Polarity Items in Type Logical Grammar.

Connection with DMG

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Contents

1. The problem

- In formal linguistic literature, one finds examples of theories based on **classifications** of items which belong to the same syntactic category but which differ in some respect. For example,
  - **generalized quantifiers** have been classified considering the different ways of distributing with respect to negation [Beghelli and Stowell'97];
  - **wh-phrases** can be divided considering their sensitivity to different weak-islands strength [Szablosci and Zwarts'97];
  - **adverbs** differ in their order relations [Ernst'01];
  - **polarity items** have been distinguished by the sort of licensors they require for grammaticality [Wouden'94,Giannakidou'97].
- In all these cases, the described typologies are based on semantically motivated **subset relations** holding among the denotations of the involved items.
- **Aim**: to show how categorial type logic can contribute to the study of linguistic typologies, and how this application sheds light on the different role of binary vs. unary operators.

2. Our proposal

- Categorial type logic provides a modular architecture to study **constants** and **variation** of grammatical composition:
  - **base logic** grammatical invariants, universals of form/meaning assembly;
  - **structural module** non-logical axioms (postulates), lexically anchored options for structural reasoning.
- Up till now, research on the constants of the base logic has focussed on binary operators. E.g.
  - Lifting theorem: $A \vdash (B/A)\backslash B$;

  While unary operators have been used to account for structural variants.
- We will show how **unary operators** can be used
  - to account for linguistic typologies encoding the **subset relations** among items of the same syntactic category, and
  - to account for **cross-linguistic** differences.

3. The logic

In [Areces, Bernardi and Moortgat] the base logic ($\mathsf{NL}(\Diamond, \cdot^{\mathbf{0}})$) consisting of residuated and Galois connected operators has been studied.

- **Language** Formulas are built from: Atoms, residuated operators: $(\backslash, \bullet, /)$, $(\Diamond\cdot, \Box^{\downarrow}\cdot)$; and unary Galois connected ones: $({}^{0}\cdot, \cdot^{0})$.
- **Models**

  Frames $F = \langle W, R_0^2, R_\Diamond^2, R_\bullet^3 \rangle$

  W: 'signs', resources, expressions

  $R_\bullet^3$: 'Merge', grammatical composition

  $R_\Diamond^2$: 'feature checking', structural control

  R_0^2: accessibility relation for the Galois connected operators

  Models $\mathcal{M} = \langle F, V \rangle$

  Valuation $V : \mathsf{TYPE} \mapsto \mathcal{P}(W)$: types as sets of expressions

4. Interpretation of the constants

$$\begin{array}{lcl} V(\Diamond A) & = & \{x \mid \exists y(R^2_\Diamond xy \ \& \ y \in V(A)\} \\ V(\Box^{\downarrow} A) & = & \{x \mid \forall y(R^2_\Diamond yx \ \Rightarrow y \in V(A)\} \end{array}$$

$$\begin{array}{lcl} V({}^{\mathbf{0}}A) & = & \{x \mid \forall y(y \in V(A) \ \Rightarrow \neg R^2_0 yx\} \\ V(A^{\mathbf{0}}) & = & \{x \mid \forall y(y \in V(A) \ \Rightarrow \neg R^2_0 xy\} \end{array}$$

$$\begin{array}{lcl} V(A \bullet B) & = & \{z \mid \exists x \exists y[R^3 zxy \ \& \ x \in V(A) \ \& \ y \in V(B)]\} \\ V(C/B) & = & \{x \mid \forall y \forall z[(R^3 zxy \ \& \ y \in V(B)) \Rightarrow z \in V(C)]\} \\ V(A \backslash C) & = & \{y \mid \forall x \forall z[(R^3 zxy \ \& \ x \in V(A)) \Rightarrow z \in V(C)]\} \end{array}$$

5. Some useful derived properties

(Iso/Anti)tonicity $A \vdash B$ implies

$$\Diamond A \vdash \Diamond B \quad \text{and} \quad \Box^{\downarrow} A \vdash \Box^{\downarrow} B$$
$$^{\mathbf{0}}B \vdash {}^{\mathbf{0}}A \quad \text{and} \quad B^{\mathbf{0}} \vdash A^{\mathbf{0}}$$
$$A/C \vdash B/C \quad \text{and} \quad C/B \vdash C/A$$
$$A \bullet C \vdash B \bullet C \quad \text{and} \quad C \bullet A \vdash C \bullet B$$

