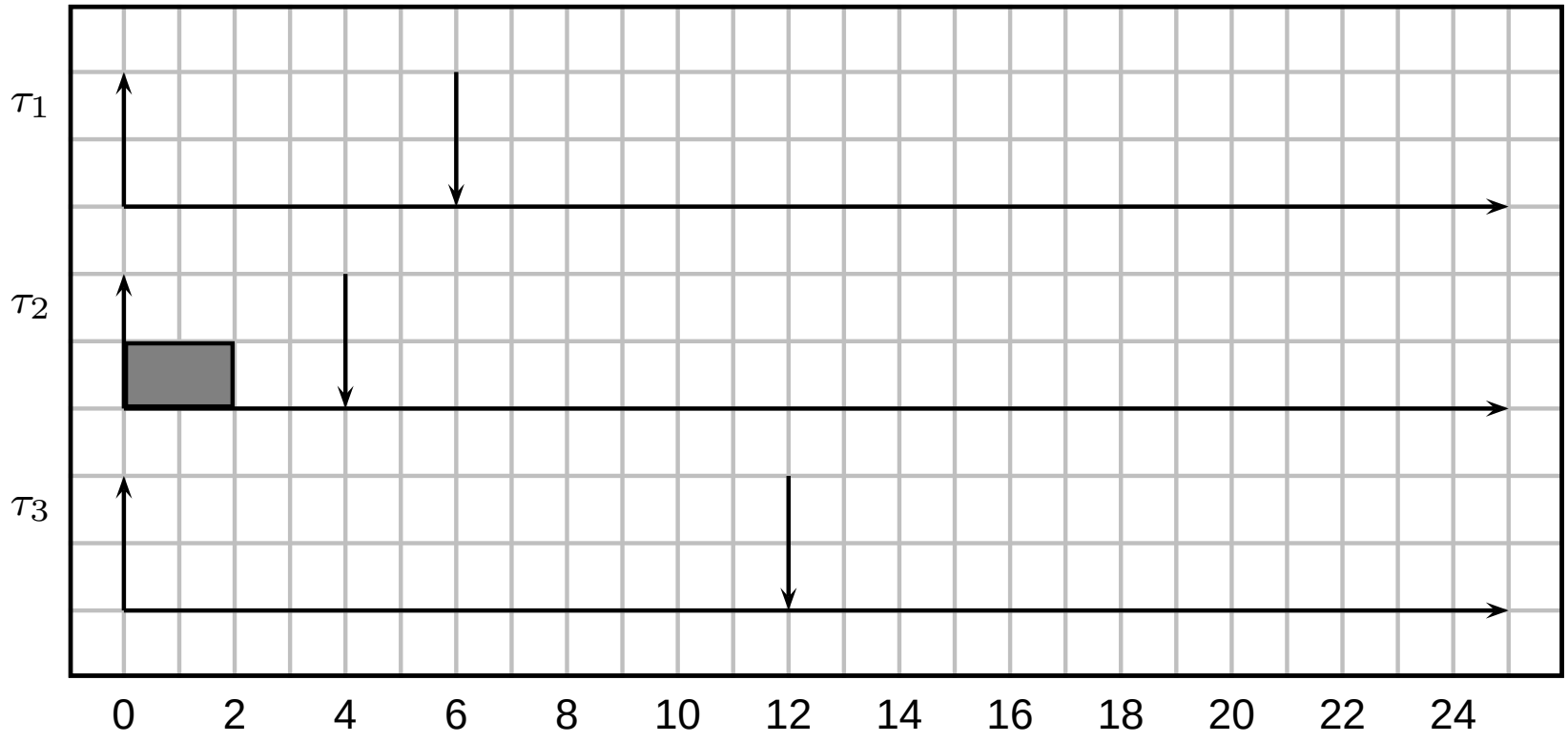

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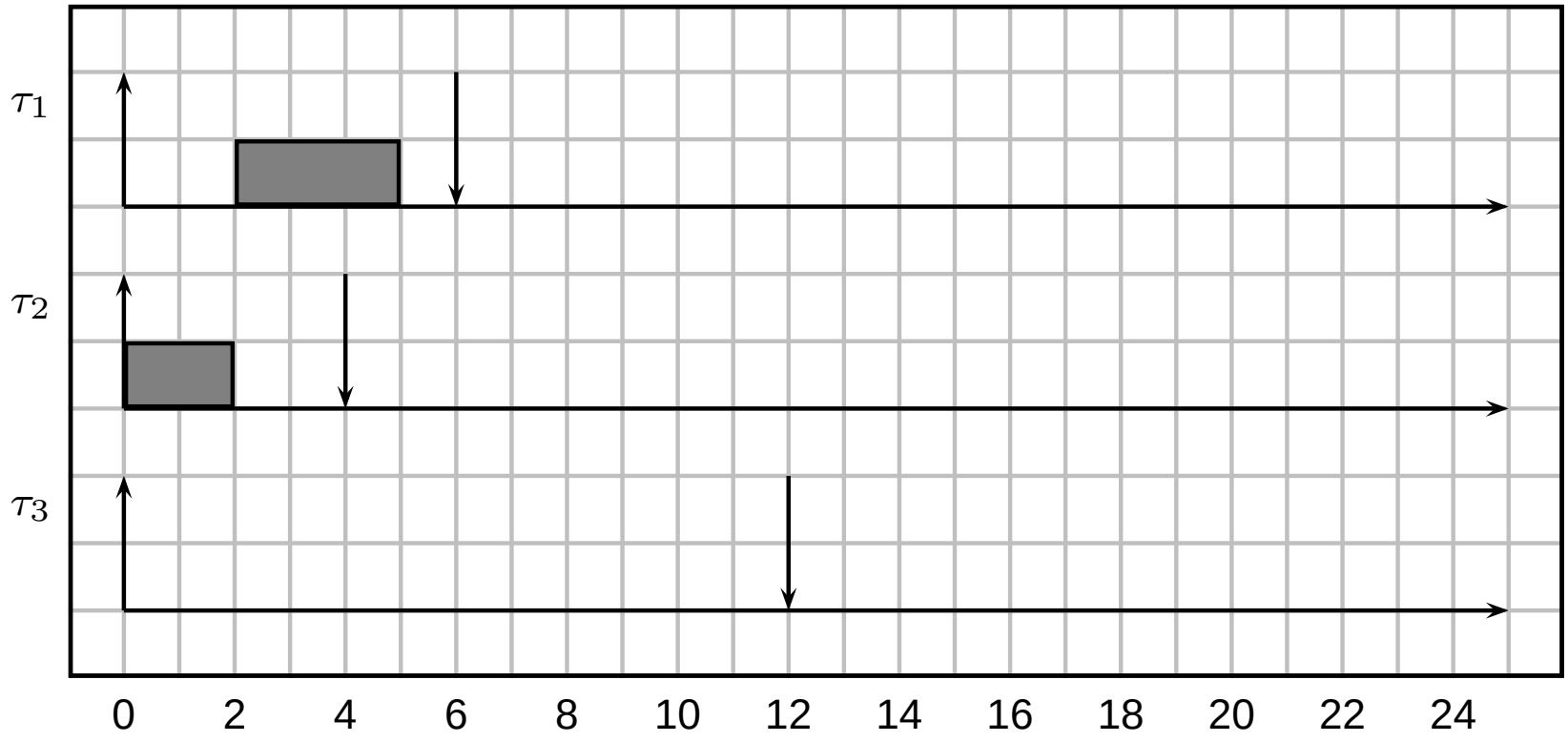

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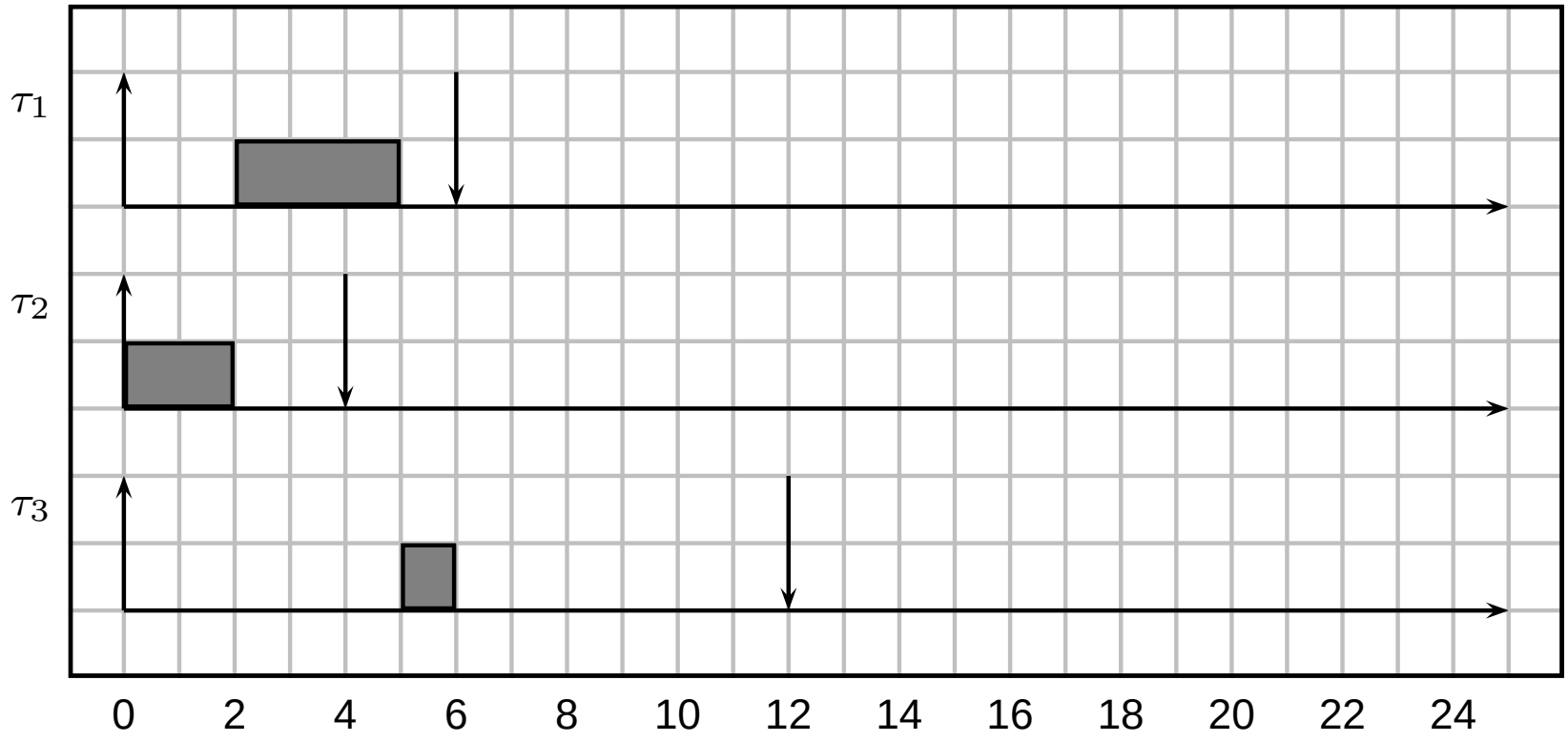

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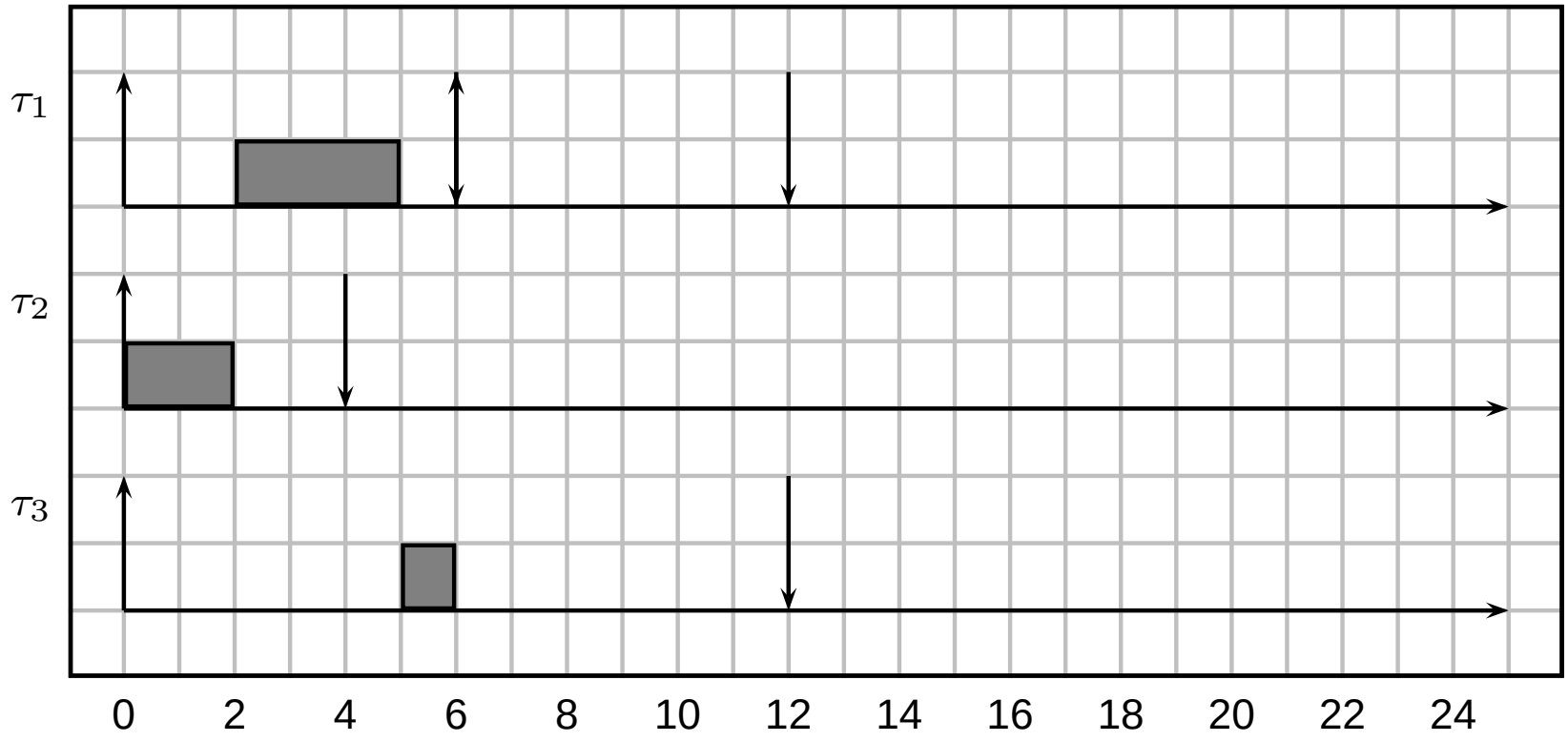

