Introduction to Formal Methods
Chapter 06: Fair CTL Model Checking

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Outline

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1. Fair CTL Model Checking: Generalities

2. Checking Fair **EG** (Explicit-State)

3. Checking Fair **EG** (Symbolic)

Fairness/Justice

An event E occurs infinitely often. Example:

- Let R_i be true iff the process i is running.
- Fairness: every process runs infinitely often.

$$\bigwedge_i \mathbf{GF} R_i$$

  (It is often used as condition for other properties.)
- You cannot express the condition in CTL:
  - $\mathbf{GF}\phi = \mathbf{AGAF}\phi$,
  - but $(\mathbf{GF}\phi) \rightarrow \psi \neq (\mathbf{AGAF}\phi) \rightarrow \psi$,
  - because $\mathbf{FG}\phi' \neq \mathbf{EFEG}\phi'$.
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Fair CTL Model Checking

- Let M be the KS $M = \langle S, S_0, R, L, AP \rangle$ and M_f the fair version $M_f = \langle S, S_0, R, L, AP, FT \rangle$, $FT = \{F_1, ..., F_n\}$.
- A path is fair iff it visits every F_i infinitely often.
- Fair CTL model checking restricts the path quantifiers to *fair paths*:
  - $M_f, s_i \models \mathbf{A}\phi$ iff $\pi \models \phi$ for every fair path $\pi = s_i, s_{i+1}, \ldots$
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- In order to solve the fair CTL model checking problem, we must be able to compute:
  - $[\mathbf{E}_f\mathbf{X}(\varphi)]$
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- Suppose we have a procedure `Check_FairEG` to compute $[\mathbf{E}_f\mathbf{G}\varphi]$.
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- We can rewrite all the other fair operators:
  - $\mathbf{E}_f\mathbf{X}(\varphi) \equiv \mathbf{E}\mathbf{X}(\varphi \land fair)$
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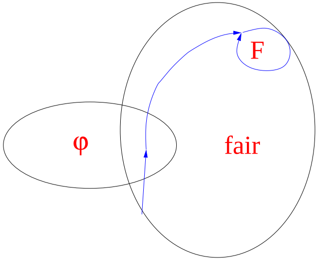

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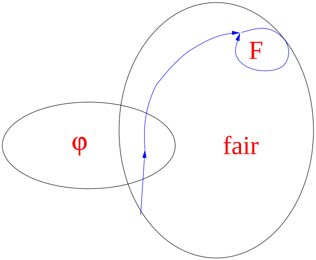

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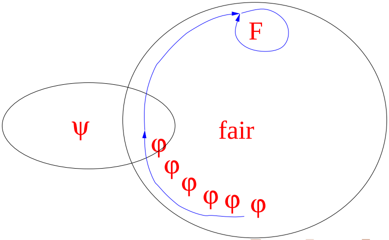

Outline

SCC-based Check_FairEG

A Strongly Connected Component (SCC) of a directed graph is a maximal subgraph s.t. all its nodes are reachable from each other.

Given a fair Kripke model M, a fair non-trivial SCC is an SCC with at least one edge that contains at least one state for every fair condition

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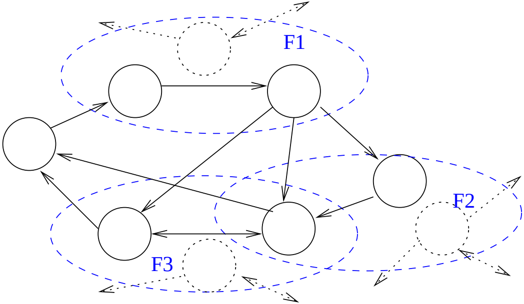

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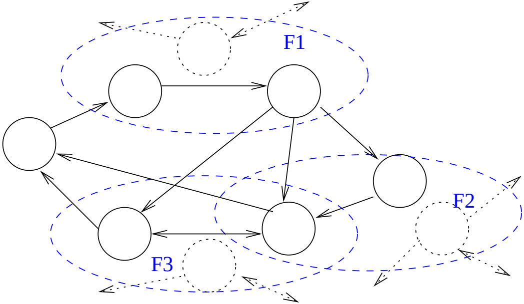

SCC-based Check_FairEG (cont.)

`Check_FairEG(`$[\phi]$`):`

(i) restrict the graph of M to $[\phi]$;

(ii) find all fair non-trivial SCCs C_i

(iii) build $C := \cup_i C_i$;

(iv) compute the states that can reach C (`Check_EU(`$[\phi]$`,`C`)`).

