Real Time Operating Systems and Middleware

Some Exercises about Task Schedulability

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Exercise

Given the following task set $T$:

<table>
<thead>
<tr>
<th>Task</th>
<th>$C_i$</th>
<th>$D_i$</th>
<th>$T_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>2</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>3</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>$\tau_4$</td>
<td>3</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

Compute the response time for all the tasks if priorities are assigned according to RM, or DM

**Solution:** If RM is used,

$$R(\tau_1) = 1 \quad R(\tau_2) = 3 \quad R(\tau_3) = 7 \quad R(\tau_4) = 18$$

If DM is used,

$$R(\tau_1) = 1 \quad R(\tau_2) = 7 \quad R(\tau_3) = 4 \quad R(\tau_4) = 18$$
Solution - I

\[ R_1 = 1 \]

\[ R_2^{(0)} = 2 \quad R_2^{(1)} = 2 + \left\lceil \frac{2}{4} \right\rceil \cdot 1 = 3 \]
\[ R_2^{(2)} = 2 + \left\lceil \frac{3}{4} \right\rceil \cdot 1 = 3 \]

\[ R_3^{(0)} = 3 \quad R_3^{(1)} = 3 + \left\lceil \frac{3}{4} \right\rceil \cdot 1 + \left\lceil \frac{3}{9} \right\rceil \cdot 2 = 6 \]
\[ R_3^{(2)} = 3 + \left\lceil \frac{6}{4} \right\rceil \cdot 1 + \left\lceil \frac{6}{9} \right\rceil \cdot 2 = 7 \quad R_3^{(3)} = 3 + \left\lceil \frac{7}{4} \right\rceil \cdot 1 + \left\lceil \frac{7}{9} \right\rceil \cdot 2 = 7 \]

\[ R_4^{(0)} = 3 \]
\[ R_4^{(1)} = 3 + \left\lceil \frac{3}{4} \right\rceil \cdot 1 + \left\lceil \frac{3}{9} \right\rceil \cdot 2 + \left\lceil \frac{3}{12} \right\rceil \cdot 3 = 9 \]
\[ R_4^{(2)} = 3 + \left\lceil \frac{9}{4} \right\rceil \cdot 1 + \left\lceil \frac{9}{9} \right\rceil \cdot 2 + \left\lceil \frac{9}{12} \right\rceil \cdot 3 = 10 \]
\[ R_4^{(3)} = 3 + \left\lceil \frac{10}{4} \right\rceil \cdot 1 + \left\lceil \frac{10}{9} \right\rceil \cdot 2 + \left\lceil \frac{10}{12} \right\rceil \cdot 3 = 13 \]
Solution - II

\[ R_4^{(4)} = 3 + \left\lfloor \frac{13}{4} \right\rfloor \cdot 1 + \left\lfloor \frac{13}{9} \right\rfloor \cdot 2 + \left\lfloor \frac{13}{12} \right\rfloor \cdot 3 = 17 \]
\[ R_4^{(5)} = 3 + \left\lfloor \frac{17}{4} \right\rfloor \cdot 1 + \left\lfloor \frac{17}{9} \right\rfloor \cdot 2 + \left\lfloor \frac{17}{12} \right\rfloor \cdot 3 = 18 \]
\[ R_4^{(6)} = 3 + \left\lfloor \frac{18}{4} \right\rfloor \cdot 1 + \left\lfloor \frac{18}{9} \right\rfloor \cdot 2 + \left\lfloor \frac{18}{12} \right\rfloor \cdot 3 = 18 \]

What happens if DM is used?
Left as a simple exercise for the reader...
Wait next week for the solution!!!
Exercise

Given the non periodic task $\tau_1$ defined as follows:

- If $j \% 2 == 0$, then $r_{1,j} = 8 \cdot \frac{j}{2}$;
- If $j \% 2 == 1$, then $r_{1,j} = 3 + 8 \cdot \left\lfloor \frac{j}{2} \right\rfloor$;
- $\forall j, c_{1,j} = 2$;
- The priority of task $\tau_1$ is $p_1 = 3$.

