7 Exercises on queueing networks

Exercise 1

Consider a closed queueing network with three queues with exponential services. Service rates are μ_1, μ_2, and μ_3 respectively. The total number of clients in the queue is $K = 3$. Routing probabilities between the queues are defined by the matrix

$$[r_{ij}] = \begin{bmatrix} 0.0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.0 \\ 0.5 & 0.5 & 0.0 \end{bmatrix}$$

1. Draw the queueing system;
2. Draw the underlying CTMC and discuss ergodicity;
3. Find the steady-state probability of having k_i clients in queue i;
4. for the case $\mu_1 = \mu_2 = \mu_3 = \mu$, find the average steady state number of customers and the average time spent by customers in each queue.

Exercise 2

Consider the queueing network shown in Fig. 27.

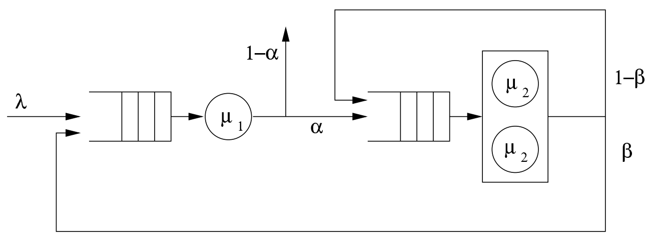

Figure 27: Queueing network of Exercise 2.

Service times at queueing stations are exponential with rates μ_1 and $\mu 2$ respectively. Clients arrive following a Poisson process with rate λ. Probabilities r_{ij} that a client visits queue j leaving queue i are:

$$r_{11} = 0;\ r_{12} = \alpha;\ r_{21} = \beta;\ r_{22} = 1 - \beta$$

1. Discuss ergodicity of the system and sketch the underlying CTMC;
2. Compute the steady-state probabilities of having k_1 customers in queue 1 and k_2 customers in queue 2;
3. Compute the average time spent in queue 1 at steady state.

Exercise 3

A closed queueing system comprises two stations, with $K = 3$ clients populating the system. Service times are exponential with uniform rate μ.

When a client ends a service, it goes deterministically to the other queue if this queue server is idle, otherwise it choses at random between the two queues.

1. Discuss ergodicity of the queueing system;
2. Draw the underlying CTMC and solve it, finding the steady-state distribution;
3. Compare the result with the solution obtained applying Gordon-Newell theorem. Are they the same? Why?

Exercise 4

Consider the queueing network shown in Fig. 28. As shown all external arrivals follow a Poisson process with rate λ, all queues have a single server with exponential service with rate μ.

1. Discuss stability of the system;
2. Find the steady state distribution of clients in queues;
3. Find the average time spent by customers in the system.

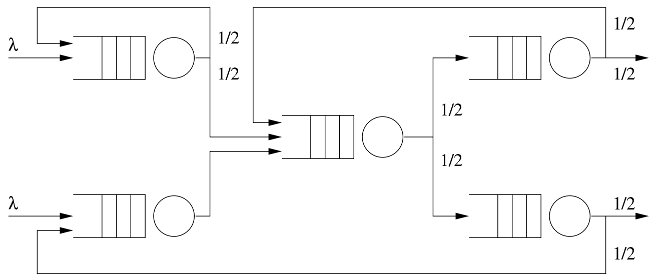

Figure 28: Queueing network of Exercise 4.

Exercise 5

Consider a cascade of $N -/D/1$ queues. Arrivals are limited to the first queue and follow a Poisson process with rate λ. Assume all services are identical with rate 1.

1. Discuss the existence of a steady-state distribution;
2. Find the average number of clients in the system at steady state;
3. Compute the average time a client takes to cross the system.

Exercise 6

Consider a closed queueing system with $N = 3$ stations and $K = 2$ clients. Stations are in cascade and from the last queue, clients recycle to the first one. All queues have exponential services with rate μ; the last queue has 2 servers, while the first two have one each.

The stations have no queuing line, thus, when a client arrives to one of the first two stations and find the server occupied, it proceeds immediately to the following queue.

1. Discuss the existence of a steady-state distribution and find it;
2. Find the average number of clients in each of the queues;
3. Find the average time spent by clients in each of the queues.