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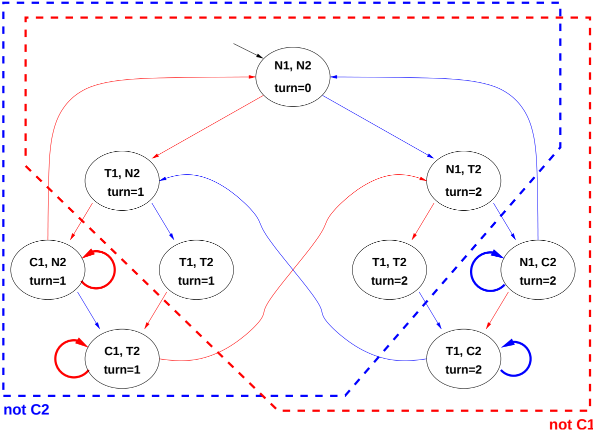
Example: Check_FairEG

F := {{ not C1},{not C2}}



$$M \models \mathbf{A}_f\mathbf{G}(T_1 \rightarrow \mathbf{A}_f\mathbf{F}C_1) = \neg\mathbf{E}_f\mathbf{F}(T_1 \wedge \mathbf{E}_f\mathbf{G}\neg C_1)$$

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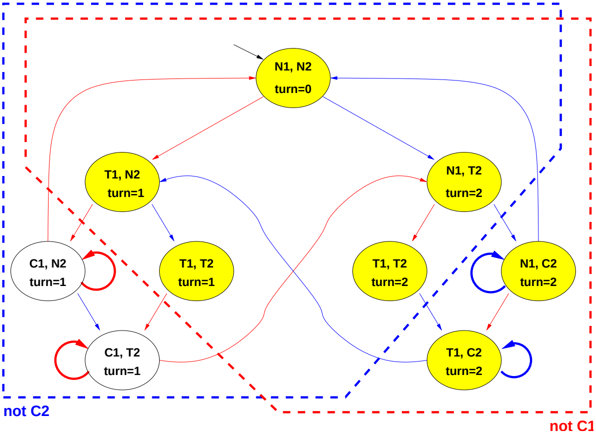

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Check_FairEG($\neg C_1$): 1. compute $[\neg C_1]$

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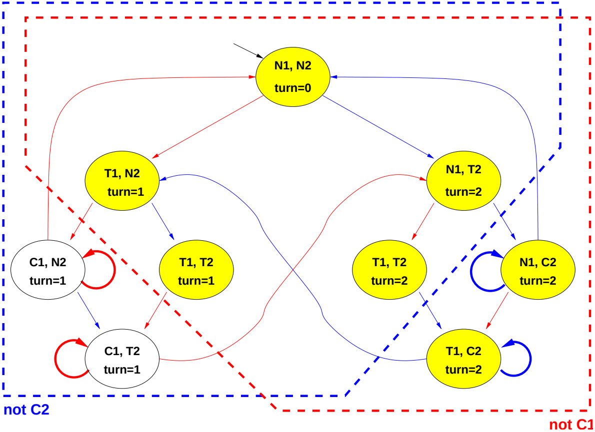

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Check_FairEG($\neg C_1$): 2. restrict the graph to $[\neg C_1]$

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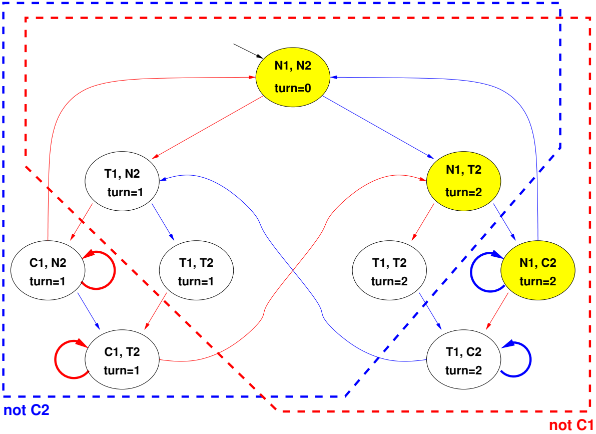

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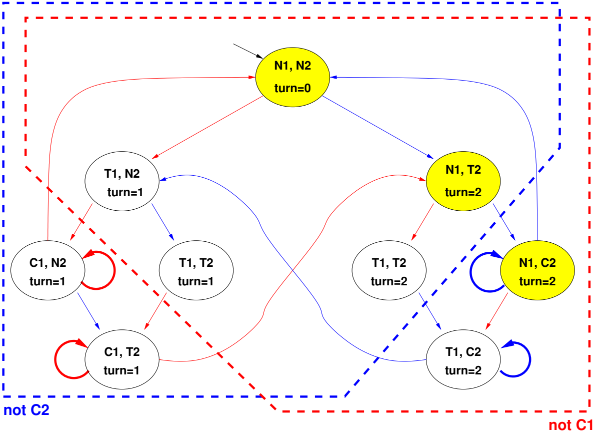

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Check_FairEG($\neg C_1$): 4. build the union C of all SCC's