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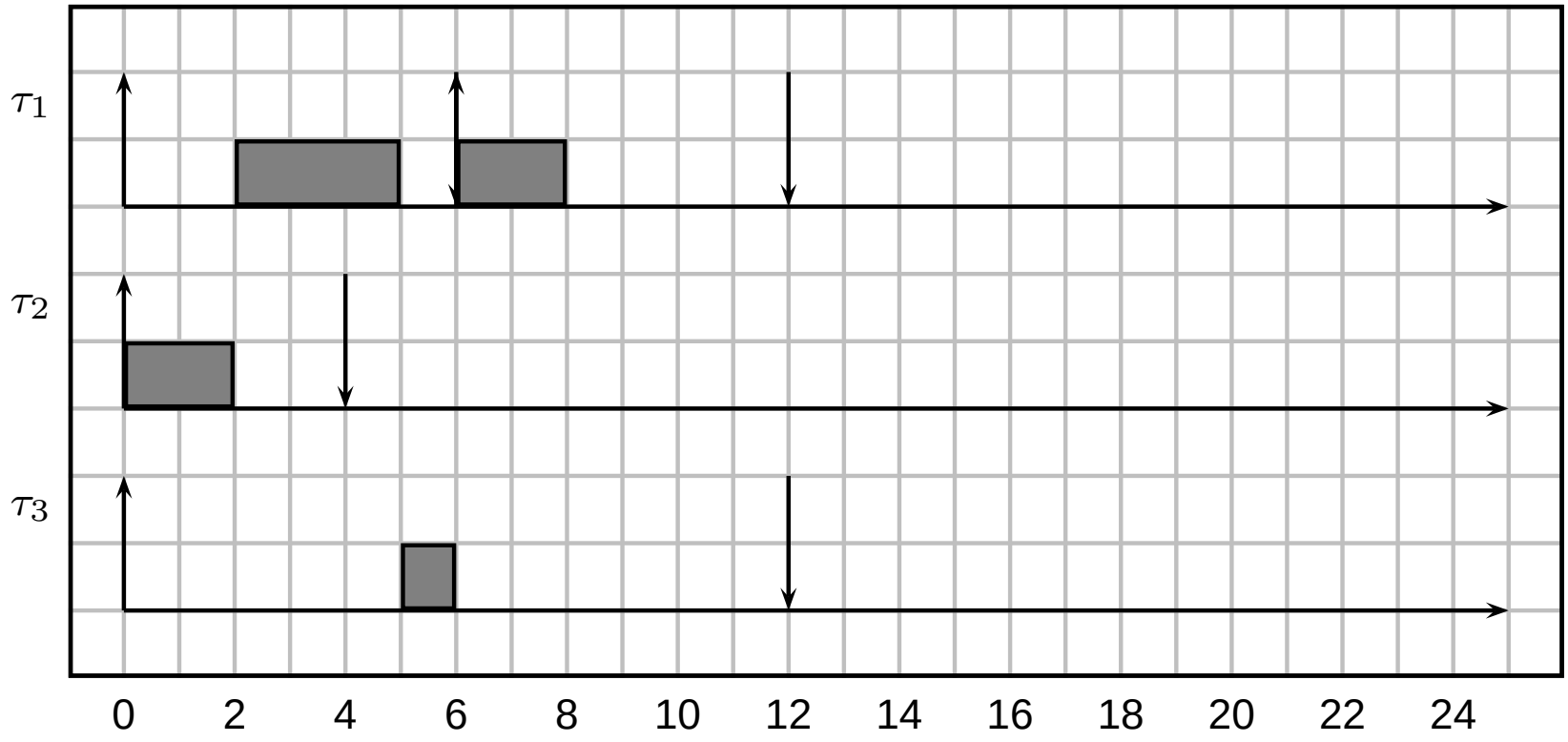

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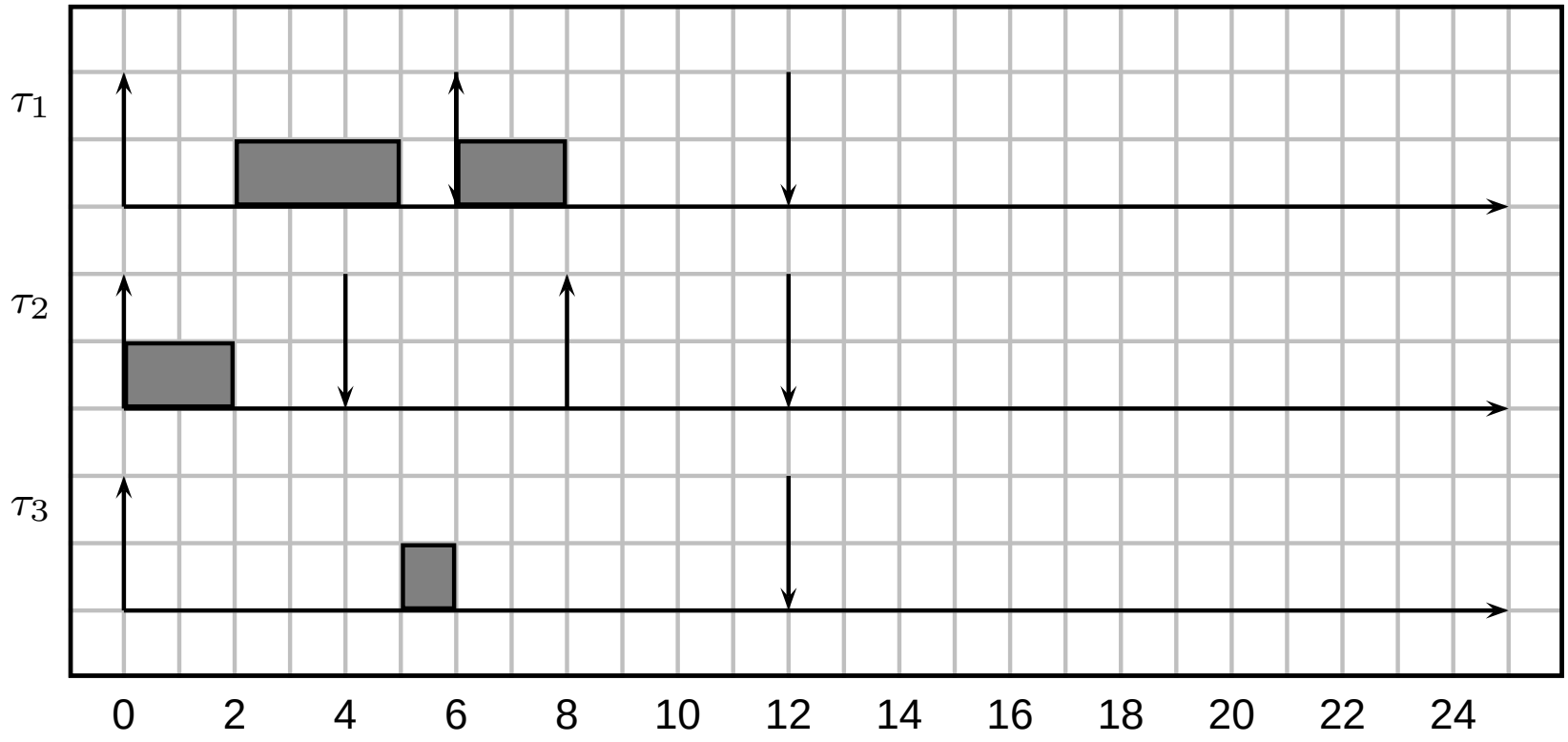

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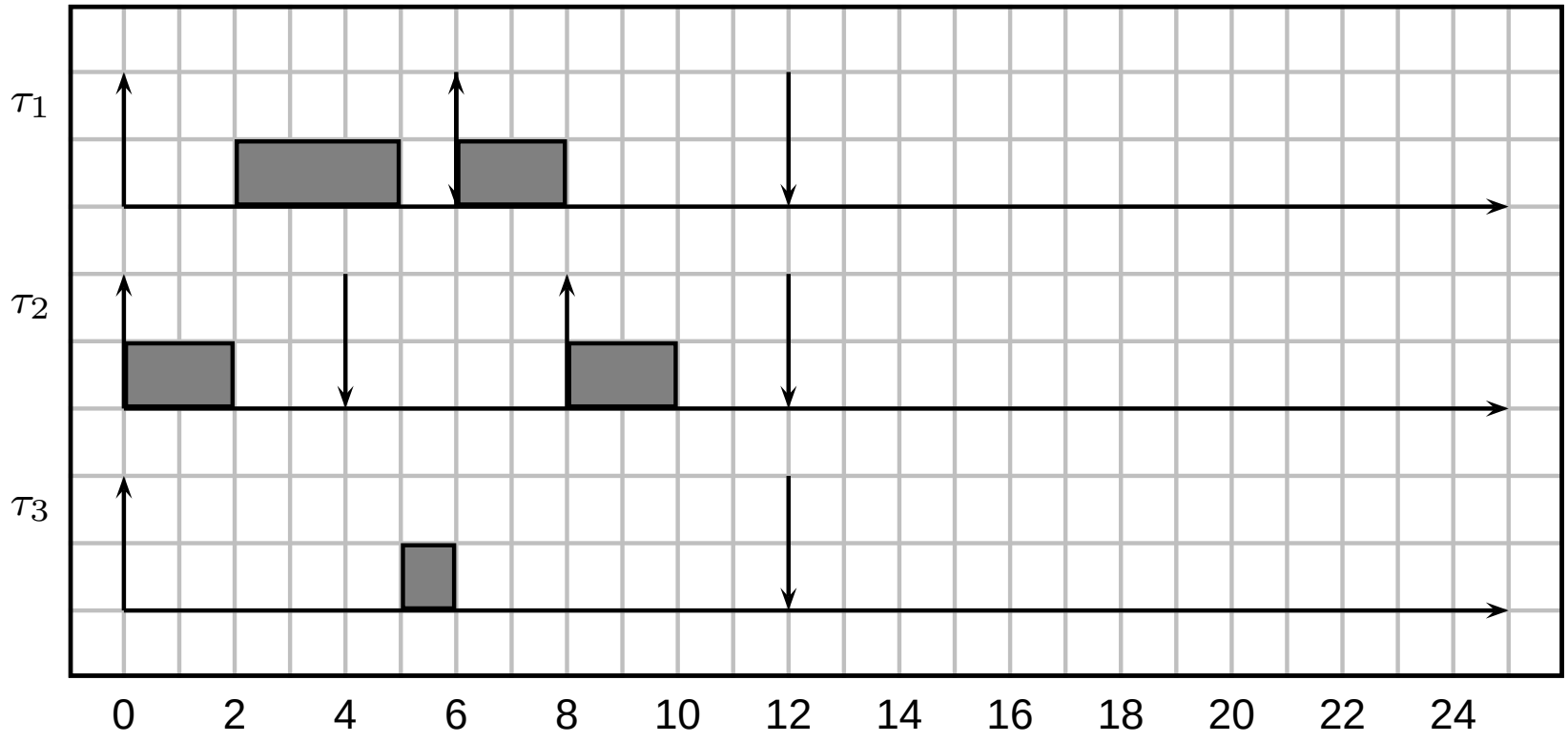

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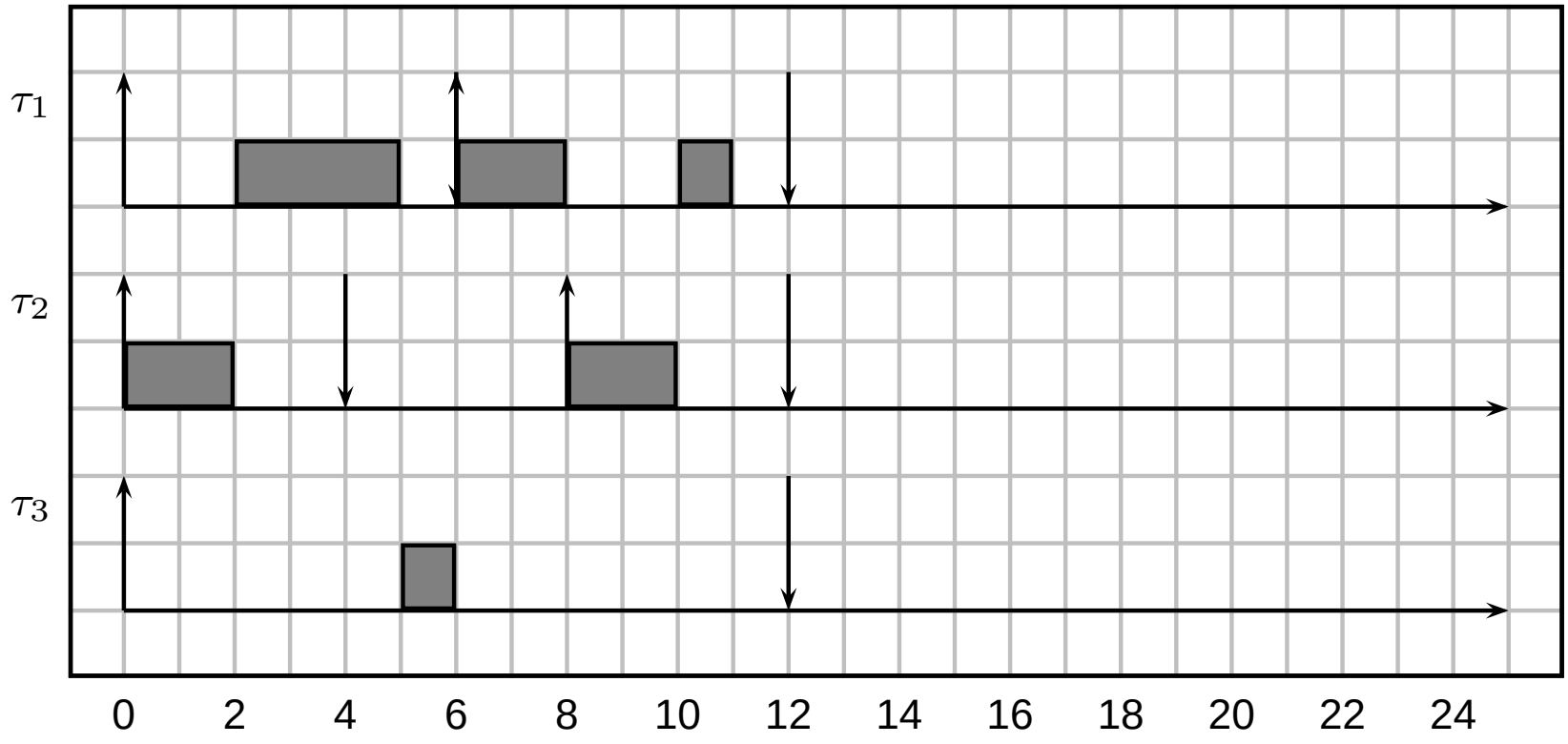

