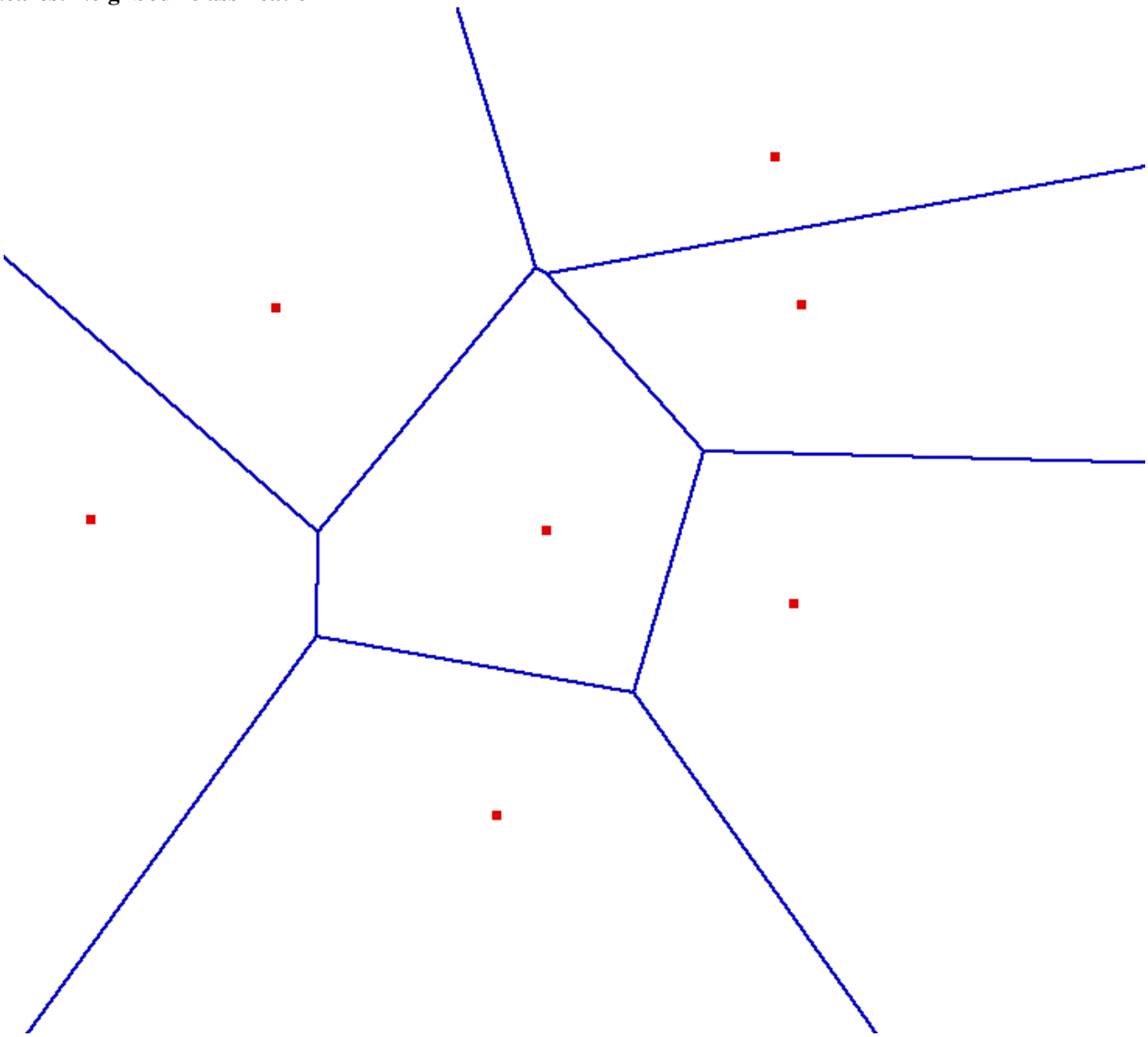
**1-Nearest Neighbour classification**



**Measuring the distance between instances**

***Metric* or *distance* definition**

Given a set $\mathcal{X}$, a function $d : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_0^+$ is a *metric* for $\mathcal{X}$ if for any $x, y, z \in \mathcal{X}$ the following properties are satisfied:

**reflexivity** $d(x, y) = 0 \quad iff \quad x = y$

**symmetry** $d(x, y) = d(y, x)$

**triangle inequality** $d(x, y) + d(y, z) \geq d(x, z)$

*E.g. Euclidean distance in* $\mathbb{R}^n$

$$d(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}$$

**k-Nearest Neighbour classification**

**The algorithm**

**for all** test examples $x$ **do**
**for all** training examples $(x_i, y_i)$ **do**
compute distance $d(x, x_i)$
**end for**
select the $k$-nearest neighbours of $x$
return class of $x$ as majority class among neighbours:

$$argmax_y \sum_{i=1}^{k} \delta(y, y_i)$$

**end for**

*Note*

$$\delta(x, y) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$$

**k-Nearest Neighbour regression**

**The algorithm**

**for all** test examples $x$ **do**
**for all** training examples $(x_i, y_i)$ **do**
compute distance $d(x, x_i)$
**end for**
select the $k$-nearest neighbours of $x$
return the average output value among neighbours:

$$\frac{1}{k} \sum_{i=1}^{k} y_i$$

**end for**

**Characteristics of k-nearest neighbour learning**

**instance-based learning** the model used for prediction is calibrated for the test example to be processed

**lazy learning** computation is mostly deferred to the classification phase

**local learner** assumes prediction should be mainly influenced by nearby instances

**uniform feature weighting** all features are uniformly weighted in computing distances

**Distance-weighted k-nearest neighbour**

**Classification**

$$argmax_y \sum_{i=1}^{k} w_i \delta(y, y_i)$$

**Regression**

$$\frac{\sum_{i=1}^{k} w_i y_i}{\sum_{i=1}^{k} w_i}$$

where: $w_i = \frac{1}{d(x,x_i)}$