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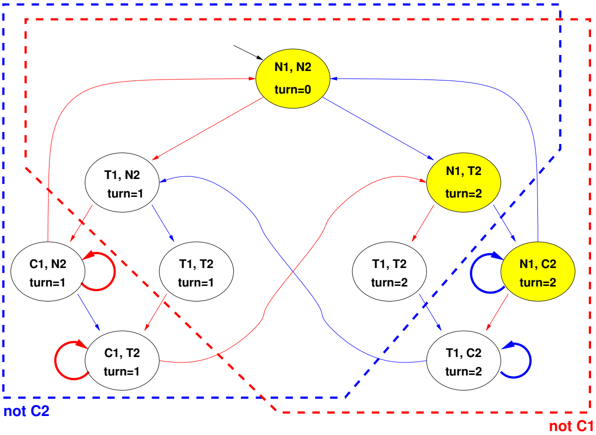

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Check_FairEG($\neg C_1$): 5. compute the states which can reach it

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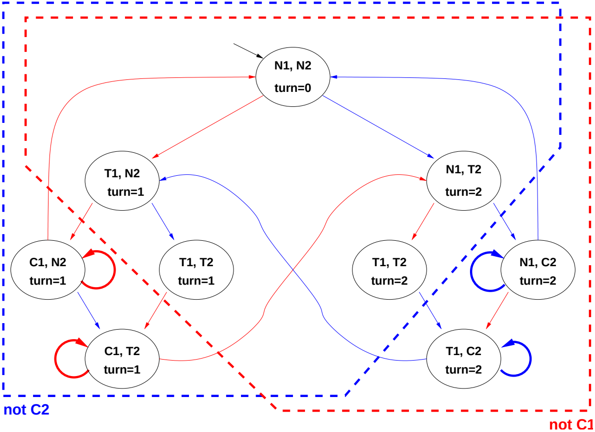

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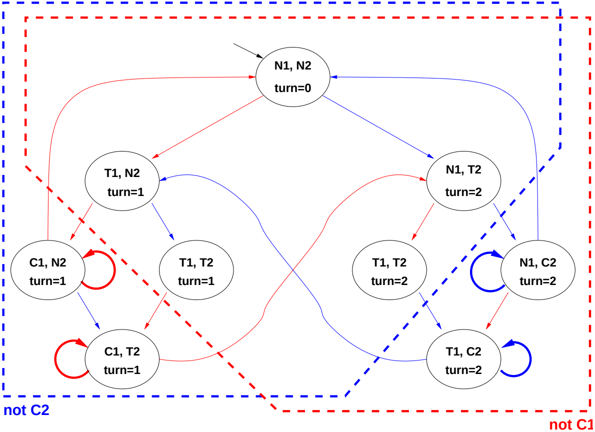

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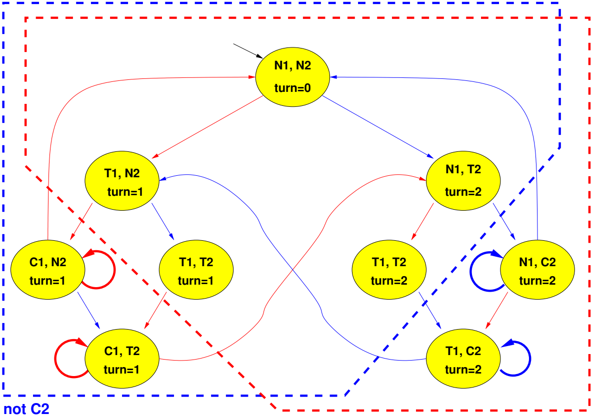

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fair=Check_FairEG($\top$)

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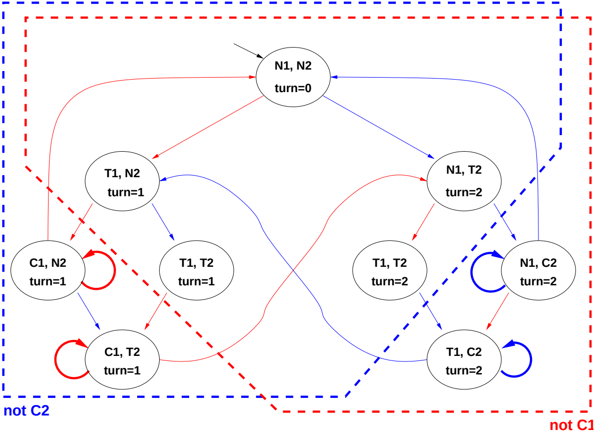