Compositions

$$\Diamond\Box^{\downarrow} A \vdash A \qquad A \vdash \Box^{\downarrow}\Diamond A$$
$$A \vdash {}^{\mathbf{0}}(A^{\mathbf{0}}) \qquad A \vdash ({}^{\mathbf{0}}A)^{\mathbf{0}}$$
$$(A/B) \bullet B \vdash A \qquad A \vdash (A \bullet B)/B$$

Closure Let $(\cdot)^*$ be $^{\mathbf{0}}(\cdot^{\mathbf{0}})$, $(^{\mathbf{0}}\cdot)^{\mathbf{0}}$, $\Box^{\downarrow}\Diamond(\cdot)$, $X/(\cdot\backslash X)$, $(X/\cdot)\backslash X$. $\forall A \in \mathsf{TYPE}$

$$A \vdash A^*, \quad A^* \vdash B^* \text{ if } A \vdash B, \quad A^{**} \vdash A^*$$

Triple Let (f_1, f_2) be either the residuated or the Galois pair, $f_1 f_2 f_1 A \longleftrightarrow f_1 A$, and similarly $f_2 f_1 f_2 A \longleftrightarrow f_2 A$. For example,

$$\Diamond\Box^{\downarrow}\Diamond A \longleftrightarrow \Diamond A \text{ and } \Box^{\downarrow}\Diamond\Box^{\downarrow} A \longleftrightarrow \Box^{\downarrow} A.$$

6. Linguistic Applications

When looking at linguistic applications $\mathsf{NL}(\Diamond, \cdot^{\mathbf{0}})$ offers operators that can be employed to:

- distinguish the distribution behavior of e.g. quantifier, polarity items etc., encoding their syntactic classification;
- represent the semantic aspects of the same items, which determine their inferential role in the language;

We will show how

- the derivability patterns of $\mathsf{NL}(\Diamond, \cdot^{\mathbf{0}})$ can be used to account for polarity items (syntatic) distribution by encoding semantic features (viz. (non-)veridicality);
- encoding of (non-)veridicality by means of unary operators sheds light on possible connections between dynamic Montague grammar and categorial type logic.

7. Non-veridical Contexts

[Zwarts 1995, Giannakidou 1997] extending the typology of PIs proposed in [van der Wouden 1994] consider polarity items sensitive to (non-)veridicality.

Definition [(Non-)veridical functions] Let f be a boolean function with a boolean argument, a definition of (non-)veridical functions can be given starting from the following basic case: $f \in (t \to t)$

- f is said to be **veridical** iff $[\![f(x)]\!] = 1$ entails $[\![x]\!] = 1$ (e.g. 'yesterday');
- f is said to be **non-veridical** iff $[\![f(x)]\!] = 1$ does not entail $[\![x]\!] = 1$ (e.g. 'usually');
- f is said to be **anti-veridical** iff $[\![f(x)]\!] = 1$ entails $[\![x]\!] = 0$ (e.g. 'It is not the case').

Note, AV functions form a proper subset of the NV onces, $\mathsf{AV} \subset \mathsf{NV}$

8. Polarity items typology

Based on these distinctions of (non-)veridical contexts, PIs have been classified as follow:

- **positive polarity items** (PPIs) can occur in veridical contexts (V) ('some N');
- **affective polarity items** (APIs) cannot occur in V, i.e. they must occur in non-veridical contexts (NV), (e.g. 'any N');
- **negative polarity items** (NPIs) cannot occur in NV, i.e. they must occur in anti-veridical contexts (AV) (e.g. 'say a word').

In type logic terms this means that

$$\begin{array}{ll} \mathrm{AV} \circ \Delta\lceil \mathrm{NPI} \rceil & {}^{*}\mathrm{NV} \circ \Delta\lceil \mathrm{NPI} \rceil, \\ \mathrm{AV} \circ \Delta\lceil \mathrm{API} \rceil & \mathrm{NV} \circ \Delta\lceil \mathrm{API} \rceil\ , \\ {}^{*}\mathrm{V} \circ \Delta\lceil \mathrm{NPI} \rceil & {}^{*}\mathrm{V} \circ \Delta\lceil \mathrm{API} \rceil. \end{array}$$

where $\circ$ is the composition operator, $\Delta\lceil X \rceil$ means that X is in the structure Δ and has wide scope in it, and * marks ungrammatical composition.