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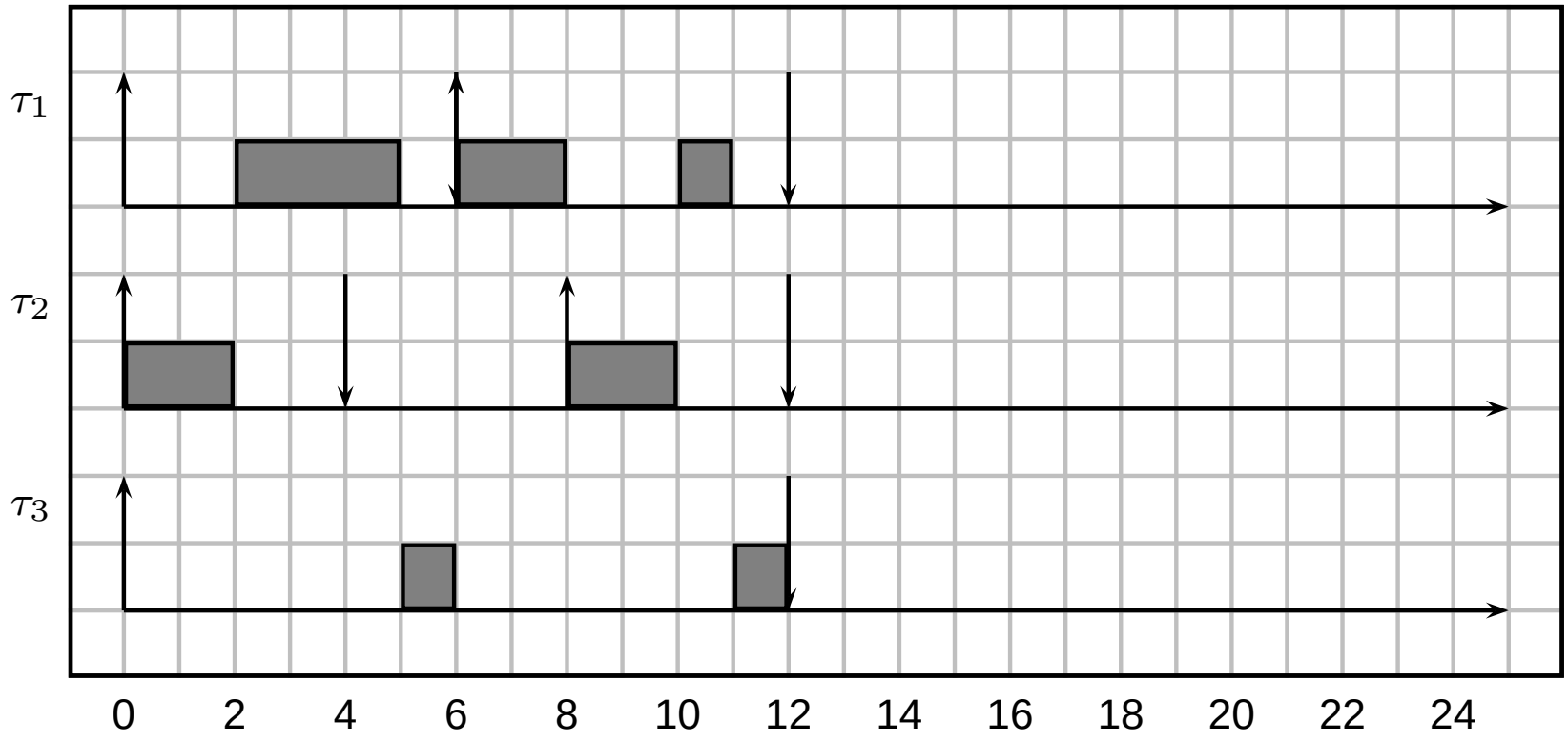

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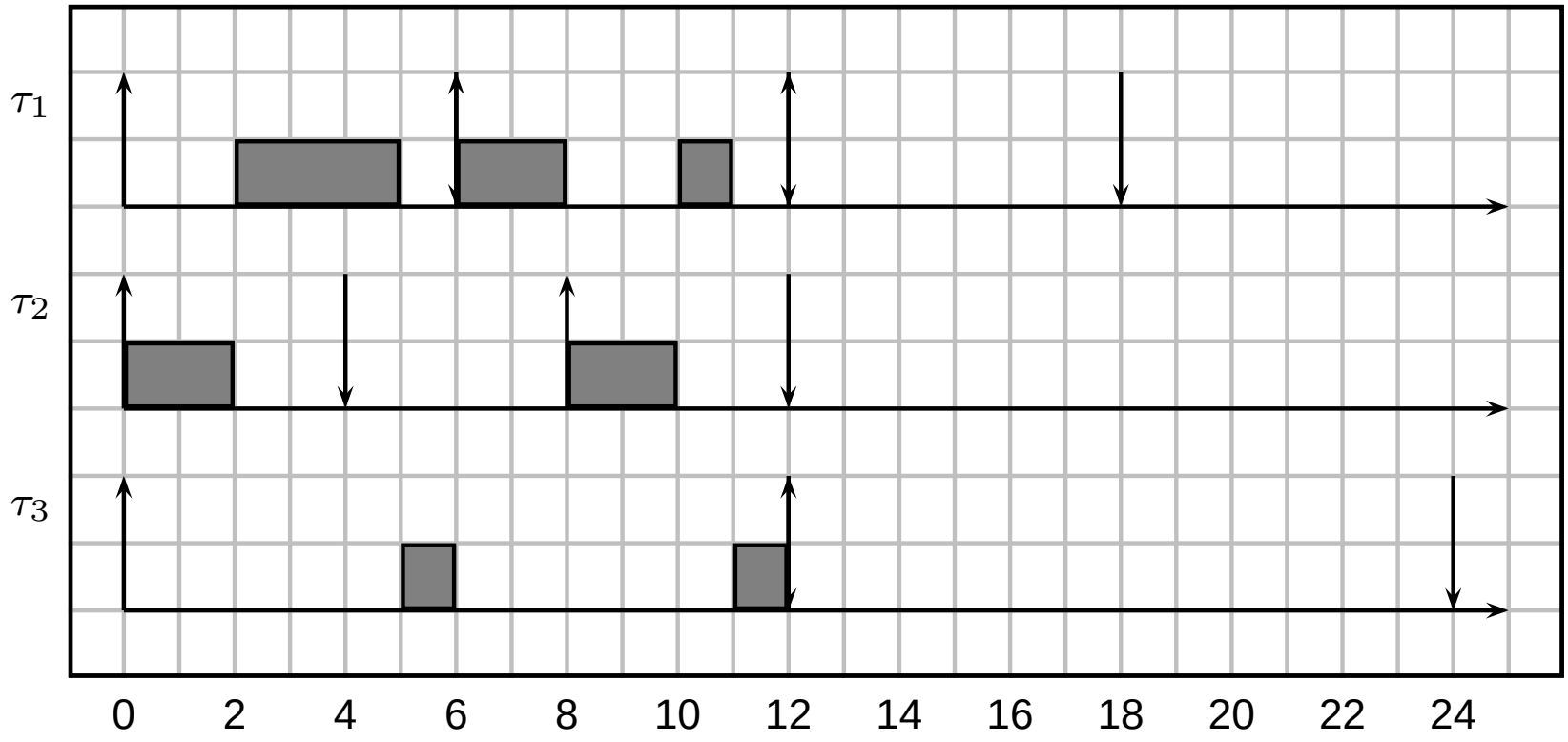

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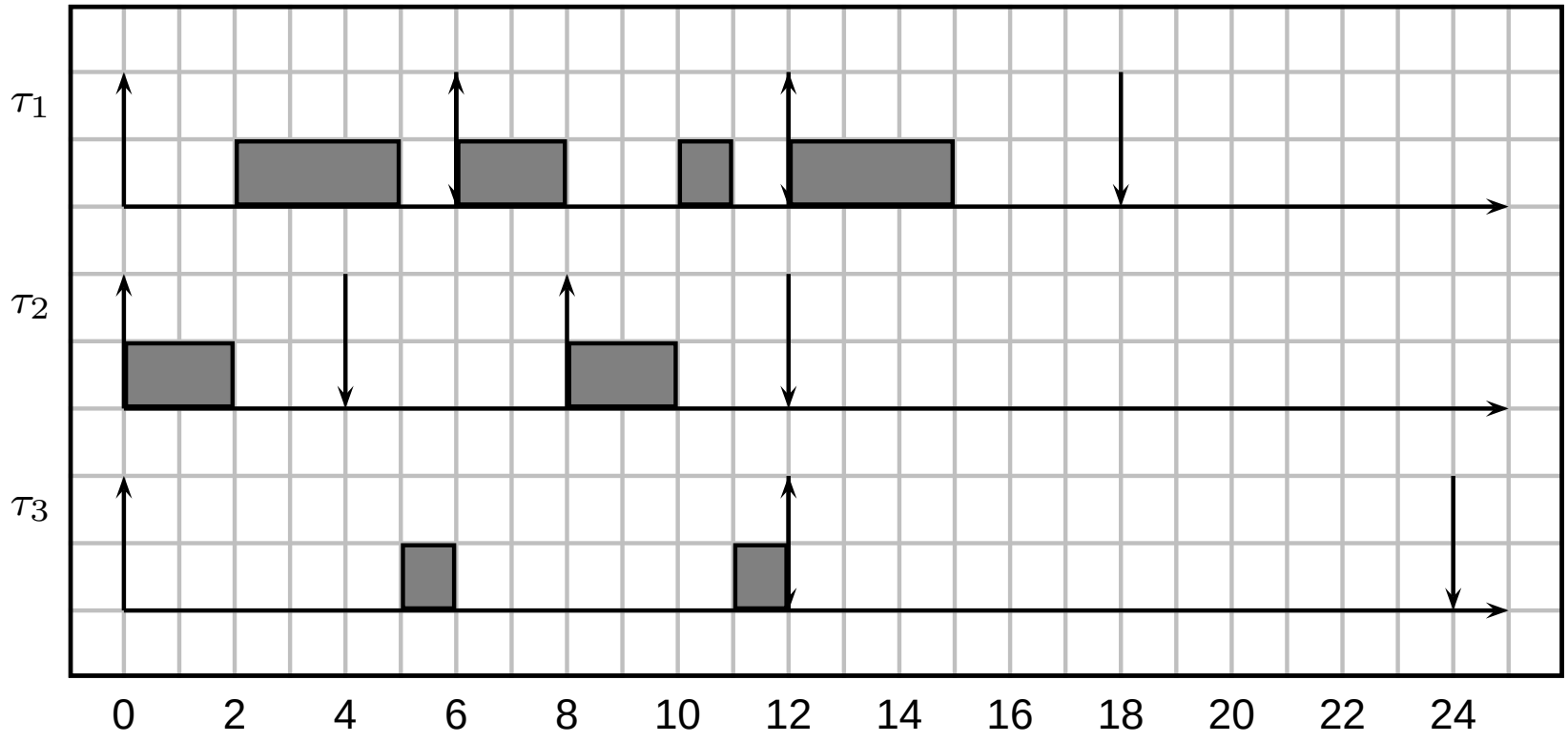

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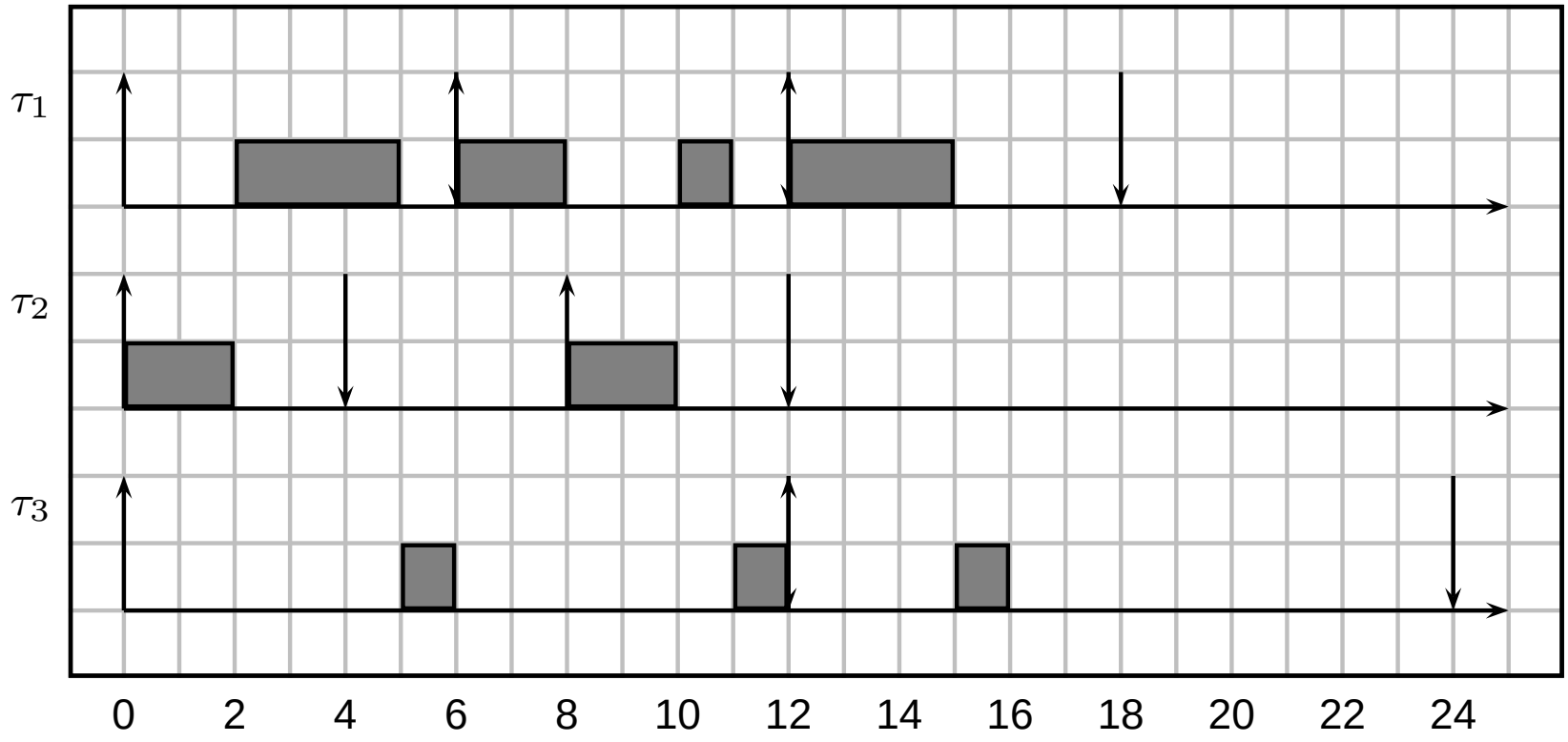

