AA 2005/2006 Performance Evaluation

Exam

September 04, 2006 – Solution time: 3 hours

- Write your name and ‘matricola’ on each sheet of paper you use.
- The solution can be done in Italian.
- Try to write the solutions describing what you’re doing; this helps in following a straightforward solution line, which in turn, enhances the value of the exam.
- The results will be available on the course site at the latest on Wednesday September 6, 19.00 PM.
- We’ll register the result Thursday September 7, 10.00 AM in my office.

Exercise 1

Consider a DTMC with state space $S = \{1, 2, 3, 4, 5\}$ and state transition probability matrix

$$\boldsymbol{P} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/3 & 0 & 0 & 2/3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 2/3 & 0 & 0 & 1/3 & 0 \end{bmatrix}$$

At time step $n = 0$ the chain is in state 1.

1. Draw the corresponding Markov chain.
2. Discuss the ergodicity and find accordingly the steady state or the stationary distribution.
3. Find the time-dependent state probability vector $\boldsymbol{\pi}(n)$ for $n = 0, 1, 2, 3, 4$, and compare the result with the distribution obtained in the previous point.
4. Find the probabilities to go from state 3 to state 5:
   (a) in exactly 3 steps;
   (b) in exactly 5 steps.

Exercise 2

A mailing list manager running on a machine controls a list with R subscribers. When a new mail is sent to the mailing list, the manager generates and sends to the mail server R copies of the mail (one for each subscriber). Suppose that the arrival rate to the mailing list is λ and the time necessary to creates the copies and to send them to the mail server is negligible. Assume that the mail server is able to process (analyze and send) one mail at a time and the processing time is a random variable exponentially distributed with mean μ. The mail server can have up to Q mails to be sent.

1. For $R = 3$ and $Q = 4$ draw the Markov chain that model the mail server.
2. Find the steady state distribution.
3. Compute the average number of mails in the system $\overline{K}$ and the average time spent by a mail in the system $\overline{T}$
4. Can the system be described as a queing system?

Exercise 3

A processing system is composed by S consecutive subsystems. A job entering in the processing system must pass through all the subsystems. The last subsystem is devoted to check the correctness of the job, i.e., if the operations made in all the subsystem are correct. The checking subsystem is able to identify the step that has generated the error. In case of error, the job must go back to the step that has generated the error (e.g., step j) and pass again through the remaining steps up to the final one (step j consider it as a new job coming from step $j-1$). Assume that a job can have a error at each step with probability $1/S$ and the last step (the checking subsystem) never makes error, so that with probability $1/S$ the job goes out from the system. Jobs arrive to the system with rate λ and the time processing of each subsystem is exponentially distributed with rate μ.

1. Draw the queueing system and discuss ergodicity;
2. Find the steady-state probability of having k_i clients in queue i, with $i = 1...S$;
3. Find the average steady state number of customers in each queue and in the whole system and the average time spent by customers in the system.
4. Assuming that each step j has a different processing rate μ_j, propose an assignment for each μ_j such that the load is balanced among subsystems.