Polarity in the Base Logic

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□ 1. Introduction

- In natural languages there exist phenomena depending on semantic motivations for grammaticality.
- A system employed as parser of linguistic strings must be able to take semantic information into consideration when working with such expressions.
- Polarity Items are an example of this class of phenomena: Their syntactic distribution depends on semantic properties of their licenser.
- The semantic information used for parsing is relevant when reasoning on the parsed sentences as well.

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Aim: To build a system able to: (1) encode the required semantic information, (2) take advantage of this information while parsing, and (3) formalize natural reasoning inferences.

□ 2. Polarity Items

Definition: Polarity Items

a. *Positive Polarity Items* (PPIs) are expressions licensed by monotone increasing functions.

b. *Negative Polarity Items* (NPIs) are expressions licensed by monotone decreasing functions.

Definition: Monotone Functions

a. f is *monotone increasing* (not. $\uparrow$Mon) iff $\forall x, y \in A$, $x \leq_A y$ implies $f(x) \leq_B f(y)$.

b. f is *monotone decreasing* (not. $\downarrow$Mon) iff $\forall x, y \in A$ $x \leq_A y$ implies $f(y) \leq_B f(x)$.

Linguistic Data

$\downarrow$**Nobody** left **yet**$^{\downarrow}$ $\uparrow$*Somebody* left *something*$^{\uparrow}$

$(*)$ $\uparrow$*Somebody* left **yet**$^{\downarrow}$ $(*)$ $\downarrow$**Nobody** left *something*$^{\uparrow}$

□ 3. Monotonic Inferences

Monotonicity is tied up to natural reasoning.

$$\frac{\text{Somebody (left something expensive)}}{\text{Somebody (left something)}} \text{ (A)}$$

$$\frac{\text{Nobody (left yet)}}{\text{Nobody (left in a hurry yet)}} \text{ (B)}$$

A substitution of an expression with something more (A) or less (B) general could be done in any position in a sentence.

If $P \leq Q$

$$\frac{N[P]}{N[Q]} \text{ (A)} \quad \text{or} \quad \frac{N[Q]}{N[P]} \text{ (B)}$$

In order to know which pattern must be applied, we need to know the polarity of the position in which the expression occurs.

□ 4. Monotonicity and Polarity

Polarity Positions:

- First order logic: Lyndon defines polarity in terms of the number of negations surrounding a subformula;

  But FOL doesn't captures the compositional behaviour needed for formalizing natural language. We need typed lambda calculus.
- Typed lambda calculus: van Benthem defines polarity in terms of functional application, i.e. number of monotone decreasing functions having scope on a subterm.

In a slogan:

Monotoncity	vs.	**Polarity**
dynamic	vs.	static
semantic	vs.	syntactic
argument positions	vs.	all positions

□ 5. The Picture up to Now

A system able to

(1) encode monotonicity markers,

(2) compute polarity positions, and

(3) use this information during parsing and inference

can account for linguistic phenomena depending on monotonic semantic properties for grammaticality and formalize (a fragment of) natural reasoning.

□ 6. Sánchez' Natural Logic

- Aim: To give a proof theoretic formalization of natural reasoning drawing inferences from parsed linguistic expressions.
- Method: Enriching the logical types of the Lambek calculus (LP) with monotonicity markers and defining monotonicity and polarity algorithms which takes an LP derivation and compute the polarity of each of its nodes.

□ 7. Parsing with Polarity

LP+Pol

$$\frac{\text{every} \in p \to ((e \to t) \to t) \qquad \text{logician} \in p}{(e \to t) \to t} \qquad \frac{\text{wanders} \in e \to t \qquad \dfrac{\text{not} \in t \to t \qquad t}{t}}{t}$$

□ 8. What has to be Fixed?

- Disadvantages:
  - Using LP as parser leads to over generation;
  - The marking algorithms are external to the parser.

    Hence, polarity markers do not play an active role in the parsing and cannot be used to account for PIs. (Even though all needed information is available in the nodes of the derivation!!)
- Advantages:
  - The polarity marking algorithm is sound.

    By interpreting proofs as lambda terms
    (1) marked functional nodes will correctly correspond to monotonic functions; and
    (2) polarity markers will correctly correspond to polarity positions in lambda terms.