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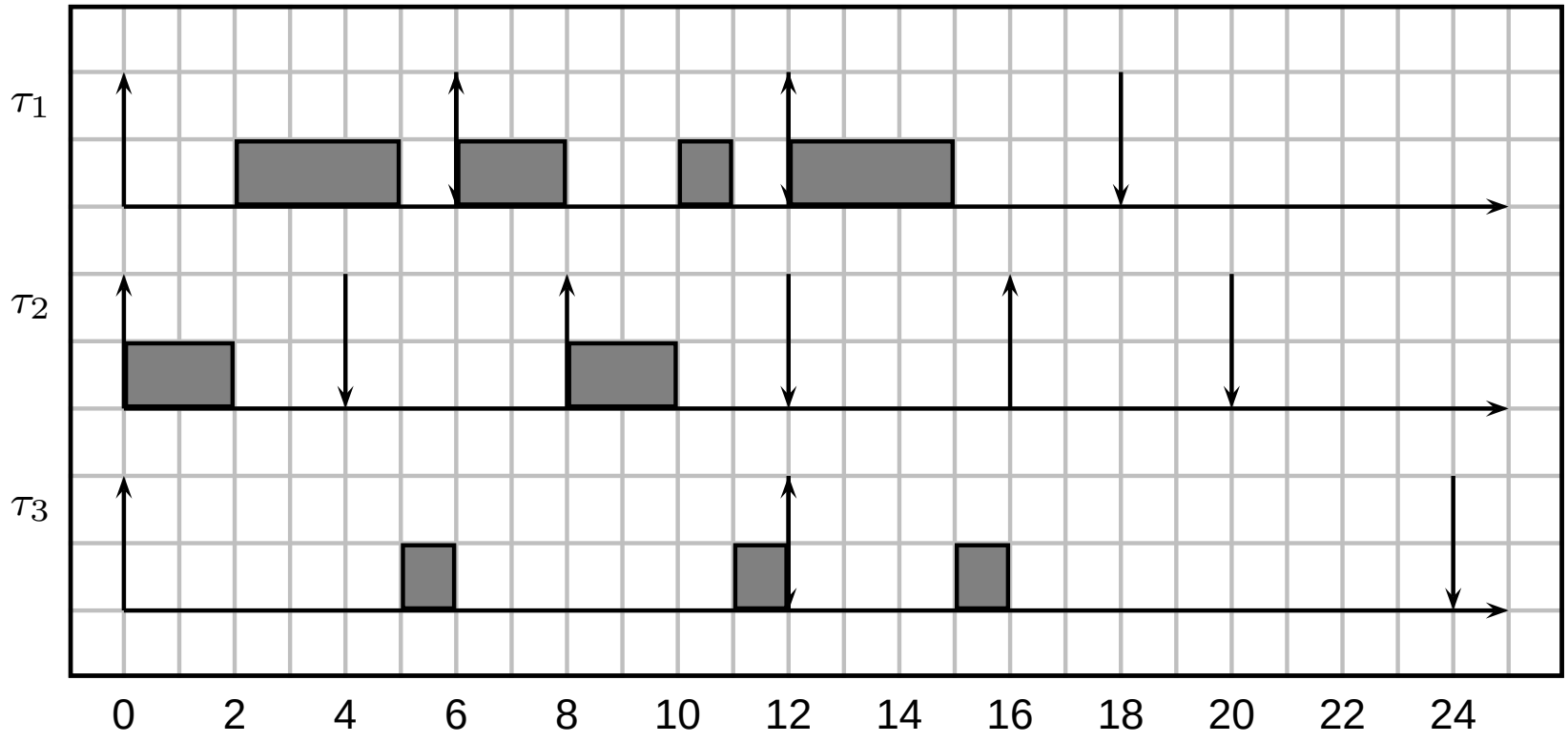

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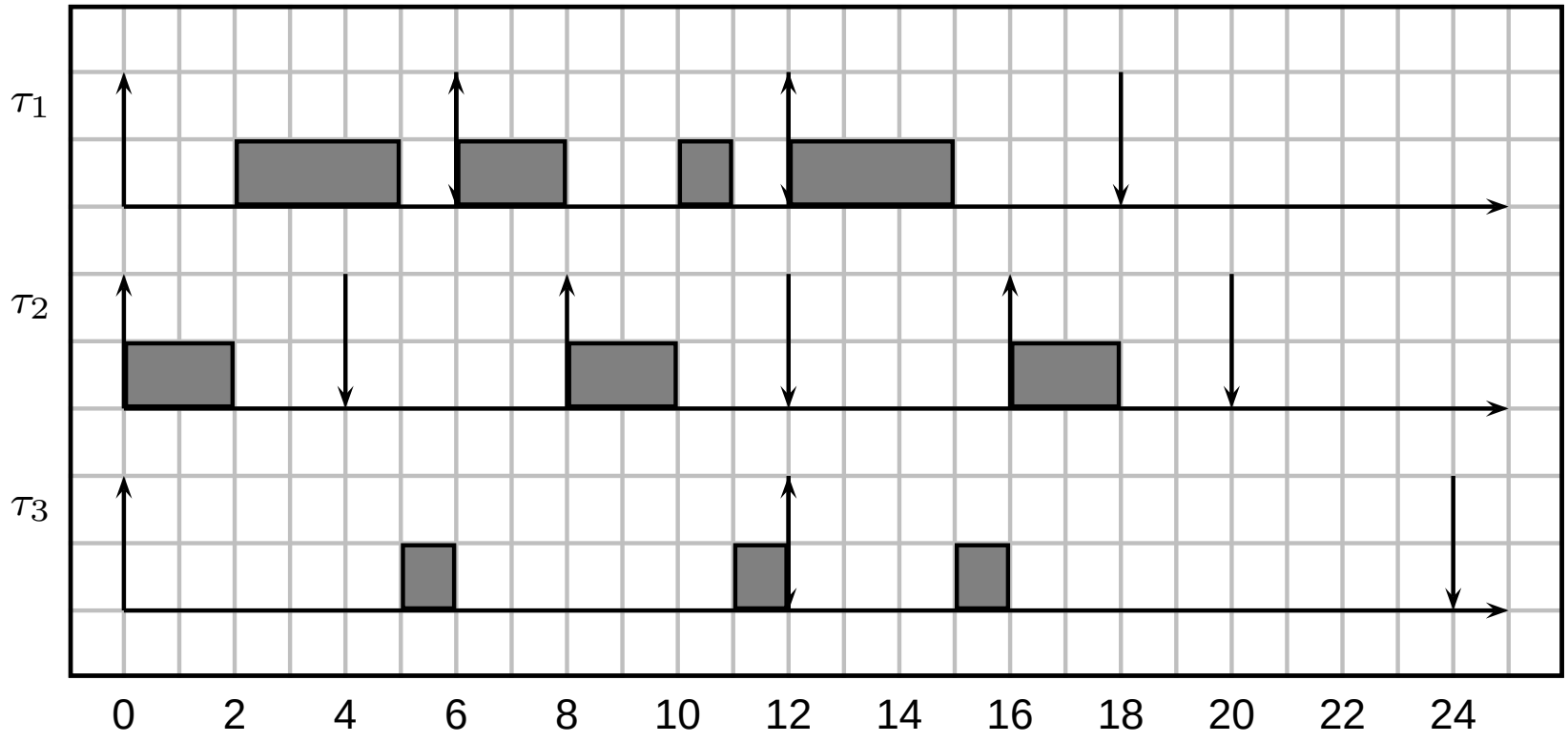

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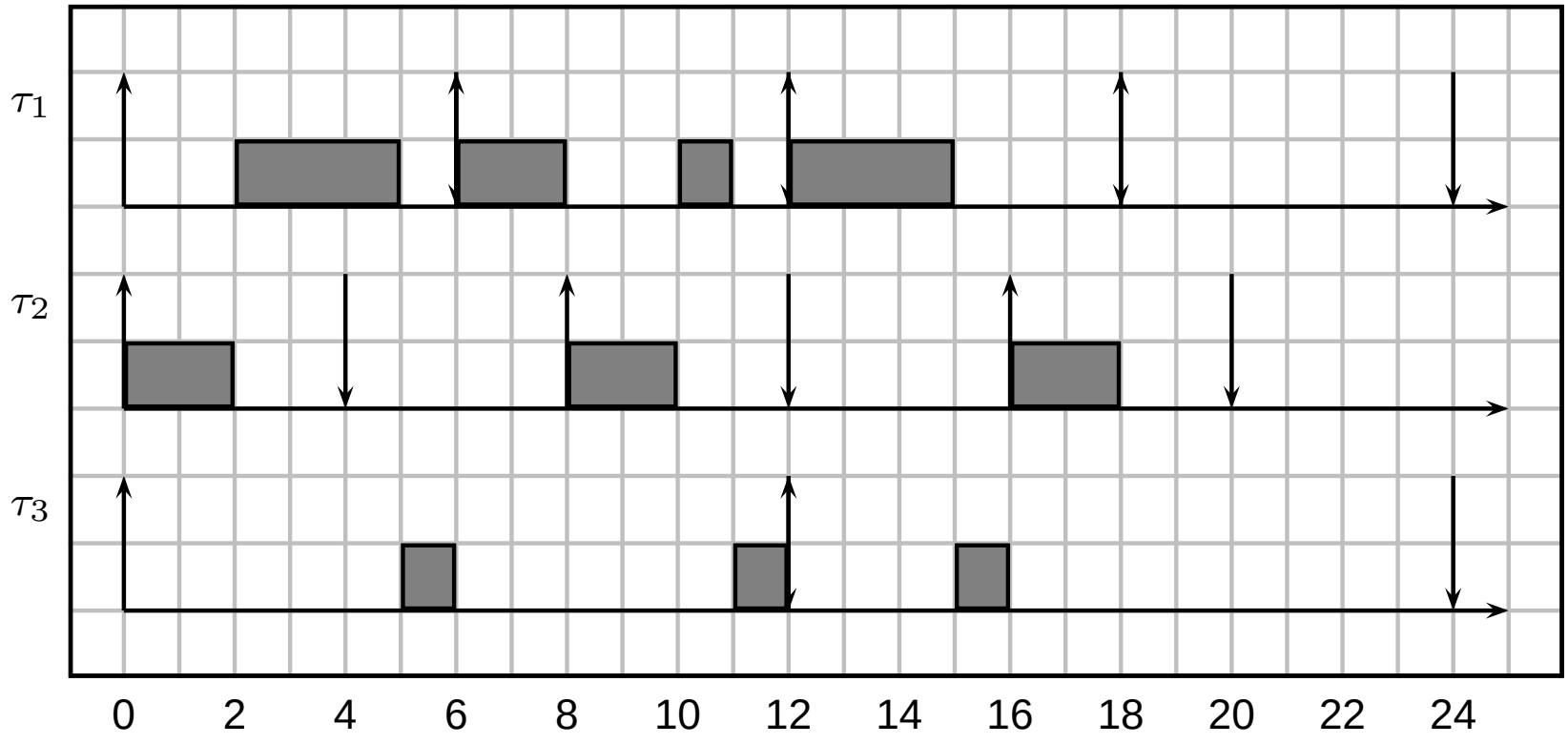

