DL-Lite with Temporalized Concepts, Rigid Axioms and Roles

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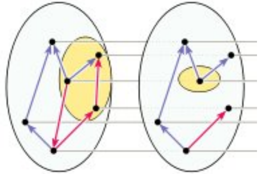
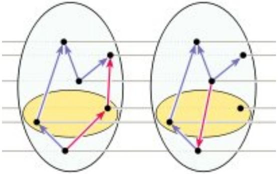
Background: complexity of temporal $\mathcal{ALC}$

When we add temporal dimension to DL we may want



temporal concepts



rigid concepts

The same applies to *roles* and *axioms*

	concepts		roles		axioms	
	rigid	temp.	rigid	temp.	rigid	temp.
undec. [GKWZ:03]	–	yes	yes	–	yes	–
2ExpTime [BGL:08]	–	yes	yes	–	–	yes
ExpSpace [GKWZ:03]	–	yes	–	yes*	–	yes
ExpTime [S:93]	–	yes	–	yes*	yes	–

* but we cannot restrict their interpretation by the language

Motivation

- We are looking for a *decidable* Temporal DL (TDL) with

	concepts		roles		axioms	
	rigid	temp.	rigid	temp.	rigid	temp.
undec.	–	yes	yes	–	yes	–

- Reason: TDL with such specifications will be in tight correspondence with Temporal Conceptual Models (ER, UML, etc.)
- Known decidable TDL weakens the temporal dimension of [GKWZ:03] from $\mathcal{LTL}$ to **S5**. It is 2ExpTime-complete [ALT:07]
- In our work we weaken the DL dimension of [GKWZ:03] from $\mathcal{ALC}$ to *DL-Lite*

Results

In this work we consider an extension of

$DL\text{-}Lite^{\mathcal{N}}_{core}$	and	$DL\text{-}Lite^{\mathcal{N}}_{bool}$
$=$ *DL-Lite*		$=$ extension of *DL-Lite* to full Boolean logic

with:

- Temporal Concepts:

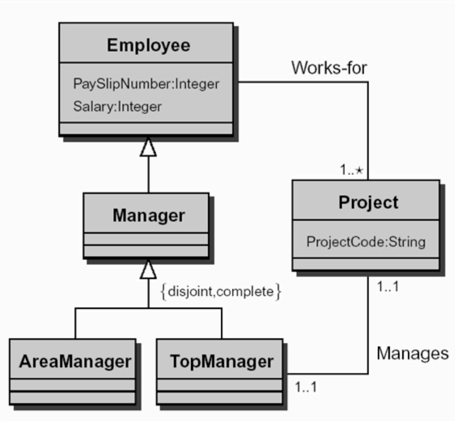
  $\Diamond$ on concepts and $\mathcal{U}$ on concepts

- Rigid Roles;
- Rigid Axioms.

Our complexity (completeness) results are

DL \ $\mathcal{LTL}$	$\Diamond$	$\mathcal{U}$
$DL\text{-}Lite^{\mathcal{N}}_{bool}$	$TDL\text{-}Lite^{\Diamond}_{bool}$[NP]	$TDL\text{-}Lite^{\mathcal{U}}_{bool}$[PSPACE]
$DL\text{-}Lite^{\mathcal{N}}_{core}$	$TDL\text{-}Lite^{\Diamond}_{core}$[NP]	$TDL\text{-}Lite^{\mathcal{U}}_{core}$[PSPACE]

The $DL\text{-}Lite_{bool}^{\mathcal{N}}$ language informally



$$
\begin{array}{rcl}
\text{Manager} & \sqsubseteq & \text{Employee} \\
\text{AreaManager} \sqcap \text{TopManager} & \sqsubseteq & \bot \\
\exists\text{WorksFor} & \sqsubseteq & \text{Employee} \\
\exists\text{WorksFor}^- & \sqsubseteq & \text{Project} \\
\geq 2\ \text{Manages} & \sqsubseteq & \bot \\
\text{Manager} & \sqsubseteq & \text{TopManager} \sqcup \\
 & & \text{AreaManager} \\
 & \vdots &
\end{array}
$$

