AA 2005/2006 Performance Evaluation

Additional Exercises on DTMC and CTMC

NOTE: exercises 1, 3 and 4 are taken from *K. S. Trivedi. Probability and Statistics with Reliability, Queuing, and Computer Science Applications. John Wiley and Sons, 2nd edition, 2001.*

Exercise 1 (examples 7.15 and 7.16 from Trivedi book)

Considering a data structure (such as linear list) being manipulated in a program. Suppose that we are interested only in the amount of memory consumed by the data structure. If the current amount of memory in use is i nodes, then we say that the state of the structure is s_i. Let probability associated with the next operation on the data structure be given by

- b_i = Pr[“next operation is an insert” | “current state is s_i”];
- d_i = Pr[“next operation is a delete” | “current state is s_i”];
- a_i = Pr[“next operation is an access” | “current state is s_i”].

When the system is in state s_0, we suppose that a delete operation has no effect on the system. Considering the special case with $b_i = b$, $d_i = d$, $a_i = a \ \forall i$,

1. draw the state diagram and find the transition probability matrix $\boldsymbol{P}$;
2. compute the limiting probability vector $\boldsymbol{\pi}$, finding the conditions for which the solution exists.

Assume that a limited number $m \geq 1$ of nodes are available for allocation. Then, is m nodes are in use, an insertion operation will give rise to an overflow. We assume that such an operation is simply ignored, leaving the system in state s_m.

1. draw the state diagram and compute limiting probability vector $\boldsymbol{\pi}$;
2. compute the probability of an overflow.

Exercise 2

Figure 1 shows the flowchart of a program. The mean execution time of each program segment is denoted by τ_i. We want to determe the mean execution time of the program.

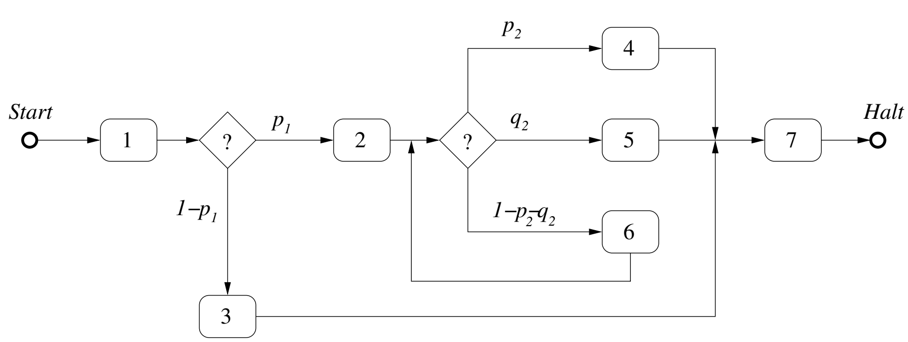

Figure 1: Program Flowchart considered in Exercise 2.

Assume that, when the program terminates, it is immediately rerun, forever. Then this corresponds to a semi-Markov chain where each state is a program segment.

1. Draw the semi-Markov chain.

2. Describe in detail the procedure that you would follow to determine the percentage of time spent in segment j.

3. What is the mean execution time of the program?

Exercise 3 (example 8.20 from Trivedi book)

Assume that the device time to failure is two-stage hyperexponential with rates λ_1 and λ_2. Assume also that the time to repair is exponentially distributed with rate μ. Further assume that an inspection is triggered after a mean duration $1/\lambda_{\text{in}}$, and takes an average time of $1/\mu_{\text{in}}$. After an inspection is completed, no action is taken if the device is found to be in a non-failure state. On the other hand, if the device is found to be in the first stage of the failure state, a preventive maintenance is carried out. We assume that the time to carry out repair is y times the time to carry out preventive maintenance.

1. Draw the state diagram.

2. Compute the steady-state distribution.

3. Compute the steady-state availability.

Exercise 4 (example 8.25 from Trivedi book)

Consider a 2-node cluster where both hardware and Operating system software (OS) failures may occur. The node hardware fails at the constant rate λ and the OS fails at the constant rate λ_{OS}. We assume here that hardware failures are permanent and hence require a repair or replacement action, while OS failures are cleared by a reboot. Repair or reboot take place at rates μ and β for the hardware and OS respectively. A node is considered down when both nodes have failed. In case of a hardware failure in one node and OS failure in the other, the OS is always recovered first.

1. Draw the state diagram.

2. Compute the steady-state distribution.

3. Compute the steady-state availability.

Exercise 5

A single repairperson looks after both machine 1 and 2. Each time it is repaired, machine i stays up for an exponential time with rate λ_i, $i = 1, 2$. When machine i fails, it requires an exponentially distributed amount of work with rate μ_i to complete its repair. The repairperson will always service machine 1 when it is down. For instance, if machine 1 fails while 2 is being repaired, then the repairperson will immediately stop work on machine 2 and start on 1.

1. Draw the state diagram.

2. Compute the steady-state distribution.

3. What proportion of time is machine 2 down?