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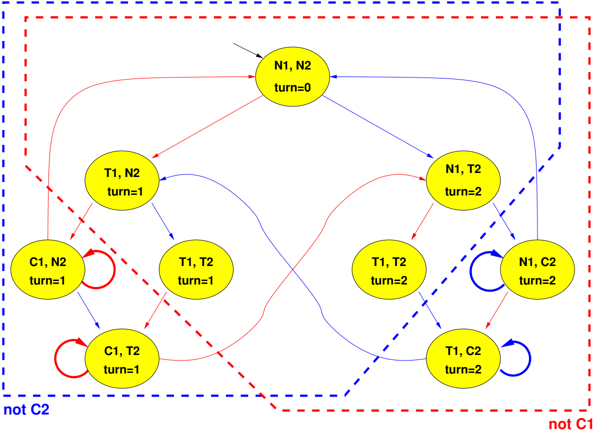

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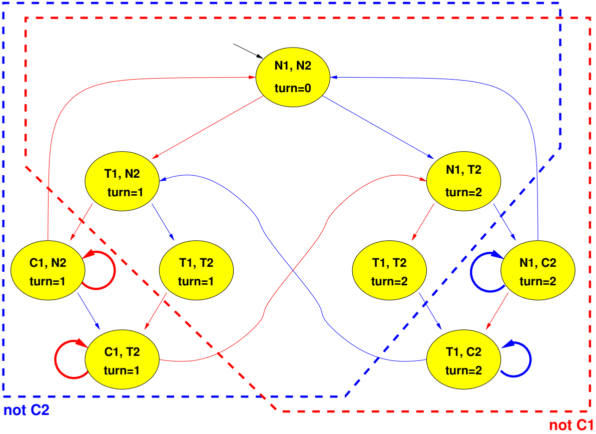

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$\Longrightarrow$ Property Verified

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- SCCs computation requires a linear ($O(\#nodes + \#edges)$) DFS (Tarjan).
- The DFS manipulates the states explicitly, storing information for every state.
- A DFS is not suitable for symbolic model checking where we manipulate sets of states.
- We want an algorithm based on the preimage computation.

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Outline

Emerson-Lei Algorithm

Recall the fixpoint characterization of **EG**:

- $[\mathbf{EG}\phi] = \nu Z.([\phi] \cap [\mathbf{EX}Z])$
  The greatest set Z s.t. every state z in Z satisfies ϕ and reaches another state in Z in one step.

We can characterize $\mathbf{E}_f\mathbf{G}$ similarly:

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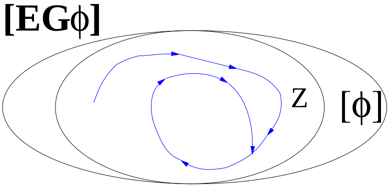

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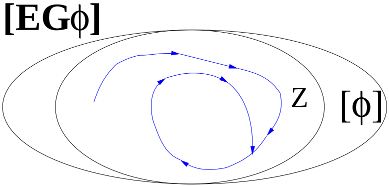

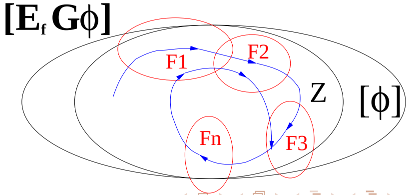

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```
state_set Check_FairEG( state_set [φ]) {
    Z' := [φ];
    repeat
      Z:= Z';
      for each Fi in FT
        Y:= Check_EU(Z,Fi∩Z);
        Z':= Z' ∩ PreImage(Y));
      end for;
    until (Z' = Z);
    return Z;
}
```

Implementation of the above formula

Emerson-Lei Algorithm

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```

Slight improvement: do not consider states in $Z \setminus Z'$

Emerson-Lei Algorithm (symbolic version)

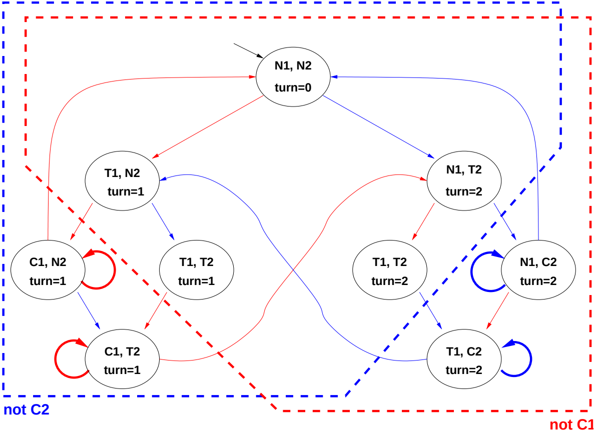
Recall: $[\mathbf{E}_f\mathbf{G}\phi] = \nu Z.([\phi] \cap \bigcap_{F_i \in FT}[\mathbf{EX}\ \mathbf{E}(Z\mathbf{U}(Z \wedge F_i))])$