Note: we use the shortcut $\exists R$ instead of $\geq 1\ R$
Note: $DL\text{-}Lite_{core}^{\mathcal{N}}$ captures conceptual modelling diagrams without *complete* operator for hierarchies

The $DL\text{-}Lite^{\mathcal{N}}_{bool}$ Language formally

$DL\text{-}Lite^{\mathcal{N}}_{bool}$ is a language over *object names* $a, b, \ldots,$ *concept names* $A, \ldots,$ *role names* $P, \ldots$:

- TBox assertions: $C_1 \sqsubseteq C_2$, with:

$$\begin{array}{rcl} C & \longrightarrow & B \mid \neg C \mid C_1 \sqcap C_2 \\ B & \longrightarrow & A \mid \geq n\, R \mid \bot \\ R & \longrightarrow & P \mid P^- \end{array}$$

- ABox assertions/Database facts:

$$A(a), \quad P(a, b), \quad \text{with } a, b \text{ objects.}$$

- Satisfiability problem is NP-complete [ACKZ:09]

Note: $DL\text{-}Lite^{\mathcal{N}}_{core}$ permits only TBox assertions of the kind $B_1 \sqsubseteq B_2$ and $B_1 \sqsubseteq \neg B_2$.The satisfiability problem is NLOGSPACE-complete

The $TDL\text{-}Lite^{\mathcal{U}}_{bool}$ language:

$$
\begin{array}{lcllllllllll}
C & ::= & B & \mid & \neg C & \mid & C_1 \sqcap C_2 & \mid & \Diamond C, & \mid & C_1 \,\mathcal{U}\, C_2 \\
B & ::= & \bot & \mid & A & \mid & \geq n\,R, & & & & \\
R & ::= & P & \mid & P^- & \mid & G & \mid & G^-, & &
\end{array}
$$

where G denotes *rigid roles*.

- TBox assertions: $C_1 \sqsubseteq C_2$,
- ABox assertions/Database facts:

$$\bigcirc^n B(a),\ \bigcirc^n R(a,b),\ \Box B(a) \text{ and } \Box R(a,b),$$

where $\bigcirc^n$ denotes the sequence of $n \geq 0$ *next-time operators*.

Note: $TDL\text{-}Lite^{\Diamond}_{bool}$ is a fragment of $TDL\text{-}Lite^{\mathcal{U}}_{bool}$ where only $\Diamond C$ subconcepts allowed

A $TDL\text{-}Lite^{\mathcal{U}}_{bool}$ *interpretation* $\mathcal{I}$ is a function over $\mathbb{N}$

$$\mathcal{I}(n) = \big(\Delta^{\mathcal{I}}, a^{\mathcal{I}}, \dots, A^{\mathcal{I}(n)}, \dots, P^{\mathcal{I}(n)}, \dots, G^{\mathcal{I}(n)}, \dots\big),$$

where:

- Objects, a, are rigid and the UNA is enforced;
- Rigid roles are time-invariant:

$$G^{\mathcal{I}(t_1)} = G^{\mathcal{I}(t_2)}, \quad \forall t_1, t_2 \in \mathbb{N}$$

- The $\mathcal{U}$ (and $\Diamond$) has an *irreflexive* semantics:

