Real Time Operating Systems and Middleware

Some Exercizes about Task Schedulability

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Exercise

Given the following task set $\mathcal{T}$:

Task	C_i	D_i	T_i
τ_1	1	4	4
τ_2	2	9	9
τ_3	3	6	12
τ_4	3	20	20

Compute the response time for all the tasks if priorities are assigned according to RM, or DM

Solution: If RM is used,

$$R(\tau_1) = 1 \quad R(\tau_2) = 3 \quad R(\tau_3) = 7 \quad R(\tau_4) = 18$$

If DM is used,

$$R(\tau_1) = 1 \quad R(\tau_2) = 7 \quad R(\tau_3) = 4 \quad R(\tau_4) = 18$$

Solution - I

$$R_1 = 1$$

$$R_2^{(0)} = 2 \qquad R_2^{(1)} = 2 + \left\lceil \frac{2}{4} \right\rceil \cdot 1 = 3$$

$$R_2^{(2)} = 2 + \left\lceil \frac{3}{4} \right\rceil \cdot 1 = 3$$

$$R_3^{(0)} = 3 \qquad R_3^{(1)} = 3 + \left\lceil \frac{3}{4} \right\rceil \cdot 1 + \left\lceil \frac{3}{9} \right\rceil \cdot 2 = 6$$

$$R_3^{(2)} = 3 + \left\lceil \frac{6}{4} \right\rceil \cdot 1 + \left\lceil \frac{6}{9} \right\rceil \cdot 2 = 7 \qquad R_3^{(3)} = 3 + \left\lceil \frac{7}{4} \right\rceil \cdot 1 + \left\lceil \frac{7}{9} \right\rceil \cdot 2 = 7$$

$$R_4^{(0)} = 3$$

$$R_4^{(1)} = 3 + \left\lceil \frac{3}{4} \right\rceil \cdot 1 + \left\lceil \frac{3}{9} \right\rceil \cdot 2 + \left\lceil \frac{3}{12} \right\rceil \cdot 3 = 9$$

$$R_4^{(2)} = 3 + \left\lceil \frac{9}{4} \right\rceil \cdot 1 + \left\lceil \frac{9}{9} \right\rceil \cdot 2 + \left\lceil \frac{9}{12} \right\rceil \cdot 3 = 10$$

$$R_4^{(3)} = 3 + \left\lceil \frac{10}{4} \right\rceil \cdot 1 + \left\lceil \frac{10}{9} \right\rceil \cdot 2 + \left\lceil \frac{10}{12} \right\rceil \cdot 3 = 13$$

Solution - II

$$R_4^{(4)} = 3 + \left\lceil \frac{13}{4} \right\rceil \cdot 1 + \left\lceil \frac{13}{9} \right\rceil \cdot 2 + \left\lceil \frac{13}{12} \right\rceil \cdot 3 = 17$$
$$R_4^{(5)} = 3 + \left\lceil \frac{17}{4} \right\rceil \cdot 1 + \left\lceil \frac{17}{9} \right\rceil \cdot 2 + \left\lceil \frac{17}{12} \right\rceil \cdot 3 = 18$$
$$R_4^{(6)} = 3 + \left\lceil \frac{18}{4} \right\rceil \cdot 1 + \left\lceil \frac{18}{9} \right\rceil \cdot 2 + \left\lceil \frac{18}{12} \right\rceil \cdot 3 = 18$$

What happens if DM is used?
Left as a simple exercise for the reader...
Wait next week for the solution!!!