```
Obdd Check_FairEG( Obdd φ) {
    Z′ := φ;
   repeat
      Z:= Z′;
     for each Fi in FT
         Y:= Check_EU(Z′,Fi∧Z′);
         Z′:= Z′ ∧ PreImage(Y));
     end for;
   until (Z′ ↔ Z);
   return Z;
}
```

Symbolic version.

Example: normal Check_EG

F := { { not C1},{not C2}}



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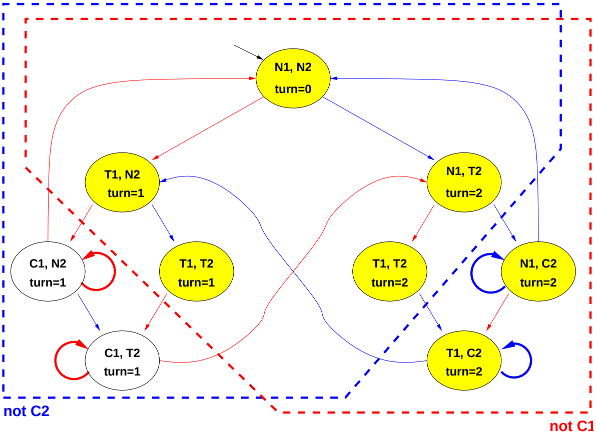

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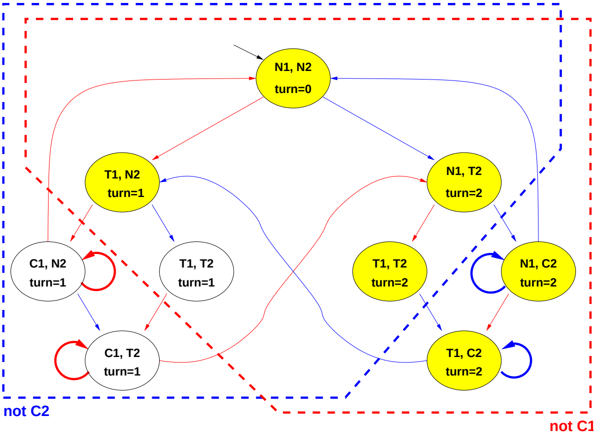

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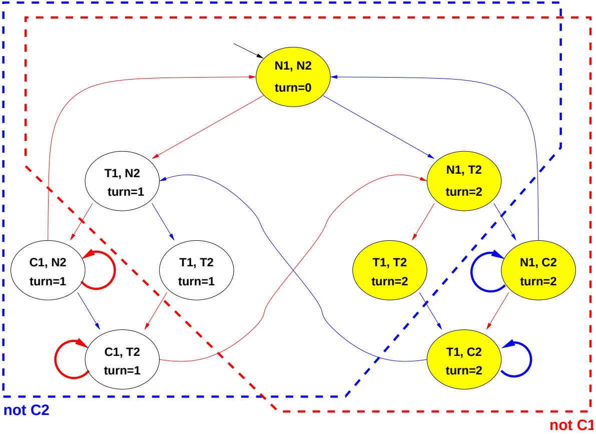

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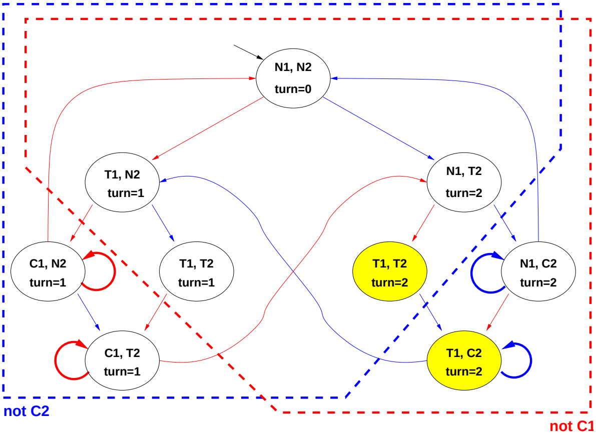