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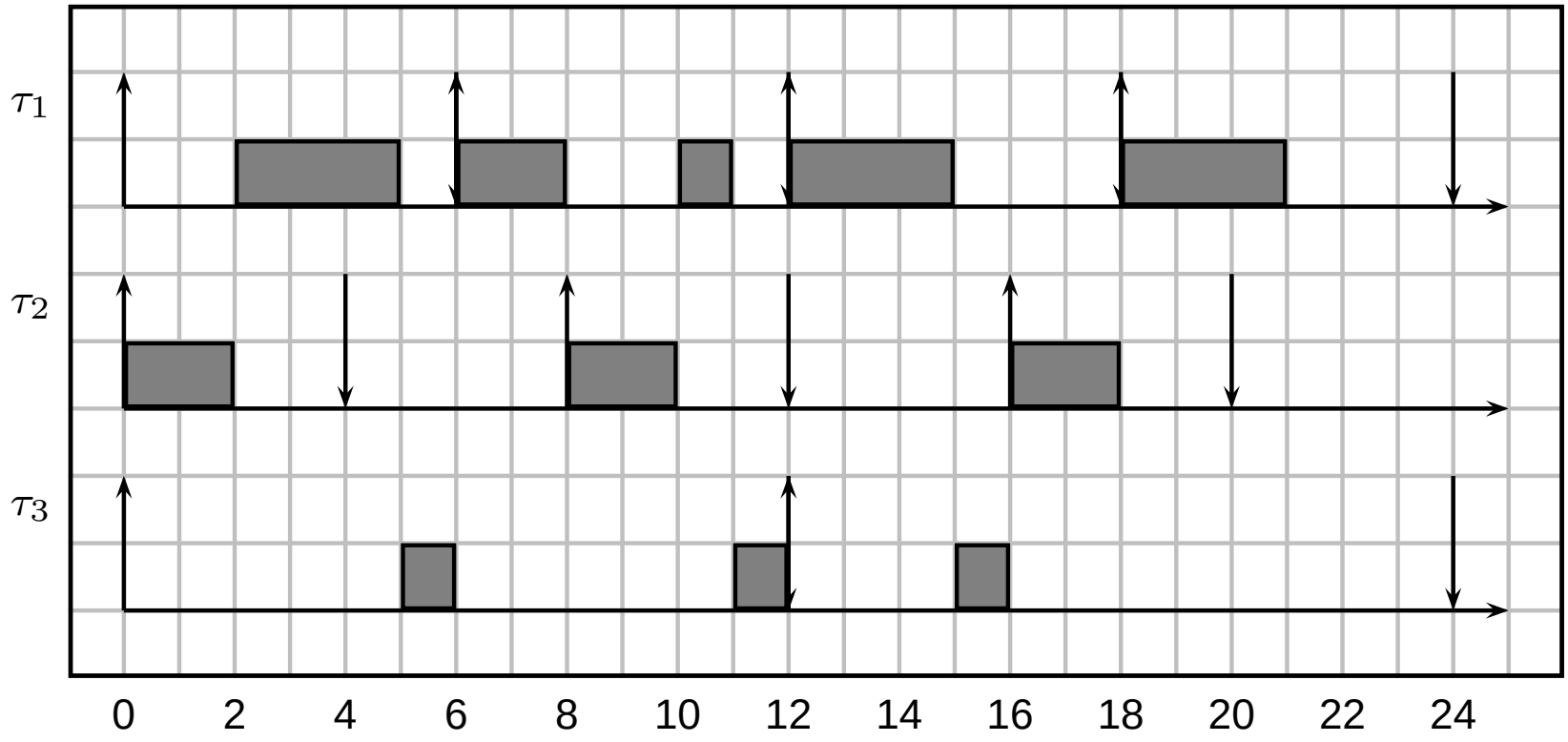

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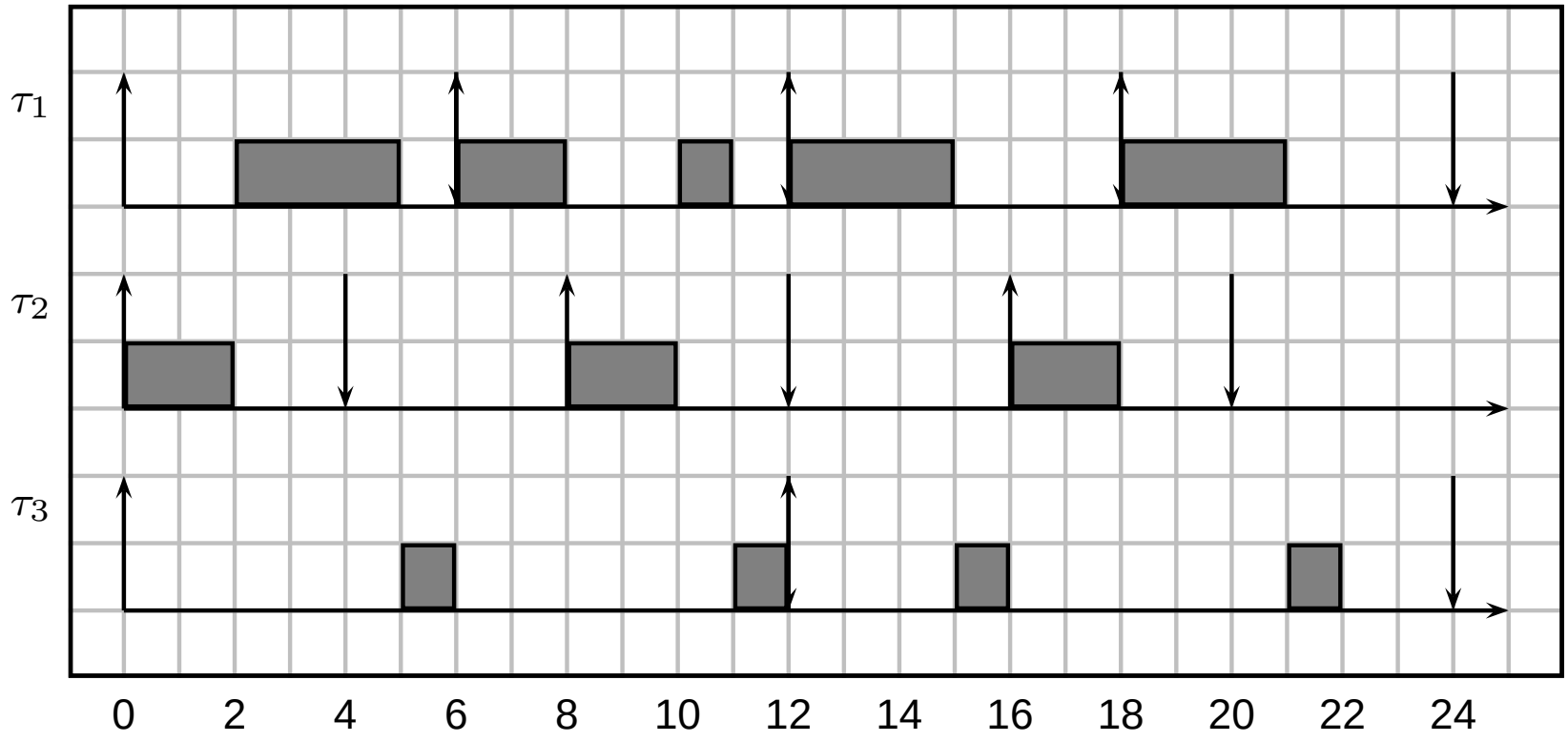


# Scheduling Analysis

# Analysis

- Given a task set, how can we guarantee if it is schedulable of not?
- The first possibility is to *simulate* the system to check that no deadline is missed;
- The execution time of every job is set equal to the WCET of the corresponding task;
  - Periodic tasks with no offsets $\Rightarrow$ sufficient to simulate the schedule until the *hyperperiod* ($H = lcm\{T_i\}$).
  - Offsets $\phi_i = r_{i,0} \Rightarrow$ simulate until $2H + \phi_{\max}$.
  - If tasks periods are prime numbers the hyperperiod can be very large!
- Note: RM $\rightarrow$ hyperperiod; Cyclic Executive $\rightarrow$ Major Cycle

# Example

- Exercise: Compare the hyperperiods of this two task sets:
  - $T_1 = 8, T_2 = 12, T_3 = 24$
  - $T_1 = 7, T_2 = 12, T_3 = 25$
- In case of sporadic tasks, we can assume them to arrive at the highest possible rate, so we fall back to the case of periodic tasks with no offsets!

# Utilisation Analysis

- In many cases it is useful to have a very simple test to see if the task set is schedulable.
- A sufficient test is based on the *Utilisation bound*:
  - The *utilisation least upper bound* for scheduling algorithm $\mathcal{A}$ is the smallest possible utilisation $U_{lub}$ such that, for any task set $\mathcal{T}$, if the task set's utilisation $U$ is not greater than $U_{lub}$ ($U \leq U_{lub}$), then the task set is schedulable by algorithm $\mathcal{A}$

# Utilisation

- Each task uses the processor for a fraction of time

$$U_i = \frac{C_i}{T_i}$$

- The total processor utilisation is

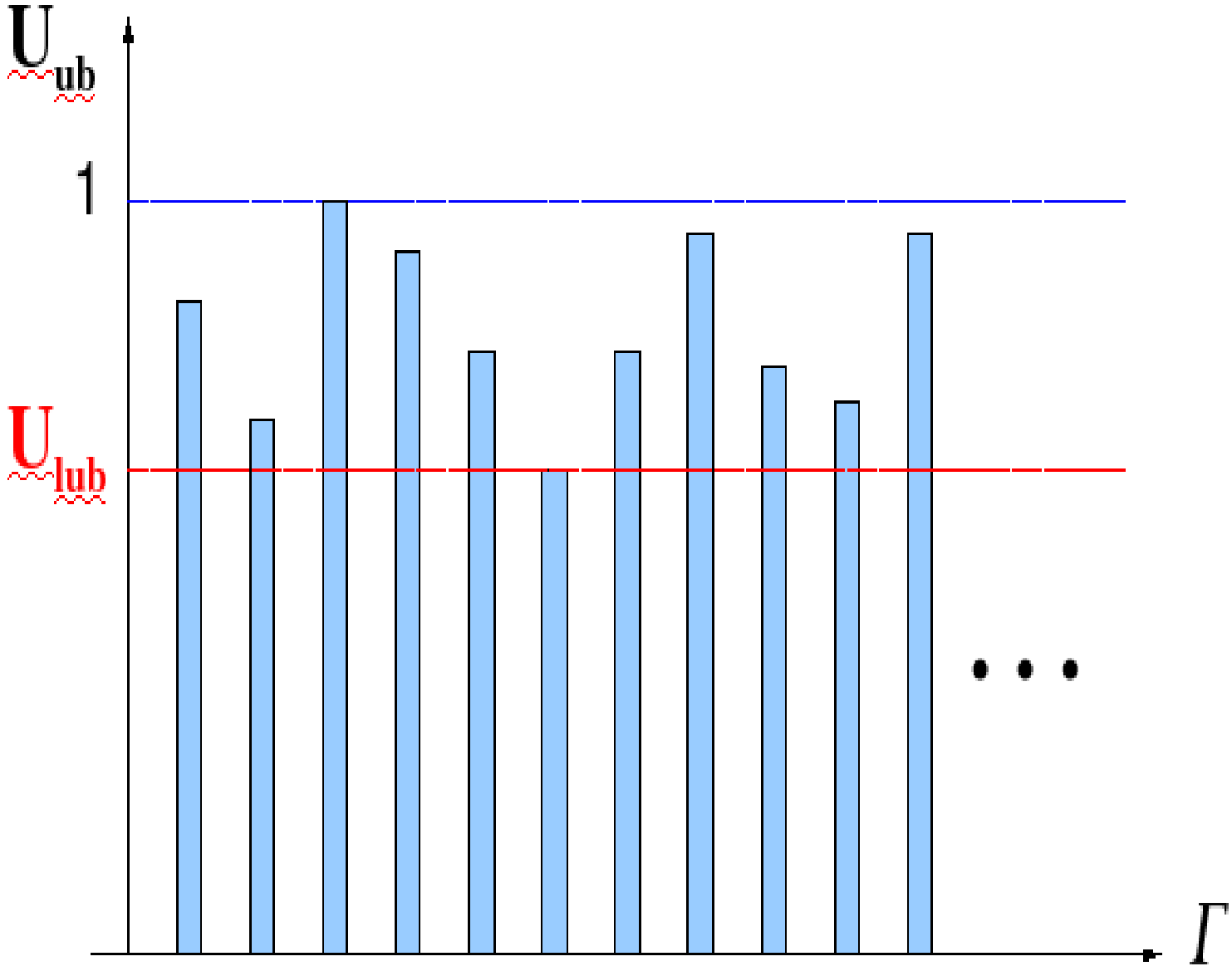
$$U = \sum_i \frac{C_i}{T_i}$$

- This is a measure of the processor's load

# Necessary Condition

- If $U > 1$ the task set is surely not schedulable
- However, if $U < 1$ the task set may or may not be schedulable ...
- If $U < U_{lub}$, the task set is schedulable!!!
  - "Gray Area" between $U_{lub}$ and $1$
  - We would like to have $U_{lub}$ near to $1$
  - $U_{lub} = 1$ would be optimal!!!

# Least Upper Bound



# Utilisation Bound for RM

- We consider $n$ periodic (or sporadic) tasks with relative deadline equal to periods.
- Priorities are assigned with Rate Monotonic;
- $U_{lub} = n(2^{1/n} - 1)$
  - $U_{lub}$ is a decreasing function of $n$;
  - For large $n$: $U_{lub} \approx 0.69$

| n | $U_{lub}$ | n | $U_{lub}$ |
|---|---|---|---|
| 2 | 0.828 | 7 | 0.728 |
| 3 | 0.779 | 8 | 0.724 |
| 4 | 0.756 | 9 | 0.720 |
| 5 | 0.743 | 10 | 0.717 |
| 6 | 0.734 | 11 | . . . |

# Schedulability Test

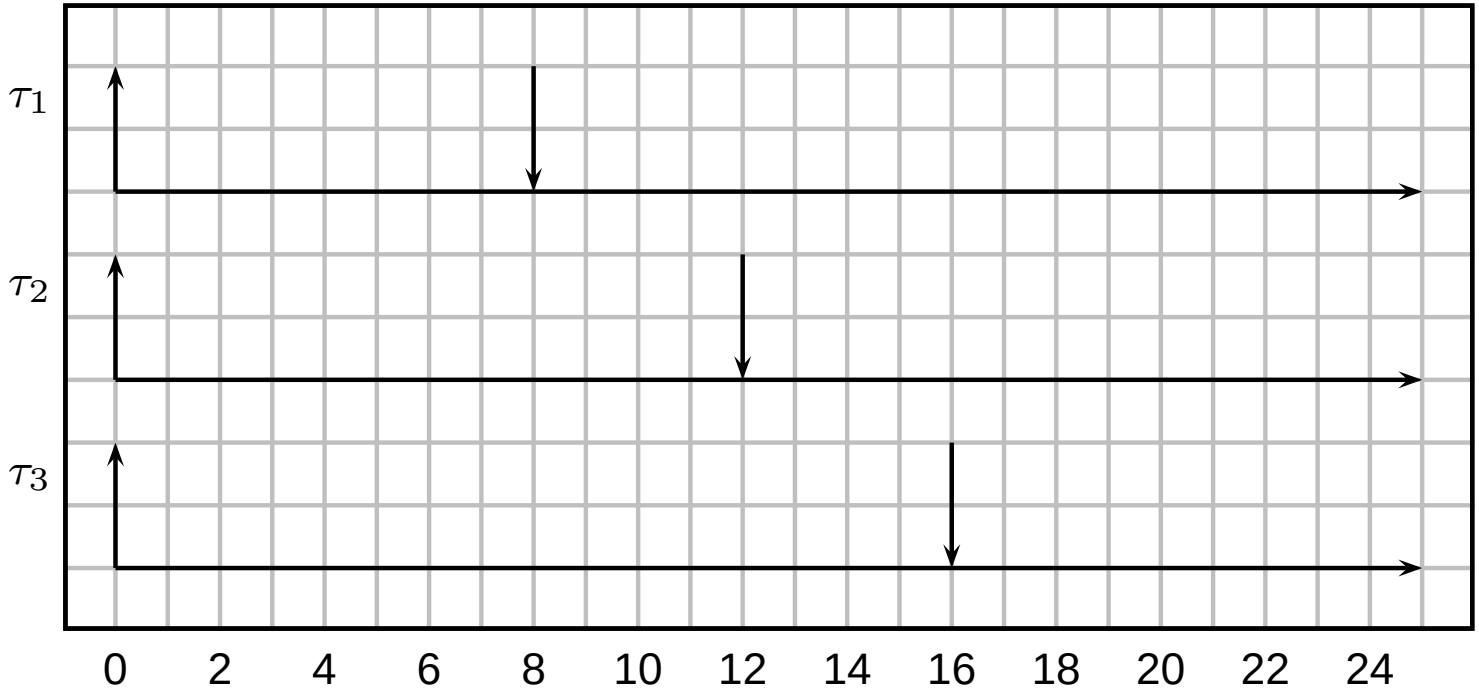
- Therefore the schedulability test consist in:
  - Computing $U = \sum_{i=1}^{n} \frac{C_i}{T_i}$
  - if $U \leq U_{lub}$, the task set is schedulable
  - if $U > 1$ the task set is not schedulable
  - if $U_{lub} < U \leq 1$, the task set may or may not be schedulable

# Example

Task set $\mathcal{T}$ composed by $3$ periodic tasks with $U < U_{lub}$: the system is schedulable.