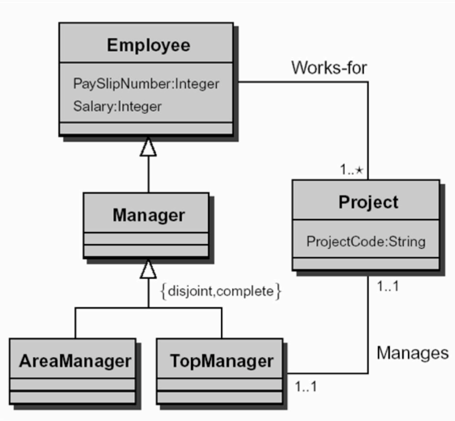
$$(C_1 \, \mathcal{U} \, C_2)^{\mathcal{I}(n)} = \{a \in \Delta^{\mathcal{I}} \mid \exists k > n \text{ s.t. } a \in C_2^{\mathcal{I}(k)} \text{ and}$$
$$\text{for all } i : n < i < k \text{ it holds } a \in C_1^{\mathcal{I}(i)}\}$$

Semantics of $TDL\text{-}Lite^{\mathcal{U}}_{bool}$ (cont.)

TBox and ABox assertions are interpreted along the following satisfaction relation:

$$
\begin{array}{rcl}
\mathcal{I} \models C \sqsubseteq D & \text{iff} & C^{\mathcal{I}(n)} \subseteq D^{\mathcal{I}(n)}, \text{ for all } n \geq 0, \\
\mathcal{I} \models \bigcirc^n B(a) & \text{iff} & a^{\mathcal{I}} \in B^{\mathcal{I}(n)}, \\
\mathcal{I} \models \Box B(a) & \text{iff} & a^{\mathcal{I}} \in B^{\mathcal{I}(n)}, \text{ for all } n > 0, \\
\mathcal{I} \models \bigcirc^n R(a,b) & \text{iff} & (a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}(n)}, \\
\mathcal{I} \models \Box R(a,b) & \text{iff} & (a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}(n)}, \text{ for all } n > 0.
\end{array}
$$

Note: TBox assertions are interpreted globally, i.e., they are rigid!



$$\text{Manager} \sqsubseteq \text{Employee}$$

Global Entities

$$\text{Employee} \equiv \Box\text{Employee}$$

Global Relations

Global Entities + Global Roles

Dynamic Entities

$$\text{AreaManager} \sqsubseteq \Diamond\text{TopManager}$$

Persistent Entities

$$\text{TopManager} \sqsubseteq \Box\text{TopManager}$$

Metric Constraints

$$\text{Manager} \sqsubseteq \bigcirc^2\text{TopManager}$$

Note: we cannot express *temporary entities* since we do not have *past-time* operators: $\text{Manager} \sqsubseteq \Diamond\neg\text{Manager} \sqcup \Diamond_{past}\neg\text{Manager}$

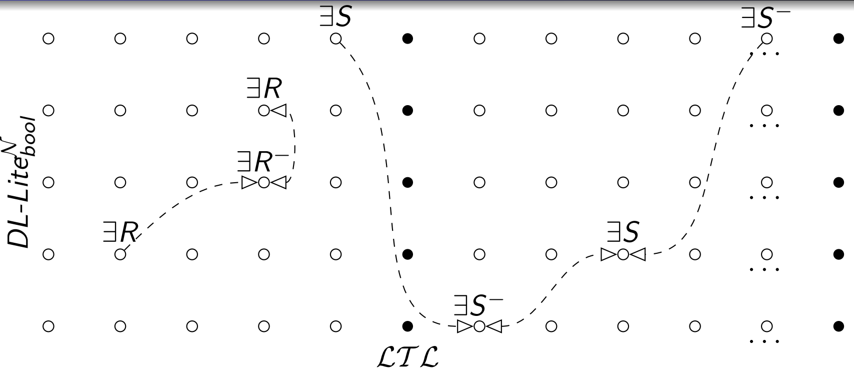
$TDL\text{-}Lite_{bool}^{\mathcal{U}}$ is not a fusion

In $TDL\text{-}Lite_{bool}^{\mathcal{U}}$ there is an interaction between the two component logics, $DL\text{-}Lite_{bool}^{\mathcal{N}}$ and $\mathcal{LTL}$.

$$\mathcal{K} = \{\exists R(a)\} \cup \{\exists R \sqsubseteq \Diamond \exists R, \Diamond \exists R^{-} \sqsubseteq \bot\}$$

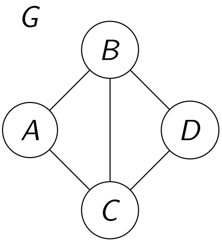
Therefore we cannot do reasoning by checking satisfiability separately in the component logics.