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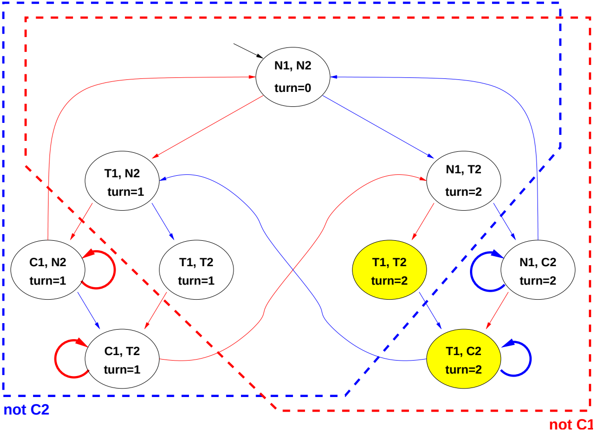

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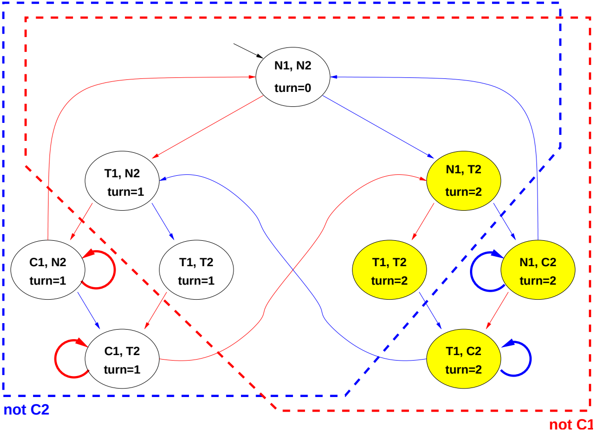

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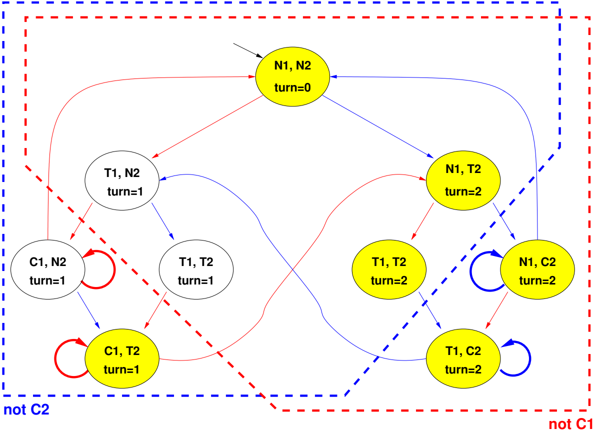

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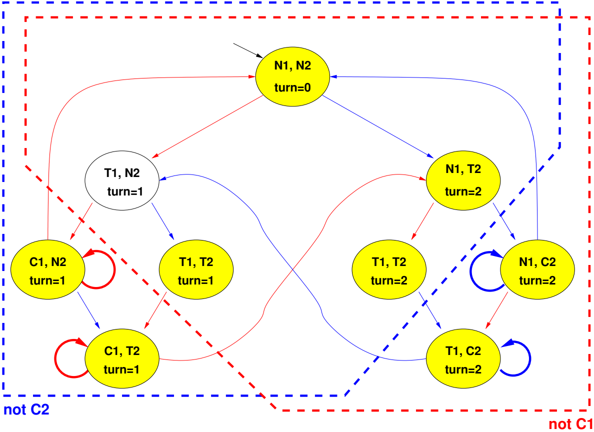

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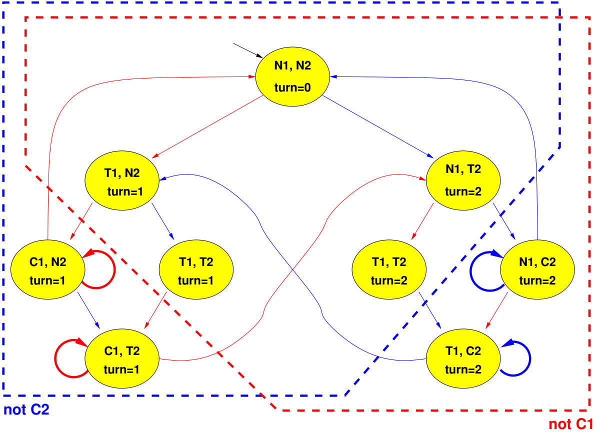

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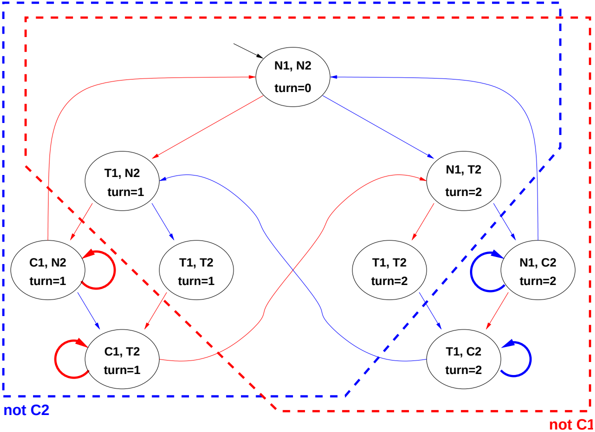

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$\Longrightarrow$ Property not verified

Example: Check_FairEG

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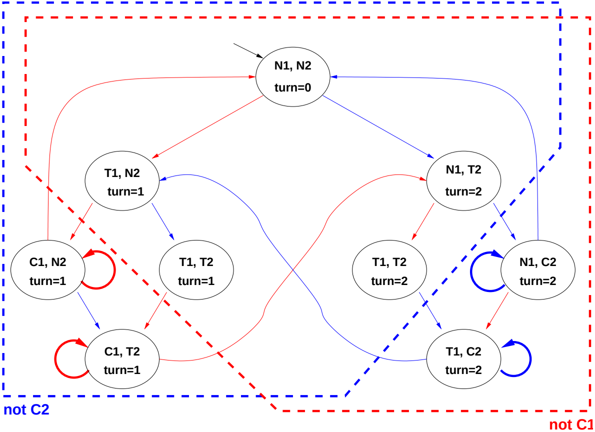

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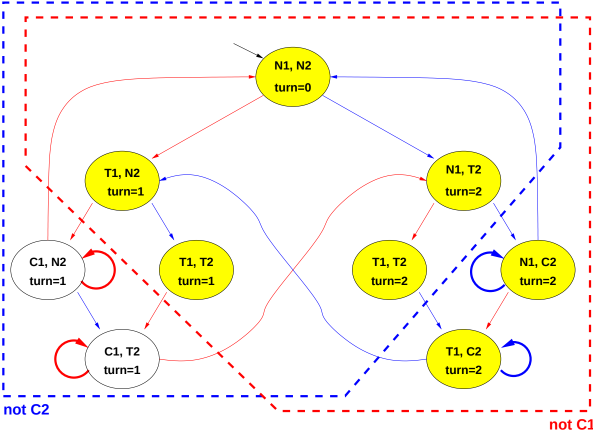

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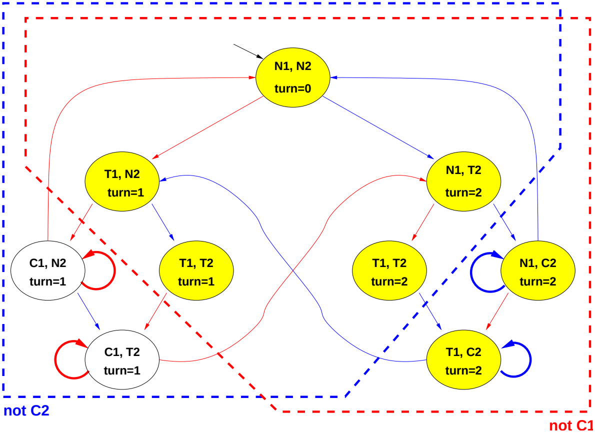

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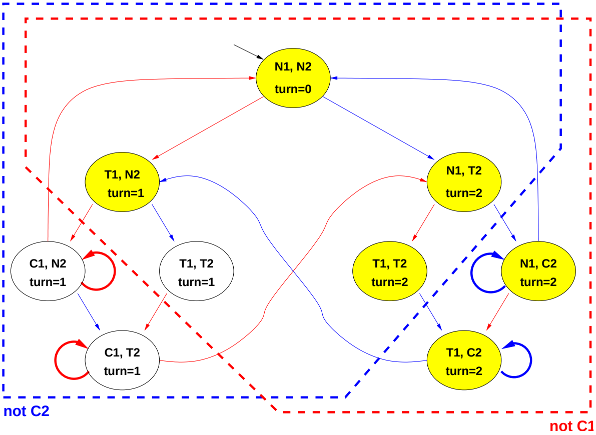

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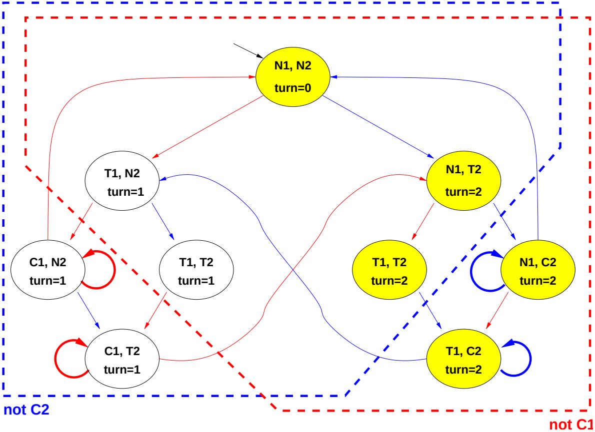

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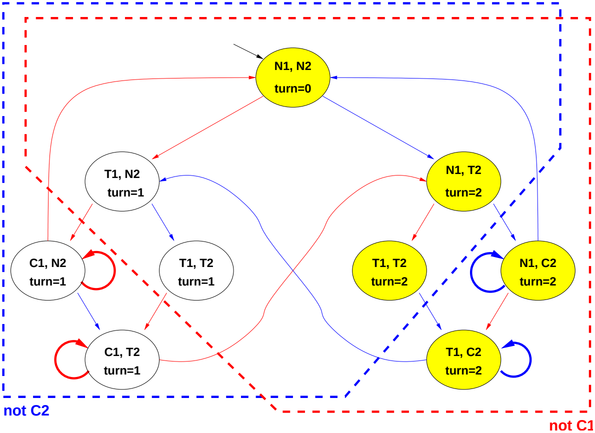