Exercise

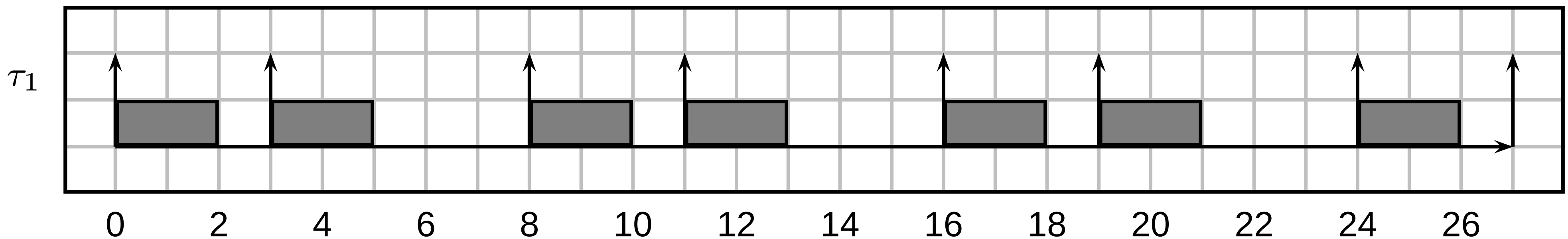
Given the non periodic task τ_1 defined as follows:

- If $j\%2 == 0$, then $r_{1,j} = 8 \cdot \frac{j}{2}$;
- If $j\%2 == 1$, then $r_{1,j} = 3 + 8 \cdot \left\lfloor \frac{j}{2} \right\rfloor$;
- $\forall j, c_{1,j} = 2$;
- The priority of task τ_1 is $p_1 = 3$.

The task set $\mathcal{T}$ is composed by τ_1, $\tau_2 = (2, 12, 12)$, and $\tau_3 = (3, 25, 25)$, with $p_2 = 2$ and $p_3 = 1$. Compute the worst case response times for τ_2 and τ_3.

Solution - I

The arrival pattern for task τ_1 is:



Since τ_1 is the highest priority task in the system, its response time is equal to $C_1 = 2$ for all its jobs.
Now the problem is computing the “interference” from τ_1 to τ_2 and τ_3.

Solution - II

The equation used for computing the response time must be extended a little bit...

$$R_i^{(k)} = C_i + \sum_{h=1}^{i-1} Ninst_h(R_i^{(k-1)})C_h$$

where $Ninst_h(t)$ is the number of instances of task τ_h arrived in $[0, t]$

For periodic tasks, we already know that $Ninst_h(t) = \left\lceil \frac{t}{T_h} \right\rceil$

For τ_1, we have

$$Ninst_1(t) = \left\lceil \frac{t}{8} \right\rceil + \left\lceil \frac{\max(0, t-3)}{8} \right\rceil$$

The first term is for $j\%2 == 0$; the second term is for $j\%2 == 1$

Solution - III

Applying the equation presented above, we have: τ_2:

$$R_2^{(0)} = 2 + 2 = 4 \qquad R_2^{(1)} = 2 + 2 \cdot 2 = 6$$
$$R_2^{(2)} = 2 + 2 \cdot 2 = 6$$

τ_3:

$$R_3^{(0)} = 3 + 2 + 2 = 7 \qquad R_3^{(1)} = 3 + 2 \cdot 2 + 1 \cdot 2 = 9$$
$$R_3^{(2)} = 3 + 3 \cdot 2 + 1 \cdot 2 = 11 \qquad R_3^{(3)} = 3 + 3 \cdot 2 + 1 \cdot 2 = 11$$