$\tau_1 = (2, 8), \tau_2 = (3, 12), \tau_3 = (4, 16)$;

$$U = 0.75 < U_{lub} = 0.77$$



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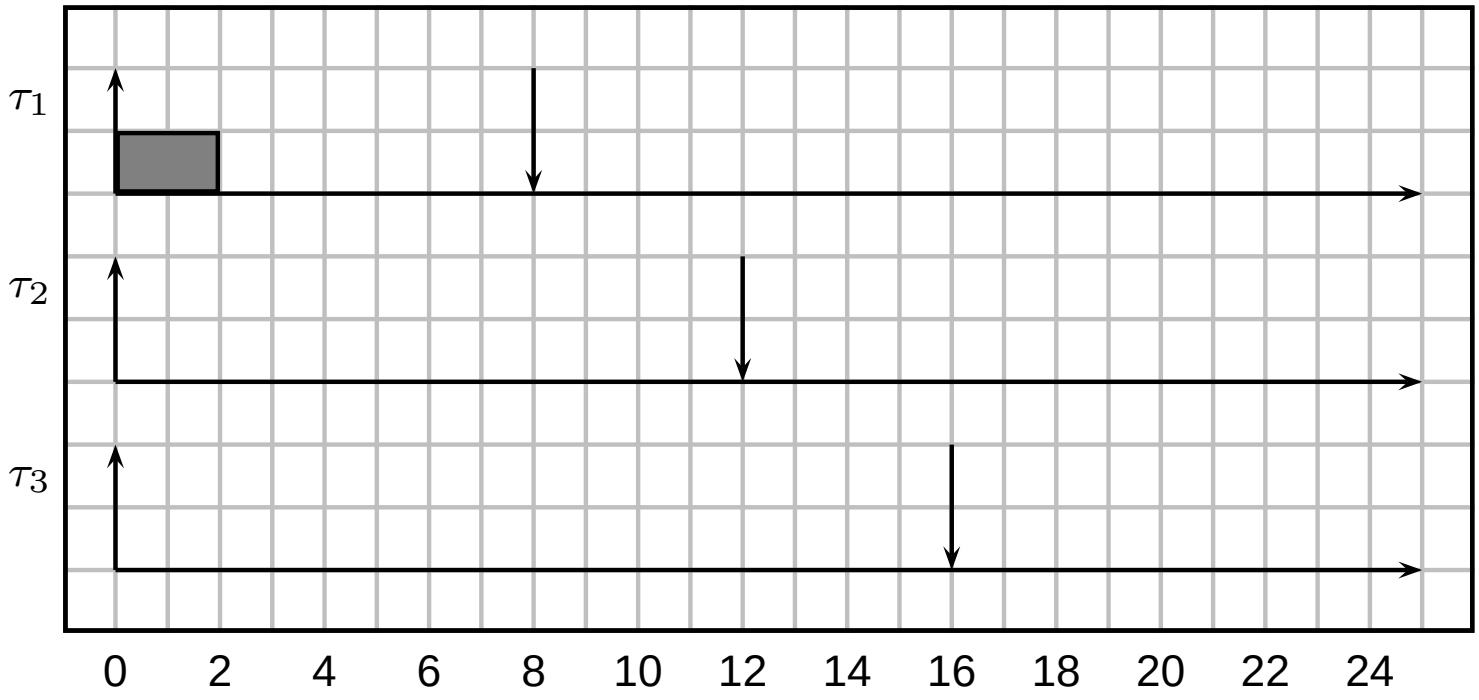

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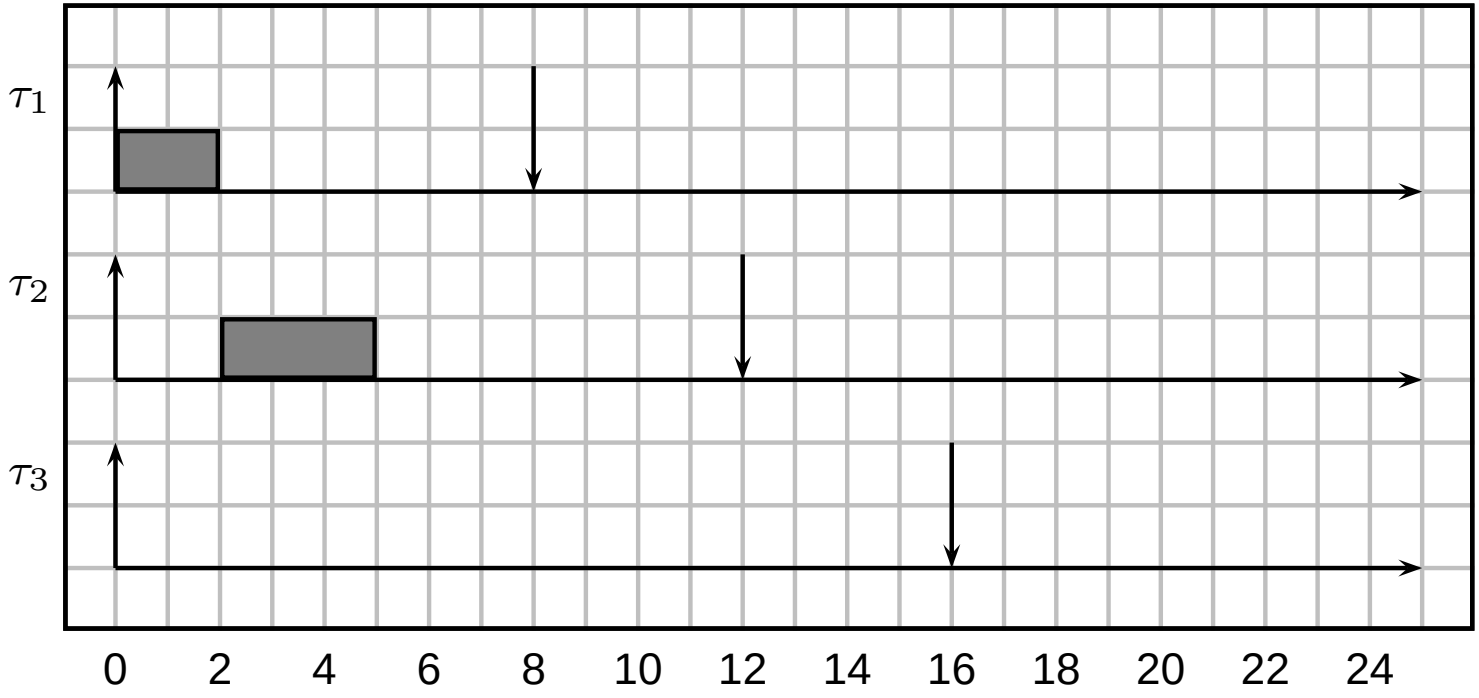

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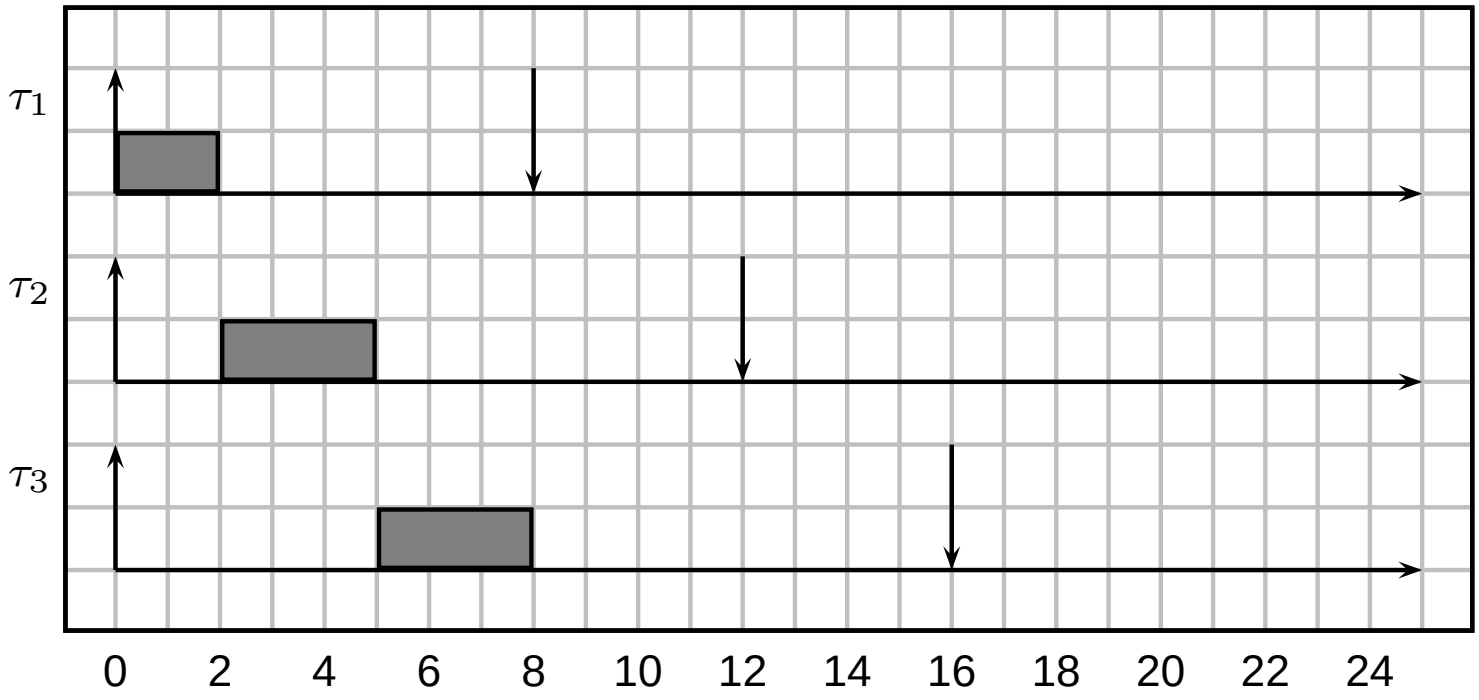

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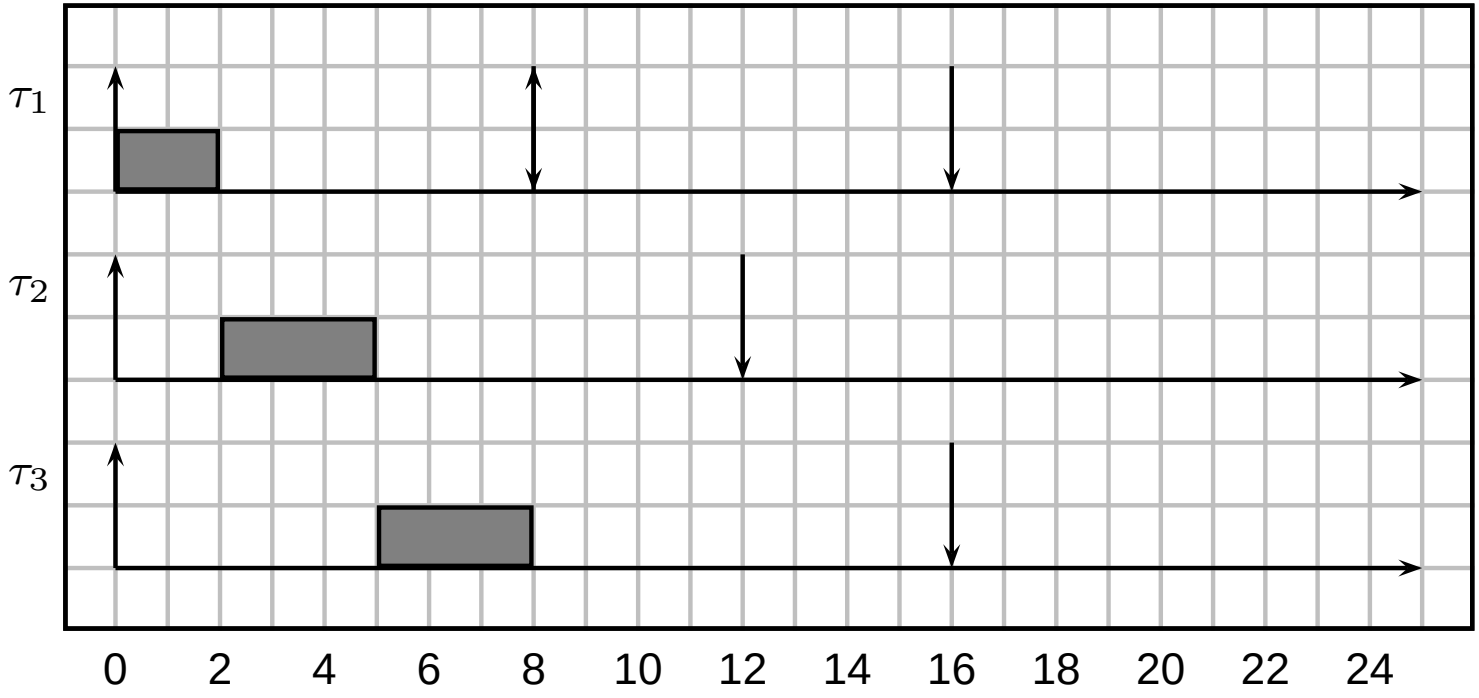

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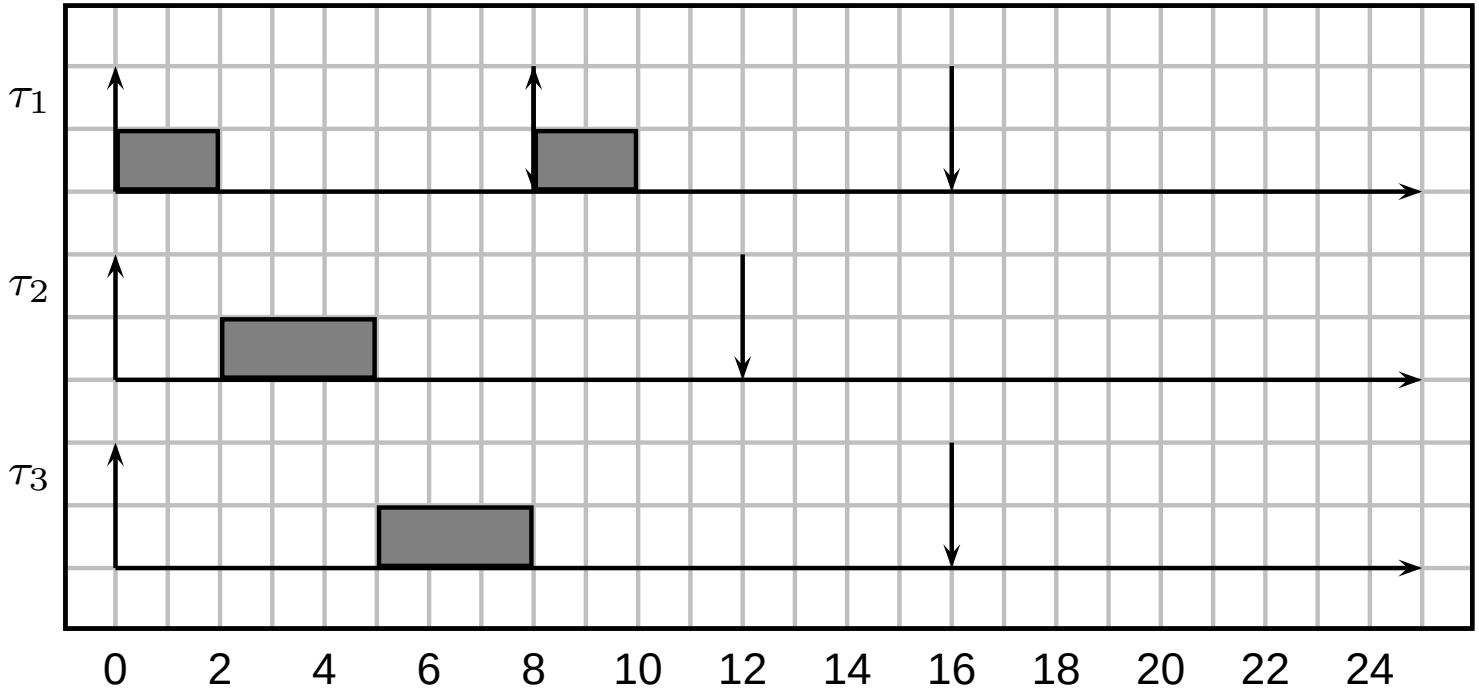

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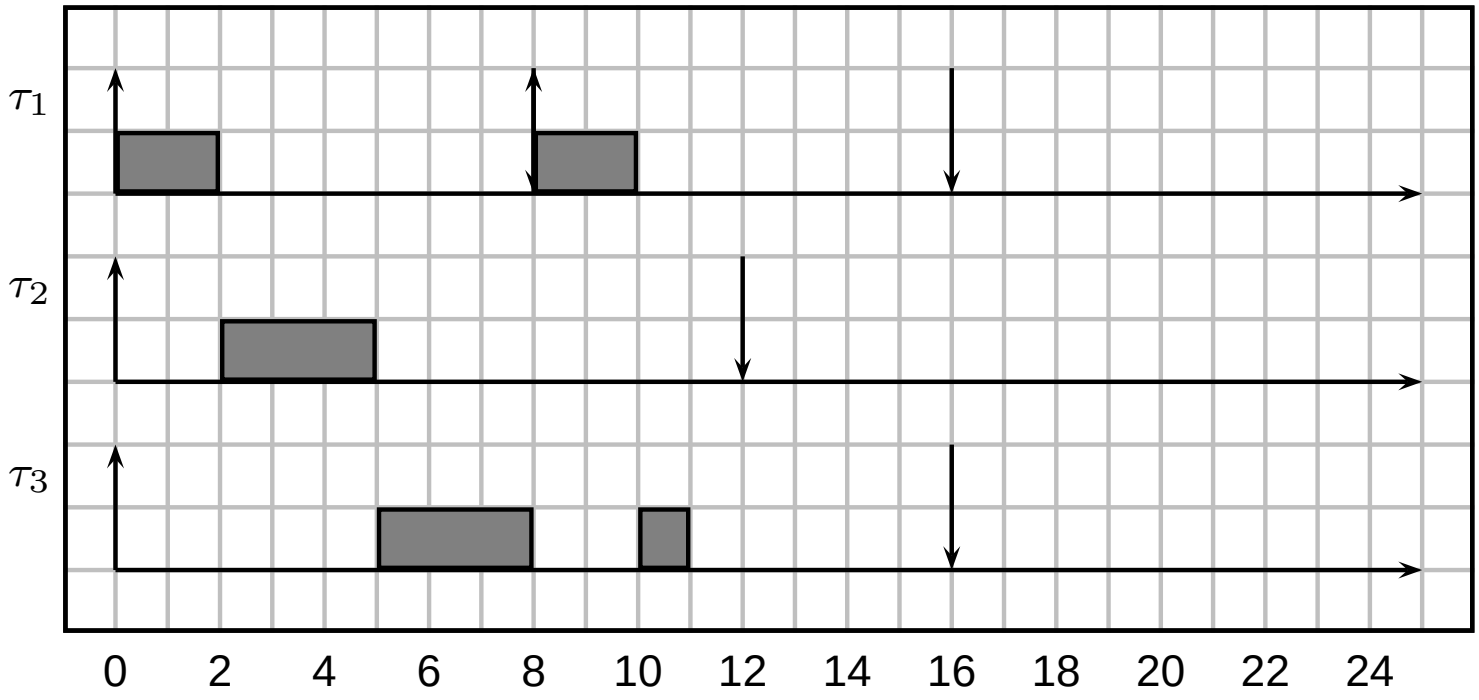

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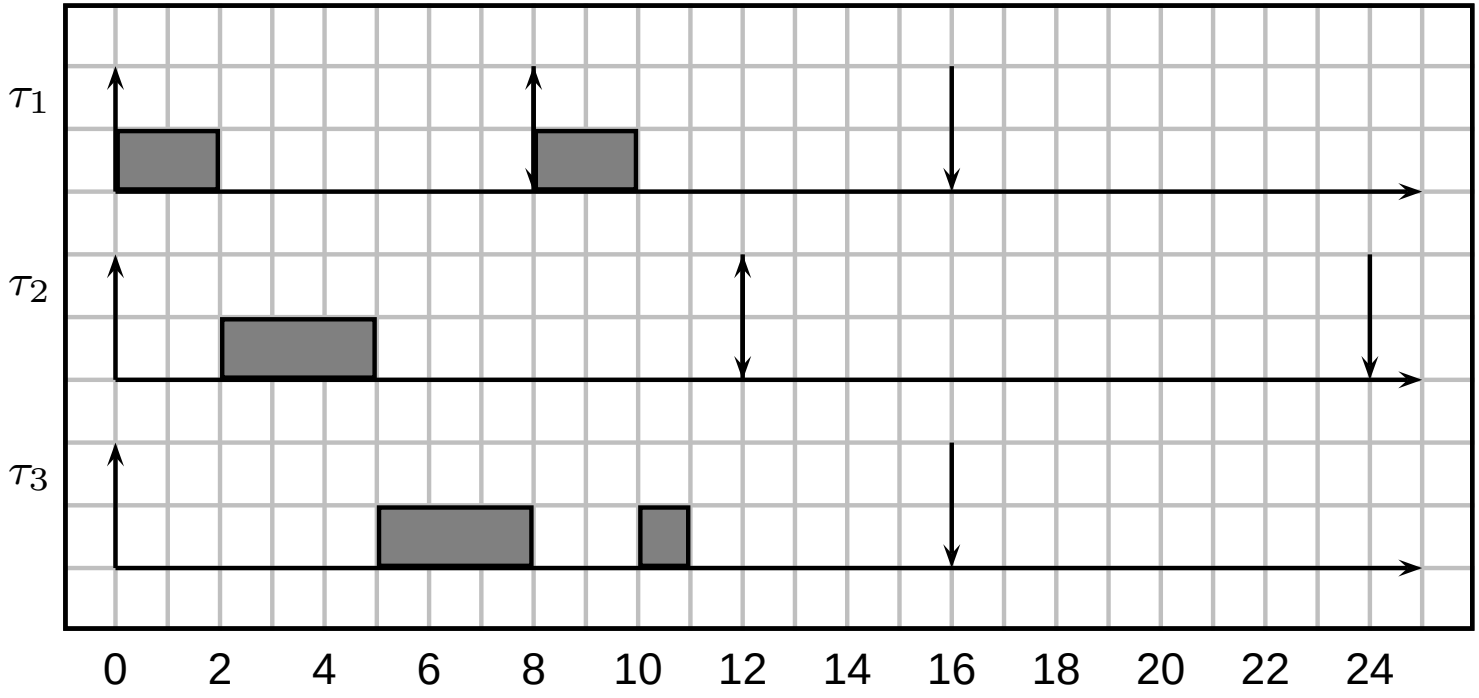

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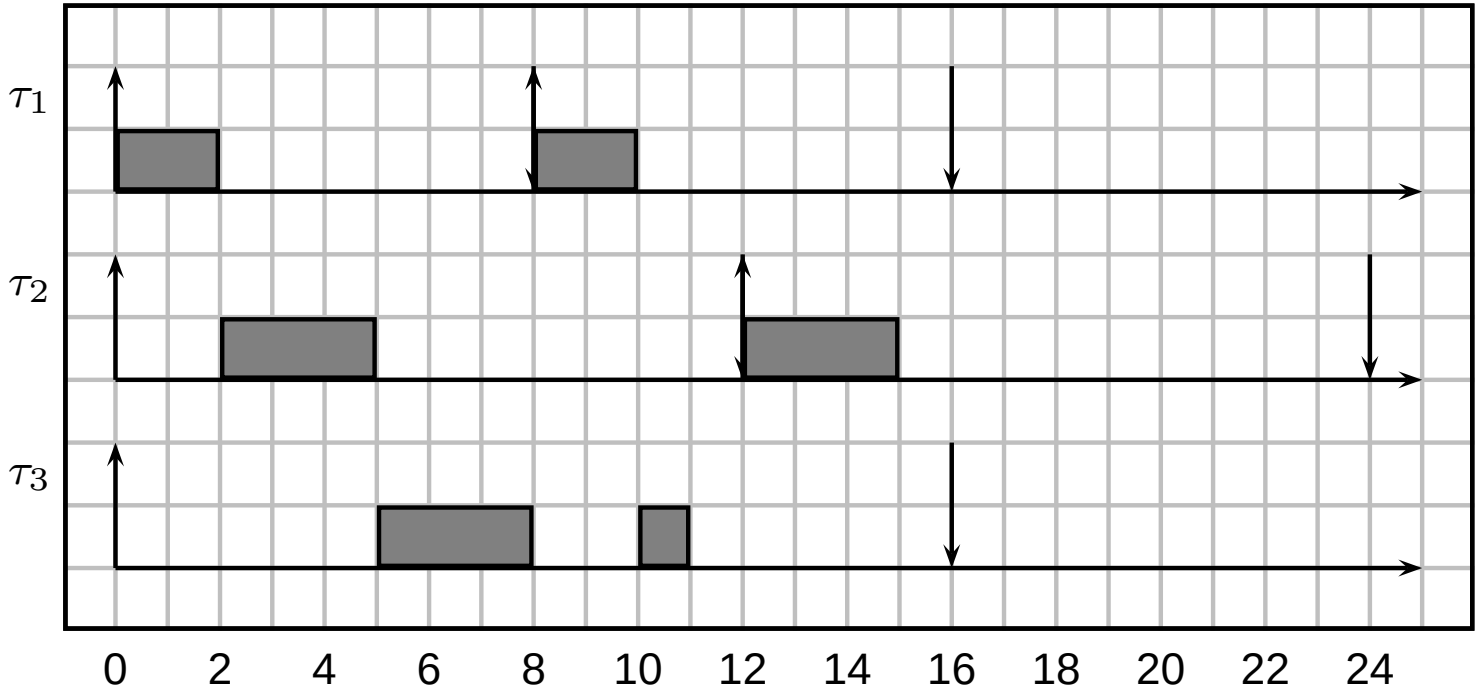