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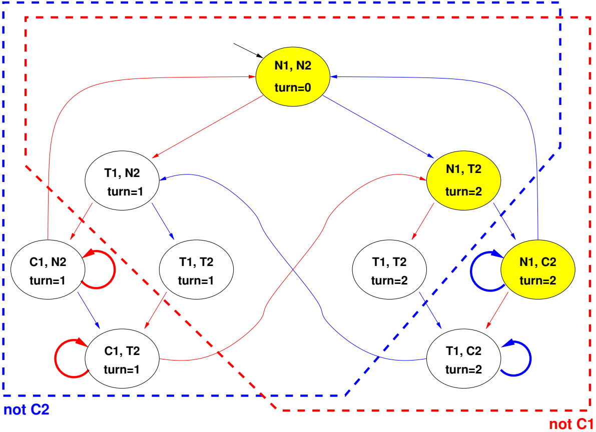

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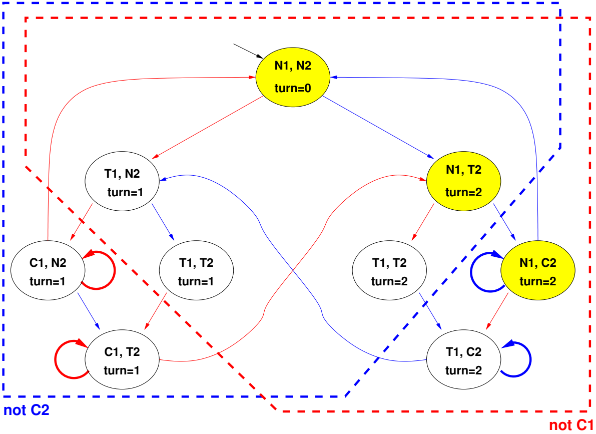

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Fixpoint reached

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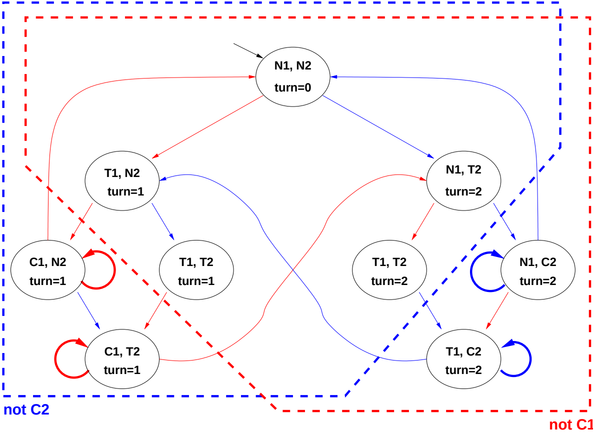

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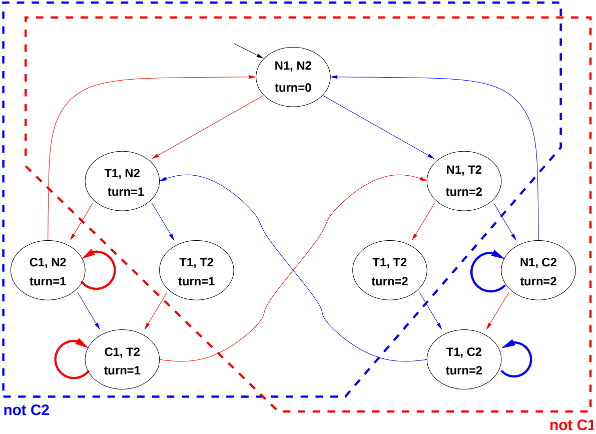

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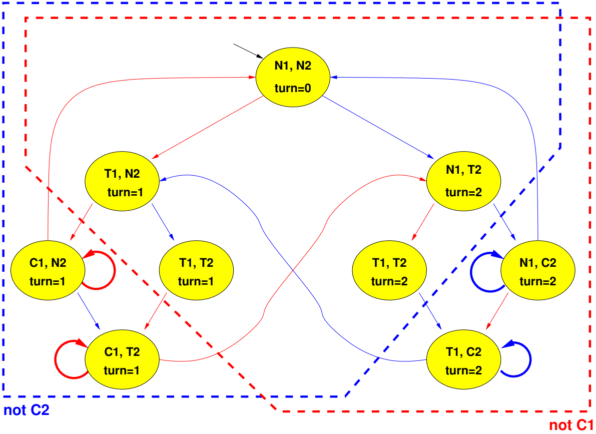

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$\Longrightarrow$ Property verified