AA 2005/2006 Performance Evaluation

Exam

June 06, 2006 – Solution time: 3 hours

- Write your name and ‘matricola’ on each sheet of paper you use.
- The solution can be done in Italian.
- Try to write the solutions describing what you’re doing; this helps in following a straightforward solution line, which in turn, enhances the value of the exam.
- The results will be available on the course site at the latest on Wednesday June 07, 8.00 AM.
- We’ll register the result Wednesday June 07, 10.00 AM in my office.

Exercise 1

Consider an application devoted to process queries. Each query is a complex request that can be represented as a composition of simple requests, called atoms. A query can have up to M atoms and the probability that a query has i atoms is $\frac{1}{M}$, $\forall i$. The application processes the queries analyzing the atoms sequentially; the analysis of each atom requires a fixed time T. Suppose that the application is able to process only one query at a time and an external entity has an infinite number of queries and provides them to the application when it is available (i.e., when it terminates the service of one query).

1. Draw the DTMC that models the application that processes the queries and discuss ergodicity.
2. Compute the limiting probability vector $\boldsymbol{\pi}$, i.e., the probability to have i atoms that has to be processed by the application.
3. Compute the mean number of atoms in the application and the mean processing time for each query.

Exercise 2

An application runs with a guaranteed priority level on a machine; with this priority level, the application is able to process jobs with a rate equal to μ. Jobs arrive at the application with rate λ. Since jobs are time-sensitive, the application should not let the queue grow too much, so when the queue reaches a threshold (number of jobs $\geq k$) the priority level is changed and the service rate increases to 2μ. The maximum number of jobs inside the system (being served and in queue) is M (with $M > k$).

1. Draw the Markov chain that model the application.
2. Find the steady state distribution.
3. Compute the average delay $\overline{T}$ experimented by jobs.

Exercise 3

A computing system has a main CPU which is able to serve jobs with service rate μ_M. When a job finishes, it can

- either leave the system with probability p_L;
- or go, with probability p_P, to a post-processing CPU, where jobs are served with rate μ_P and then leave the system;
- or go, with probability p_D, to a data manipulation system, which serves jobs at rate μ_D; after manipulation, jobs are sent back to the main CPU as if they were new jobs.

Jobs arrives at the CPU with rate λ.

1. Draw the queueing system and discuss ergodicity;
2. Find the steady-state probability of having k_i clients in queue i, with $i = M, P, D$;
3. Find the average steady state number of customers and the average time spent by customers in each queue.