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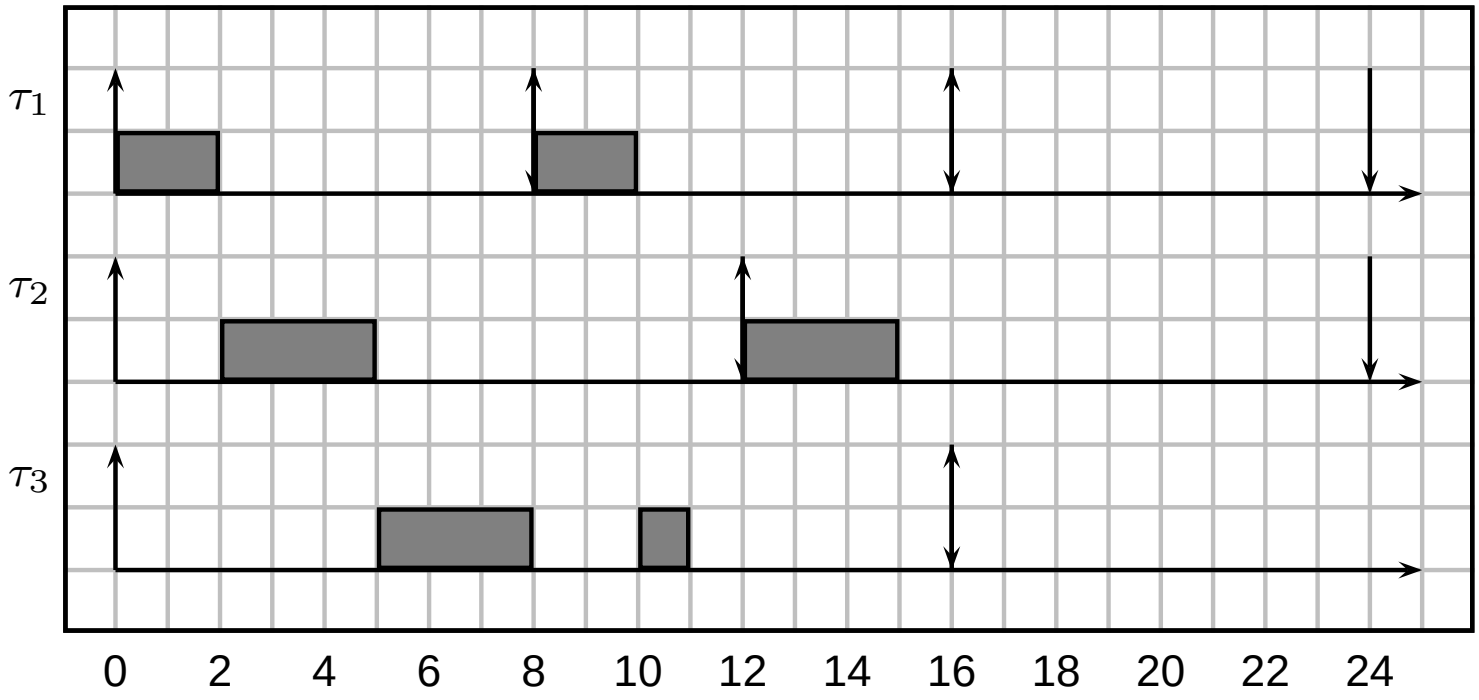

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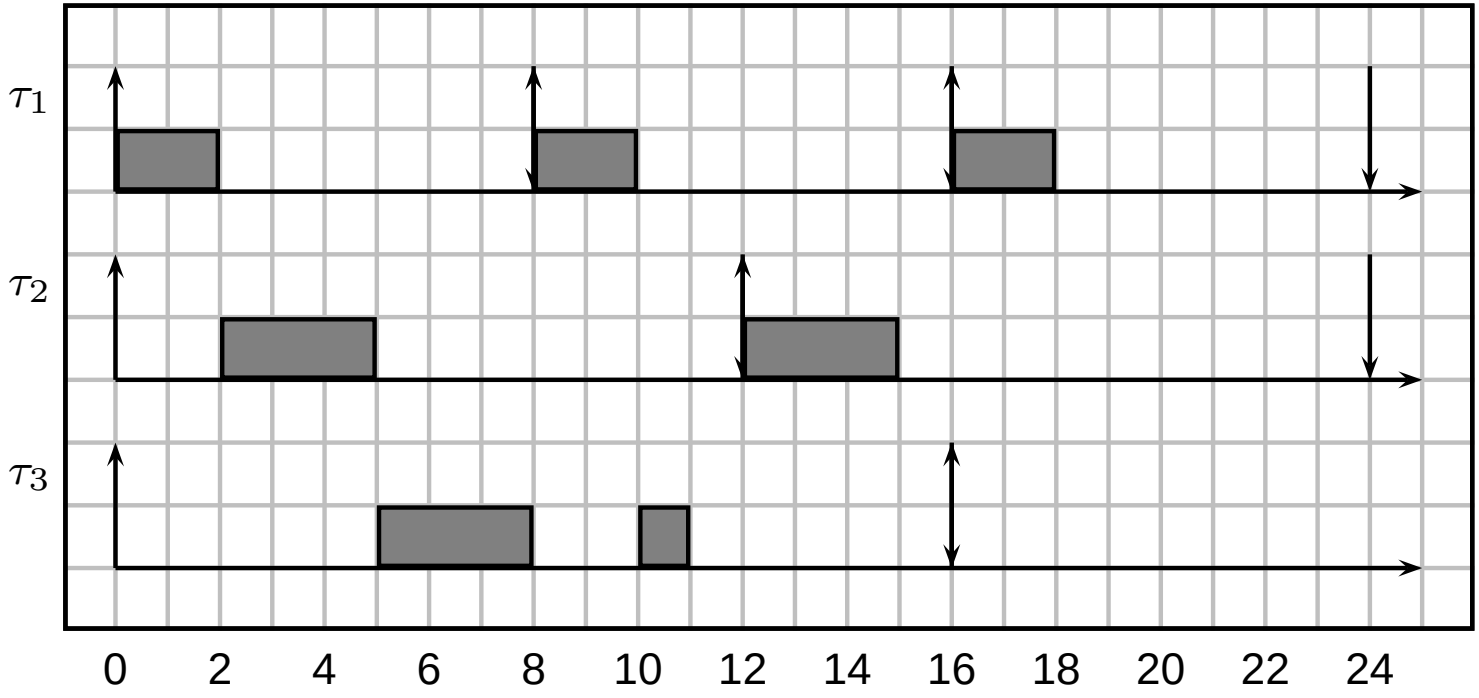

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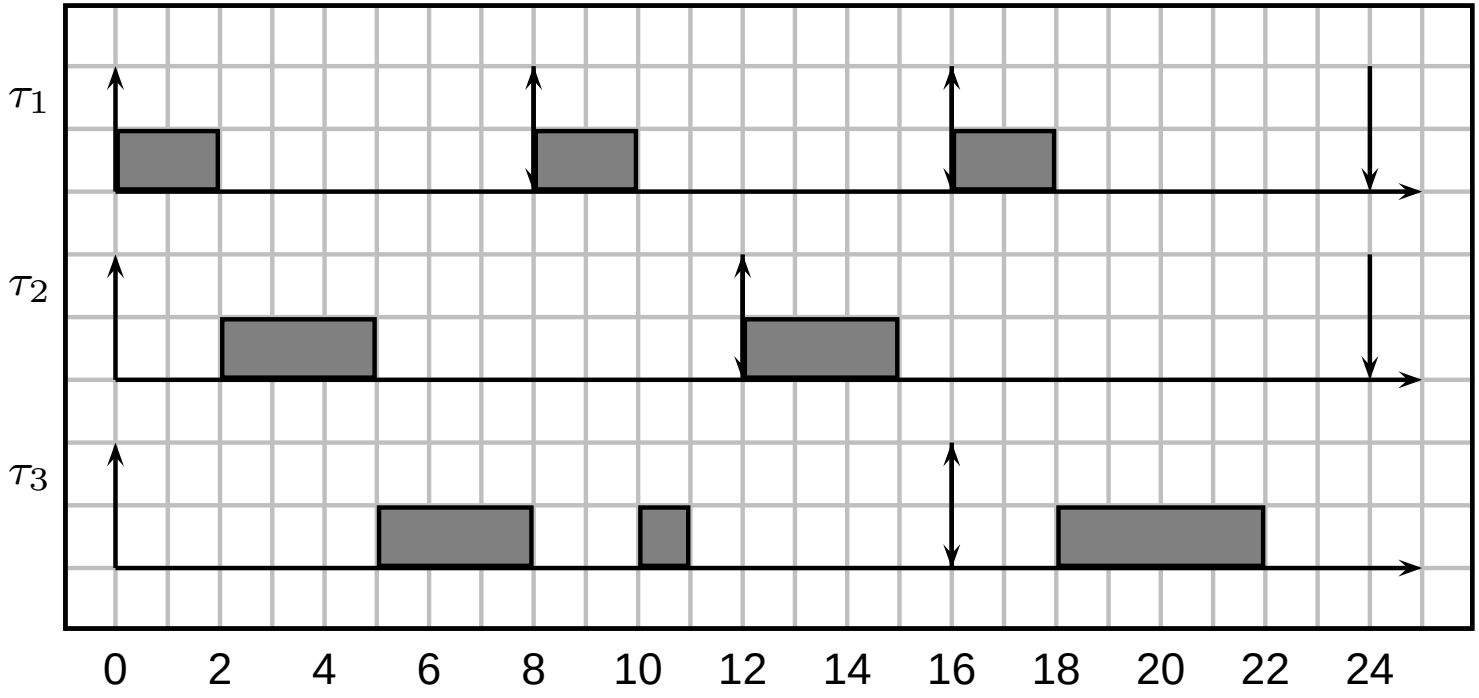
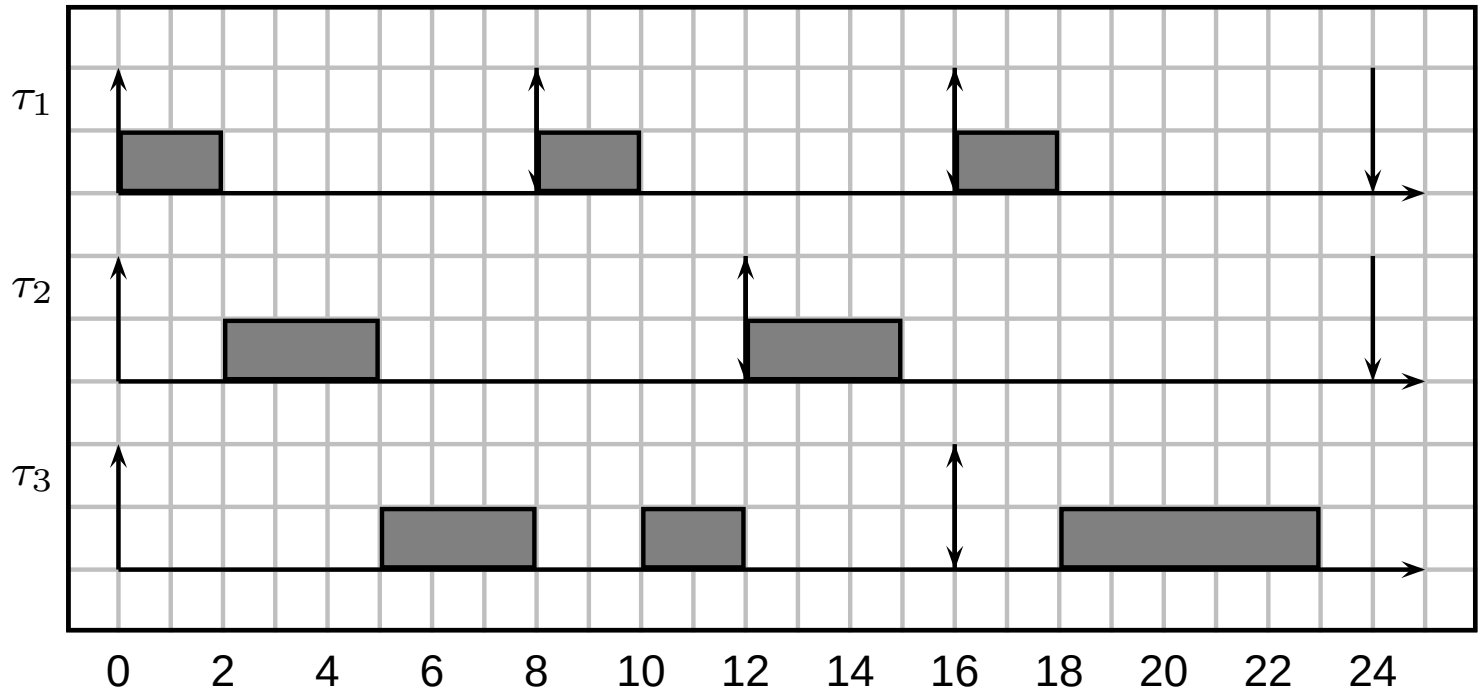


# Example 2

By increasing the computation time of task $\tau_3$, the system may still be schedulable

$\tau_1 = (2, 8), \tau_2 = (3, 12), \tau_3 = (5, 16)$;

$$U = 0.81 > U_{lub} = 0.77$$



# Utilisation Bound for DM

- If relative deadlines are less than or equal to periods, instead of considering $U = \sum_{i=1}^{n} \frac{C_i}{T_i}$, we can consider:

$$U' = \sum_{i=1}^{n} \frac{C_i}{D_i}$$

- Then the test is the same as the one for RM (or DM), except that we must use $U'$ instead of $U$.
- Idea: $\tau = (C, D, T) \rightarrow \tau' = (C, D, D)$
  - $\tau'$ is a "worst case" for $\tau$
  - If $\tau'$ can be guaranteed, $\tau$ can be guaranteed too

# Pessimism

- The bound is very pessimistic: most of the times, a task set with $U > U_{lub}$ is schedulable by RM.
- A particular case is when tasks have periods that are *harmonic*:
  - A task set is *harmonic* if, for every two tasks $\tau_i, \tau_j$, either $T_i$ is multiple of $T_j$ or $T_j$ is multiple of $T_i$.
- For a harmonic task set, the utilisation bound is $U_{lub} = 1$
- In other words, Rate Monotonic is an *optimal* algorithm for harmonic task sets

# Example of Harmonic Task Set

$\tau_1 = (3, 6)$, $\tau_2 = (3, 12)$, $\tau_3 = (6, 24)$;

$U = 1$;