□ 9. Internalizing Markers
MCTL

We can use the logical unary operators of Multimodal Categorial Type Logic (MCTL) to carry the markers, and structural unary operators to record their effect.

Translation of logical types:
$ltr : \mathcal{L}(\mathsf{LP{+}Pol}) \rightarrow \mathcal{L}(\mathsf{MCTL})$

$$\begin{aligned} ltr((\psi_1)^s \rightarrow \psi_2) &= \boxplus^{\downarrow}(ltr(\psi_2) / \diamondsuit_s ltr(\psi_1)); \\ ltr(e) &= \boxplus^{\downarrow} np; \\ ltr(p) &= \boxplus^{\downarrow} n; \\ ltr(t) &= \boxplus^{\downarrow} s. \end{aligned}$$

Translation of nodes: *Nodes* $\rightarrow$ *Structures*

$$\frac{\begin{array}{cc} \text{every} & \text{logician} \\ + & - \end{array}}{\begin{array}{cc} \text{every} & \text{logician} \\ & + \end{array}} \rightsquigarrow \frac{\langle \text{every} \rangle^{+} \qquad \langle \langle \text{logician } \rangle^{+} \rangle^{-}}{\langle \langle \text{every} \rangle^{+} \circ \langle \langle \text{logician } \rangle^{+} \rangle^{-} \rangle^{+}}$$

□ 10. Internalizing Marker Propagation
MCTL

To simulate LP+Pol we have to *extend* the multimodal basic logic (NL($\Diamond$)) with (1) Associativity and Permutation; (2) polarity structural rules [Pol_s^+], [Pol_{uv}] and [Esc]; and (3) we have to modify [/E] and [/I]:

Distribution

$$\frac{\Gamma[\langle \Delta_1 \circ \Delta_2 \rangle^s] \vdash A}{\Gamma[\langle \langle \Delta_1 \rangle^s \circ \langle \Delta_2 \rangle^s \rangle^+] \vdash A}\ [\text{Pol}_s^+]$$

Computation

$$\frac{\Gamma[\langle \langle \Delta \rangle^u \rangle^v] \vdash A}{\Gamma[\langle \Delta \rangle^s] \vdash A}\ [\text{Pol}_{u,v}]$$

$$\frac{\Gamma[\langle \Delta \circ \langle x \rangle^s \rangle^+] \vdash A}{\Gamma[\langle \Delta \rangle^+ \circ \langle x \rangle^s] \vdash A}\ [\text{Esc}]$$

$$\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\langle \Gamma \circ \Delta \rangle^+ \vdash A}\ [/\text{E}^+] \qquad \frac{\begin{array}{c} x \vdash B \\ \vdots \\ \Gamma \circ x \vdash A \end{array}}{\langle \Gamma \rangle^+ \vdash A/B}\ [/\text{I}^+]$$

□ 11. How it Works

Summing up, monotonicity and polarity are displayed as follows:

To express that:	type
A function is s-monotone	$B/\Diamond_s A$
An item must have an s polarity	$\boxdot_s^{\downarrow} A$
A structure has polarity s	$\langle\cdot\rangle^s$

- Lexical items are initially marked $\boxdot_s^{\downarrow} A$ and they will pass this information to the structure in the first inference step:

$$\frac{\mathsf{LEX} \vdash \boxplus^{\downarrow} A}{\langle \mathsf{LEX} \rangle^{+} \vdash A}\ [\boxplus^{\downarrow}\mathrm{E}]$$

- Application of a monotone function implies the propagation of the marker from the function to the argument:

$$\frac{\dfrac{\Delta \vdash B}{\langle \Delta \rangle^s \vdash \Diamond_s B}\ [\Diamond_s \mathrm{I}] \qquad \Gamma \vdash \Diamond_s B \backslash A}{\langle \langle \Delta \rangle^s \circ \Gamma \rangle^{+} \vdash A}\ [\backslash \mathrm{E}^{+}]$$

- The polarity is computed "on-demand" by means of the polarity structural rules.

□ 12. What we have Learned

- We can simulate LP+Pol in MCTL. Not all the markers contributes to computing the polarity. The only relevant markers are the $\boxplus^{\downarrow}$ on the lexical entries and the $\diamondsuit_s$ on the argument position; the rest can be dropped.
- There is no need of marking each node with a polarity marker. Hence there is no need of modifying [/E], [/I] and adding [Esc];
- Associativity and Permutation do not affect the polarity assigned to the substructures involved in their applications. Their use is independent from polarity computation.