Alternative Solution

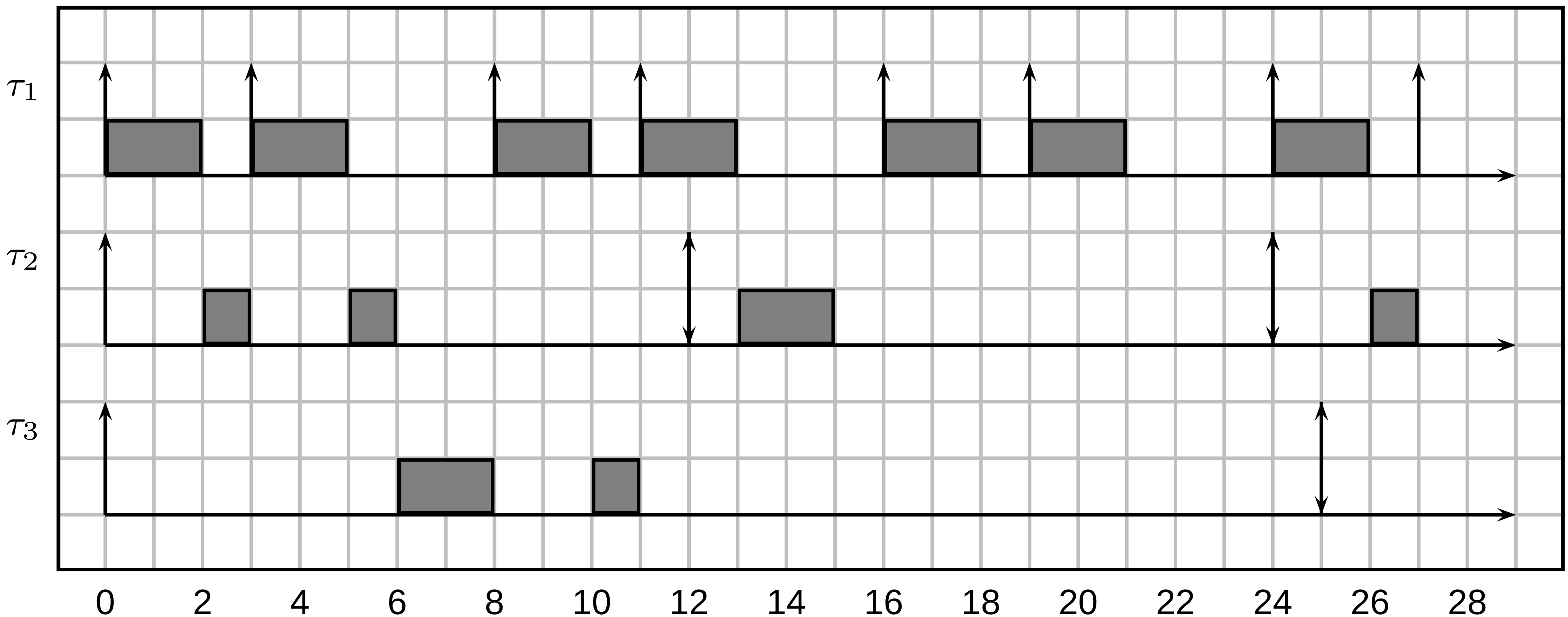
A periodic task $\tau_1' = (2, 3)$ consumes more time than $\tau_1 \Rightarrow$ the response times for τ_2 and τ_3 computed using τ_1' are pessimistic bounds...

$$R_2^{(0)} = 2 + 2 = 4 \qquad R_2^{(1)} = 2 + \left\lceil \frac{4}{2} \right\rceil \cdot 2 = 6$$
$$R_2^{(2)} = 2 + \left\lceil \frac{6}{2} \right\rceil \cdot 2 = 6$$

$$R_3^{(0)} = 3 + 2 + 2 = 7 \qquad R_3^{(1)} = 3 + \left\lceil \frac{7}{3} \right\rceil \cdot 2 + \left\lceil \frac{7}{12} \right\rceil \cdot 2 =$$
$$R_3^{(2)} = 3 + \left\lceil \frac{11}{3} \right\rceil \cdot 2 + \left\lceil \frac{11}{12} \right\rceil \cdot 2 = 13 \qquad R_3^{(3)} = 3 + \left\lceil \frac{13}{3} \right\rceil \cdot 2 + \left\lceil \frac{13}{12} \right\rceil \cdot 2 =$$
$$R_3^{(4)} = 3 + \left\lceil \frac{17}{3} \right\rceil \cdot 2 + \left\lceil \frac{17}{12} \right\rceil \cdot 2 = 19 \qquad R_3^{(3)} = 3 + \left\lceil \frac{19}{3} \right\rceil \cdot 2 + \left\lceil \frac{19}{12} \right\rceil \cdot 2 =$$
$$R_3^{(3)} = 3 + \left\lceil \frac{21}{3} \right\rceil \cdot 2 + \left\lceil \frac{21}{12} \right\rceil \cdot 2 = 21$$

Schedule

By the way, this is the resulting schedule:



Schedule’

This would have been the schedule of the resulting task set: