AA 2006/2007 Performance Evaluation

Exam

July 18, 2007 – Solution time: 3 hours

- Write your name and 'matricola' on each sheet of paper you use.
- The solution can be done in Italian.
- Try to write the solutions describing what you're doing; this helps in following a straightforward solution line, which in turn, enhances the value of the exam.
- The results will be available on the course site at the latest on Tuesday July 24, 9.00 AM.
- We'll register the result Wednesday July 25, 9.00 AM in my office.

Exercise 1

Consider a DTMC with state space $S = \{1, 2, 3, 4, 5\}$ and state transition probability matrix

$$\boldsymbol{P} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 & 0 \\ 2/3 & 0 & 0 & 1/3 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

At time step $n = 0$ the chain is in state 1.

1. Draw the corresponding Markov chain.
2. Discuss the ergodicity and find accordingly the steady state or the stationary distribution.
3. Find the time-dependent state probability vector $\boldsymbol{\pi}(n)$ for $n = 0, 1, 2, 3, 4$, and compare the result with the distribution obtained in the previous point.
4. Find the probabilities to go from state 3 to state 5:
   (a) in exactly 3 steps;
   (b) in exactly 5 steps.

Exercise 2

A processing system is composed by S consecutive subsystems. A job entering in the processing system must pass through all the subsystems. The last subsystem is devoted to check the correctness of the job, i.e., if the operations made in all the subsystem are correct. The checking subsystem is able to identify the step that has generated the error. In case of error, the job must go back to the step that has generated the error (e.g., step j) and pass again through the remaining steps up to the final one (step j consider it as a new job coming from step $j-1$).
Assume that a job can have an error at each step with probability $1/S$ and the last step (the checking subsystem) never makes error. Jobs arrive to the system with rate λ and the time processing of each subsystem is exponentially distributed with rate μ.

1. Draw the queueing system and discuss ergodicity;
2. Find the steady-state probability of having k_i clients in queue i, with $i = 1...S$;
3. Find the average steady state number of customers in each queue and in the whole system and the average time spent by customers in the system.
4. Assuming that each step j has a different processing rate μ_j, propose an assignment for each μ_j such that the load is balanced among subsystems.

Exercise 3

Find the probability P_b that a customer arriving to an M/M/3/3 finds all servers busy and is lost. Set the service rate $\mu = 1$ and plot P_b as a function of λ.
Compare the result with the probability of loosing customers in an M/M/1/3 queue with the same service capacity, i.e., $\mu = 3$.