9. Types for PIs and their licensors

The needed types are;

$$\text{AV} \in A/npi \quad \text{NV} \in A/api, \quad \text{V} \in A/ppi$$

$$api \longrightarrow npi \qquad npi \not\longrightarrow ppi \qquad api \not\longrightarrow ppi.$$

A concrete example

'Yesterday', 'usually' and 'it is not the case' are all denoted in the domain $D_t^{D_t}$, hence their (syntatic) category is s/s. However,

1. (a) *Yesterday I spoke with anybody I met. $\quad$ $*\text{V} \circ \Delta\lceil\text{API}\rceil$
   (b) *Yesterday I said a word. $\quad$ $*\text{V} \circ \Delta\lceil\text{NPI}\rceil$
2. (a) **Usually** I speak with anybody I meet. $\quad$ $\text{NV} \circ \Delta\lceil\text{API}\rceil$
   (b) *Usually I say a word. $\quad$ $*\text{NV} \circ \Delta\lceil\text{NPI}\rceil$

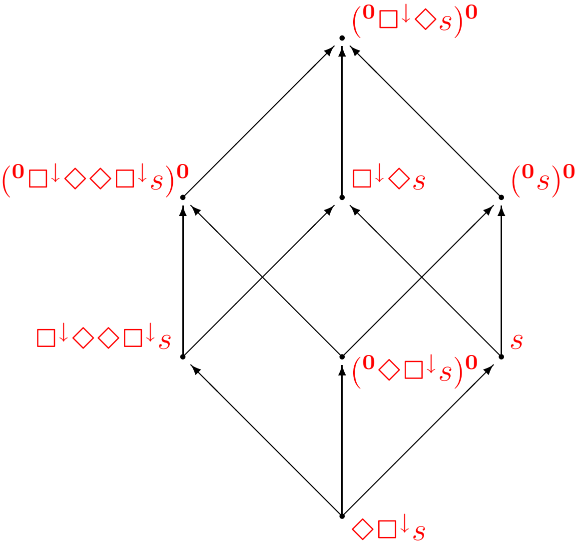
It is not...	$\in s/({}^{\mathbf{0}}s)^{\mathbf{0}}$	(AV)
Usually	$\in s/({}^{\mathbf{0}}(\Diamond\Box^{\downarrow}s))^{\mathbf{0}}$	(NV)
Yesterday	$\in s/\Box^{\downarrow}\Diamond s$	(V)

where $api : ({}^{\mathbf{0}}(\Diamond\Box^{\downarrow}s))^{\mathbf{0}} \longrightarrow npi : ({}^{\mathbf{0}}s)^{\mathbf{0}}$, $ppi : \Box^{\downarrow}\Diamond s$. Note, $AV \longrightarrow NV$.

10. Reflection: Curry-Howard Correspondence

- **Fact** Lambek calculus is in a Curry-Howard correspondence with (a fragment of) typed lambda calculus. The latter is based purely on functional application and the language can represent either atomic or functional expressions.
- **Observation** The syntatic behavior of some linguistic phenomena is influenced by semantic properties, which cannot be accounted for simply by means of functional applications. Unary operators seem to provide the right expressivity, distinguishing functions denoted in domains which are connected by subset relations.
- **Question** Should the syntatic types classification have any effect on the semantic representation, and if so which are the proper interpretations of the used unary operators?

11. Options for cross-linguistic variation



12. Greek (I)

NPI: **ipe leksi**, PI: **kanenan**, FCI: **opjondhipote**

1.	**Dhen** idha kanenan. (tr. I didn't see anybody)	Neg > PI
2.	**Dhen** ipe leksi oli mera (tr. He didn't say a word all day)	Neg > NPI
3.	*Dhen idha opjondhipote (tr. I didn't see anybody)	*Neg > FCI
4.	Opjosdhipote fititis **bori** na lisi afto to provlima. (tr. Any student can solve this problem.)	Modal > FCI
5.	**An** dhis tin Elena [puthena/optudhipote], . . . (tr. If you see Elena anywhere, . . .)	Cond > PI/FCI
6.	**An** pis leksi tha se skotoso. (tr. If you say a word, I will kill you)	Cond > NPI

13. Greek (II)

The data presented above can be summarized as follows:

Greek	FCI	PI	NPI
Veridical	*	*	*
Negation	*	Yes	Yes
Modal verb	Yes	Yes	*
Conditional	Yes	Yes	Yes

Lexicon

PPI: $q(np, s_4, s_4)$, **kapjos**
PI: $q(np, s'_1, s'_1)$, **kanenan**
modal: $(((s'_4/np)\backslash s'_4)\backslash s_1)/(np\backslash s'_4)$, **bori**
cond.: $(s_1/s'_1)/s'_3$, **an**

NPI: $np\backslash s'_2$, **ipe leksi**
FCI: $q(np, s'_4, s'_4)$, **optudhipote**
neg.: $(np\backslash s_1)/(np\backslash s'_2)$, **dhen**