The task set $\mathcal{T}$ is composed by $\tau_1$, $\tau_2 = (2, 12, 12)$, and $\tau_3 = (3, 25, 25)$, with $p_2 = 2$ and $p_3 = 1$. Compute the worst case response times for $\tau_2$ and $\tau_3$. 

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The arrival pattern for task $\tau_1$ is:

Since $\tau_1$ is the highest priority task in the system, its response time is equal to $C_1 = 2$ for all its jobs. Now the problem is computing the “interference” from $\tau_1$ to $\tau_2$ and $\tau_3$. 
The equation used for computing the response time must be extended a little bit...

\[ R_i^{(k)} = C_i + \sum_{h=1}^{i-1} N_{\text{inst}}(R_i^{(k-1)})C_h \]

where \( N_{\text{inst}}(t) \) is the number of instances of task \( \tau_h \) arrived in \([0, t]\)

For periodic tasks, we already know that \( N_{\text{inst}}(t) = \left\lceil \frac{t}{T_h} \right\rceil \)

For \( \tau_1 \), we have

\[ N_{\text{inst}}_1(t) = \left\lfloor \frac{t}{8} \right\rfloor + \left\lfloor \frac{\max(0, t - 3)}{8} \right\rfloor \]

The first term is for \( j \% 2 = 0 \); the second term is for \( j \% 2 = 1 \)
Applying the equation presented above, we have: $\tau_2$:

\[
\begin{align*}
R_2^{(0)} &= 2 + 2 = 4 \\
R_2^{(1)} &= 2 + 2 \cdot 2 = 6 \\
R_2^{(2)} &= 2 + 2 \cdot 2 = 6
\end{align*}
\]

$\tau_3$:

\[
\begin{align*}
R_3^{(0)} &= 3 + 2 + 2 = 7 \\
R_3^{(1)} &= 3 + 2 \cdot 2 + 1 \cdot 2 = 9 \\
R_3^{(2)} &= 3 + 3 \cdot 2 + 1 \cdot 2 = 11 \\
R_3^{(3)} &= 3 + 3 \cdot 2 + 1 \cdot 2 = 11
\end{align*}
\]
A periodic task $\tau'_1 = (2, 3)$ consumes more time than $\tau_1 \Rightarrow$ the response times for $\tau_2$ and $\tau_3$ computed using $\tau'_1$ are pessimistic bounds...

$$R^{(0)}_2 = 2 + 2 = 4 \quad R^{(1)}_2 = 2 + \left\lceil \frac{4}{2} \right\rceil \cdot 2 = 6$$
$$R^{(2)}_2 = 2 + \left\lceil \frac{6}{2} \right\rceil \cdot 2 = 6$$

$$R^{(0)}_3 = 3 + 2 + 2 = 7 \quad R^{(1)}_3 = 3 + \left\lceil \frac{7}{3} \right\rceil \cdot 2 + \left\lceil \frac{7}{12} \right\rceil \cdot 2 = 11$$
$$R^{(2)}_3 = 3 + \left\lceil \frac{11}{3} \right\rceil \cdot 2 + \left\lceil \frac{11}{12} \right\rceil \cdot 2 = 13 \quad R^{(3)}_3 = 3 + \left\lceil \frac{13}{3} \right\rceil \cdot 2 + \left\lceil \frac{13}{12} \right\rceil \cdot 2 = 17$$
$$R^{(4)}_3 = 3 + \left\lceil \frac{17}{3} \right\rceil \cdot 2 + \left\lceil \frac{17}{12} \right\rceil \cdot 2 = 19 \quad R^{(3)}_3 = 3 + \left\lceil \frac{19}{3} \right\rceil \cdot 2 + \left\lceil \frac{19}{12} \right\rceil \cdot 2 = 21$$
$$R^{(3)}_3 = 3 + \left\lceil \frac{21}{3} \right\rceil \cdot 2 + \left\lceil \frac{21}{12} \right\rceil \cdot 2 = 21$$
By the way, this is the resulting schedule:
This would have been the schedule of the resulting task set:

\[
\tau_1, \tau_2, \tau_3
\]