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NL($\diamondsuit$) with [Pol$_s$] and [Pol$_{uv}$] is the basic fragment we need to compute structural polarity. It does (1) encode monotoncity markers, (2) compute polarity positions, and (3) take advantage of this information whenever needed.

□ 13. Polarity Items

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□ 14. Soundness
What we know

In order to guarantee the soundness of the system we have to use the unary operators $\diamondsuit_S$, $\boxdot_S^{\downarrow}$ as explained above. The set of formulas below satisfies this requirement:

$\mathsf{SAFE} = \{\mathsf{FORM}_1, \mathsf{FORM}_3, \mathsf{FORM}_4\}$

where:

$$
\begin{aligned}
\mathsf{FORM}_1 = \quad & \boxplus^{\downarrow}\mathsf{ATOM} \mid \boxplus^{\downarrow}(\mathsf{FORM}_2/\mathsf{FORM}_3) \mid \\
& \boxplus^{\downarrow}(\mathsf{FORM}_3\backslash\mathsf{FORM}_2) \mid \boxdot_S^{\downarrow}\diamondsuit_S\mathsf{FORM}_4 \mid \\
& \boxplus^{\downarrow}(\boxdot_S^{\downarrow}\diamondsuit_S\mathsf{FORM}_2/\boxdot_S^{\downarrow}\diamondsuit_S\mathsf{FORM}_3) \mid \\
& \boxplus^{\downarrow}(\boxdot_S^{\downarrow}\diamondsuit_S\mathsf{FORM}_3\backslash\boxdot_S^{\downarrow}\diamondsuit_S\mathsf{FORM}_2); \\
\mathsf{FORM}_2 = \quad & \mathsf{FORM}_4 \mid \boxdot_S^{\downarrow}\diamondsuit_S\mathsf{FORM}_4; \\
\mathsf{FORM}_3 = \quad & \diamondsuit_S\mathsf{FORM}_4; \\
\mathsf{FORM}_4 = \quad & \mathsf{ATOM} \mid \mathsf{FORM}_2/\mathsf{FORM}_3 \mid \mathsf{FORM}_3\backslash\mathsf{FORM}_2
\end{aligned}
$$

To be proved: A proof in $\mathsf{NL}(\diamondsuit)$ of a safe formula contains only safe types.

□ 15. Soundness

How we want to proceed

- **Fact**: NL(◇) proofs can be translated into LP when forgetting the unary operators.
- **Aim**: Polarity markers in NL(◇)+Pol could be proved to be sound using the the soundness result for LP+Pol markers.

Needed ingredients:

(1) Structural negative markers $\langle\cdot\rangle^-$ are "introduced" only when applying a type $A/\Diamond B$ (or $\Diamond B\backslash A$);

(2) Given a proof D in NL(◇)+Pol of $\Gamma \vdash A$ for any substructure Γ_i of Γ, if Γ_i has positive (negative) polarity, Γ_i in D is in the scope of an even (odd) number of $A/\Diamond B$ (or $\Diamond B\backslash A$).

(3) Let D be a proof in NL(◇)+Pol, D' the corresponding proof in LP and D^m the proof D' decorated with monotonicity markers. Each node in D of type $A/\Diamond B$ (or $\Diamond B\backslash A$) has a corresponding node of type $ltr(B)^- \to ltr(A)$ in D^m.

□ 16. Further Research

Further research can be done on both the parsing and natural reasoning level of the described system, and on the interplay of the two.

(1) Parsing:

   i. So far we have been working with a free-product system. Which formulas built by means of the • are safe?

   ii. The standard lifting property does not account for monotonicity information. Which is the proper reformulation?

(2) Reasoning:

   i. The Natural Logic patterns of monotonic reasoning remind the weakening rule. Which is the precise connection?

(3) Parsing and reasoning.

   i. In the actual system, parsing and reasoning are not integrated. How can the two levels interact?

   ii. Some PIs can be licensed by "the meaning conveyed" by the sentence they occur in. Can an integrated system account for such PIs?