14. Italian (I)

NPI: **nessuno**, PI: **mai**, FCI: **chiunque**

1.	**Non** gioco mai (tr. I don't play ever)	Neg > PI
2.	**Non** ho visto nessuno (tr. I haven't seen anybody)	Neg > NPI
3.	***Non** ho visto chiunque (tr. I haven't seen anybody)	*Neg > FCI
4.	Chiunque **puó** risolvere questo problema (tr. Anybody can solve this problem)	Modal > FCI
5.	*Puoi giocare mai (tr. You can play ever)	*Modal > PI
6.	*Puoi prendere in prestito nessun libro (tr. You can borrow any book)	*Modal > NPI
7.	**Se** verrai mai a trovarmi, . . . (tr. If you ever come to visit me, . . .)	Cond > PI

15. Italian (II)

The data presented above can be summarized as follows:

Italian	FCI	PI	NPI
Veridical	*	*	*
Negation	*	Yes	Yes
Modal verb	Yes	*	*
Conditional	*	Yes	*

Lexicon

PPI: $q(np, s_4, s_4)$, **qualcuno**
PI: $(np\backslash s_1)\backslash(np\backslash s_1')$, **mai**
modal: $(((s_4''/np)\backslash s_4'')\backslash s_1)/(np\backslash s_4'')$, **puó**
cond: $(s_1/s_1')/s_4'$, **se**

NPI: $q(np, s_2', s_2')$, **nessuno**
FCI: $q(np, s_4'', s_4'')$, **chiunque**
neg.: $(np\backslash s_1)/(np\backslash s_2')$, **non**

□

16. The point up till now

These two examples show that the type hierarchy given by Galois and residuated unary operators

- helps carry out cross-linguistic analysis;
- predicts the existence of non-veridical contexts which do not license polarity items: The two non-veridical levels $(^{\mathbf{0}}(\cdot^{\mathbf{0}}), (^{\mathbf{0}}\cdot)^{\mathbf{0}})$, express syntactically different items, which have the same semantic interpretation, e.g. 'possibly' is non-veridical but behaves differently from other non-veridical contexts, viz. does not license API.
- predicts the existence of some contexts shared by (negative) polarity items and positive ones.

17. Connection with DMG

Non veridical (and therefore also anti-veridical) sentences do not allow anaphoric links. Veridical ones do.

1. This house have a bathtub.
   (a) It is upstairs.
2. This house **does not** have a bathtub.
   (a) *It is upstairs.
   (b) *It **might/could/should** be upstairs.
3. This house **might/could/should** have a bathtub.
   (a) *It's green.
   (b) It **might/could/should** be green.
4. This house **allegedly/possibly** has a bathtub.
   (a) *It's green.
   (b) It is **allegedly/possibly** green.

However, while AV contexts (2) close anaphoric links permantely, NV do not.

18. Conjecture and Questions

Conjecture

- If an expression is in the scope of ${}^{\mathbf{0}}(\cdot^{\mathbf{0}})$ (or $({}^{\mathbf{0}}\cdot)^{\mathbf{0}}$) it is closed;
- if it is in the scope of $\Box^{\downarrow}\Diamond\cdot$ anaphoric links are allowed.

Translating this into dynamic Montague grammar terms:

$$\begin{array}{lll} \Box^{\downarrow}\Diamond & \rightsquigarrow & \uparrow & \text{where } \uparrow\phi =_{def} \lambda p.(\phi \wedge {}^{\vee}p) \\ {}^{\mathbf{0}}(\cdot^{\mathbf{0}}), \text{ (and } ({}^{\mathbf{0}}\cdot)^{\mathbf{0}}) & \rightsquigarrow & \downarrow & \text{where } \downarrow\psi =_{def} \psi({}^{\wedge}\mathsf{true}) \end{array}$$

Questions

- Can the connection with DMG help understanding the semantics of $({}^{\mathbf{0}}\cdot, \cdot^{\mathbf{0}})$?
- Is there any logic connection between Galois and non-veridicality vs. residuation and veridicality?

19. Conclusions

We have shown that

- categorial type logic can contribute to the study of linguistic typologies. More precisely, **unary operators** can be used
  - to account for linguistic typologies encoding the **subset relations** among items of the same syntactic category, and
  - to account for **cross-linguistic** differences.
- the derivability patterns which characterize Galois connected and residuated operators give a proper typology of PIs and show new directions for linguistic investigation;
- on the other hand, the linguistic application considered opens the way to further logic research, sheding light on new connections between dynamic Montague grammar and categorial type logic.