We need to develop satisfiability checking procedure!



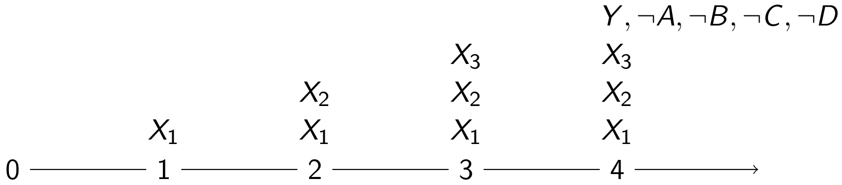
- We show that a $TDL\text{-}Lite^{\mathcal{U}}_{bool}$ KB is satisfiable iff there exists a *quasimodel* for it "exponentially long" in the $\mathcal{LTL}$ dimension and "polynomially wide" in the $DL\text{-}Lite^{\mathcal{N}}_{bool}$ dimension
- For $TDL\text{-}Lite^{\Diamond}_{bool}$ the quasimodel is "polynomially long" in the $\mathcal{LTL}$ dimension and "polynomially wide" in the $DL\text{-}Lite^{\mathcal{N}}_{bool}$ dimension

$TDL\text{-}Lite^{\Diamond}_{core}$ is NP-hard: reduction from 3-colorability



$$\mathcal{K} = \{ O \sqsubseteq \Diamond A \sqcap \Diamond B \sqcap \Diamond C \sqcap \Diamond D,$$
$$A \sqsubseteq \Box X_1, X_1 \sqsubseteq \Box X_2, X_2 \sqsubseteq \Box X_3,$$
$$X_3 \sqsubseteq \Box Y,$$
$$Y \sqsubseteq \neg A \sqcap \neg B \sqcap \neg C \sqcap \neg D,$$
$$A \sqsubseteq \neg B, B \sqsubseteq \neg C, B \sqsubseteq \neg D$$
$$C \sqsubseteq \neg D, C \sqsubseteq \neg A,$$
$$O(a)\}$$

The graph G is 3-colorable iff $\mathcal{K}$ is satisfiable



Conclusions & future work

We investigated languages which are useful to capture temporal data models (ER, UML, etc.)
Our results are the complexity of reasoning in these languages:

DL $\backslash \mathcal{LTL}$	$\Diamond$	$\mathcal{U}$
$\textit{DL-Lite}_{bool}^{\mathcal{N}}$	$\textit{TDL-Lite}_{bool}^{\Diamond}$[NP]	$\textit{TDL-Lite}_{bool}^{\mathcal{U}}$[PSPACE]
$\textit{DL-Lite}_{core}^{\mathcal{N}}$	$\textit{TDL-Lite}_{core}^{\Diamond}$[NP]	$\textit{TDL-Lite}_{core}^{\mathcal{U}}$[PSPACE]

Interesting directions of future work

- Adding *past-time* operators to $\textit{TDL-Lite}_{bool}^{\mathcal{U}}$ to capture *Temporary* Entities, Relations and Attributes:

$$E \sqsubseteq \Diamond_F \neg E \sqcup \Diamond_P \neg E$$

- Restricting $\textit{TDL-Lite}_{bool}^{\mathcal{U}}$ and $\textit{TDL-Lite}_{bool}^{\Diamond}$ to Core does not make them computationally easier. Weaken temporal dimension from $\mathcal{LTL}$ to simpler models of time, **S5**?
- Extention of $\textit{DL-Lite}_{core}^{\mathcal{N}}$ with $\Diamond$ and $\bigcirc$ operators (instead of $\mathcal{U}$) in NP?