AA 2006/2007 Performance Evaluation

Exam

August 30, 2007 – Solution time: 3 hours

- Write your name and 'matricola' on each sheet of paper you use.
- The solution can be done in Italian.
- Try to write the solutions describing what you're doing; this helps in following a straightforward solution line, which in turn, enhances the value of the exam.
- The results will be available on the course site at the latest on Tuesday September 4, 8.00 PM.
- We'll register the result Tuesday September 11, 11.00 AM in my office.

Exercise 1

Consider a simple game between two players called "scissor-paper": at each try, each player has to show to the other player (at the same time) his/her hand, and the hand can have only two possible "configurations" – scissor (two fingers are straight) or paper (all the fingers are straight). The scissor beats the paper, while the same configuration (two scissors or two papers) is a tie.
Each player starts with a score of N points and each time a player is able to beat the other player, the winner gain one point, the loser loses one point (in case of a tie, the points remain unchanged). Assume that, when one of the two players wins, the game starts from the beginning, with each player with a score of N points.

1. Draw the DTMC that models this simple game and find the transition probability matrix $\boldsymbol{P}$.
2. Find the condition under which the DTMC is ergodic and, for $N = 2$, compute the limiting probability vector $\boldsymbol{\pi}$.
3. Compute the mean number of tries before one of the two players win.
4. Compute the mean number of tries between two consecutive wins of a given player.

Exercise 2

A processing system is composed by two specialized CPUs and a classifier. One CPU is devoted to the processing of type-1 jobs, while the other CPU is devoted to the processing of type-2 jobs. The classifier receives the jobs from the external devices and send them to CPU-1 or CPU-2 according to the type of the job.
Assume that jobs arrive with rate λ and that the fraction of type-1 jobs is equal to the fraction of type-2 jobs. Assume also that the rate of the two CPUs is the same and equal to μ, while the service rate of the classifier is μ_c. Suppose that, with probability α the classifier makes a wrong classification and send the job of one type to the wrong CPU. When the CPU processes a job, if it is not of the right type, there can be two options:

- Option A: the CPU sends back the job to the classifier;

- Option B: the CPU sends the job to the other CPU.

1. Draw the two queueing systems that correspond to the processing system and the two possible options, A and B, and discuss ergodicity.
2. Find the average time spent by queries in the systems (for the two possible options, A and B);
3. Discuss which option performs better and explain why.

Exercise 3

Personal Computers (PCs) can be infected by viruses. Using an antivirus software the infection can be prevented (if the PC was not infected) or solved (if the PC was infected). The PC is said *susceptible* if it can be infected, i.e., it has not updated the antivirus and it has not been infected. The PC is said *infected* if it has the virus. The PC is said *recovered* if it has the updated anti-virus (independently on whether it has been infected before or not).
Assume that the rate λ at which PCs can be infected (each individual PC) is independent from the number of infected PC. Once a PC is infected, the time it takes to download and install the update for the antivirus is exponentially distributed with parameter μ. A non-infected PC updates the antivirus with rate γ.
With rate β a new virus appears, so that all the recovered PCs becomes immediately susceptible. The infected PC, instead, take advantage of the fact that they are updating the antivirus, so that, when recovered, they will be immune also from the new virus.
Consider the simple case of a company with 2 PCs.

1. Draw the Markov chain that describes the system and find the steady state distribution (finding the correct system, without solving it, is enough).
2. Compute the average number of susceptible, infected and recovered PCs using the probability vector π.
3. Compute the mean time before a single susceptible PC becomes recovered.