Non-Classical Logics for Natural Language: Introduction to Substructural Logics

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Contents

1. Course Overview

- **Today**:

  1. `(Formal) Linguistic Background`. First of all, we introduce the concept of Formal Grammar for Natural Languages by presenting some linguistic background and the **challenges** such a grammar should face. Furthermore, we motivate the use of a **Logical Grammar** to address such challenges.
  2. `Introduction to Substructural Logics`. First of all, we will introduce **non-classical logics** by underlining the differences with respect to classical logics. Then we move to introduce **substructural logics** and we will briefly look at some well known of them – Relevant Logics, Linear Logic... Then, we will focus attention on a sub-family of Substructural Logics known as **Lambek Calculi** or Logics of Residuation.

- **Tomorrow**:

  1. `Lambek Calculus`. We start presenting both the Model Theoretical and Proof Theoretical aspects of the Lambek Calculi. Then, we look at its application to natural language parsing.
  2. `Syntax-Semantics Interface.` In the second part, we consider the application of the Lambek Calculi and the Lambda calculus to natural language analysis. We start by introducing some background notions of **Formal Semantics** focusing on the set-theoretical perspective first, and then on the corresponding functional perspective. Then we will show how the Lambek Calculi account for the composition of linguistic resources while simultaneously allowing parsing and the construction of meaning.

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- understand how semantic structures are built
- model **syntax-semantic** interface
- find the universal **core** of all natural languages
- find natural language variations

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- **Dependency**: Categories are interdependent, for example

Ryanair services $[\text{Pescara}]_{np}$	Ryanair flies $[\text{to Pescara}]_{pp}$
*Ryanair services $[\text{to Pescara}]_{pp}$	*Ryanair flies $[\text{Pescara}]_{np}$

  the verbs services and flies determine which category can/must be juxtaposed. If their constraints are not satisfied the structure is ungrammatical.

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2.3. Relative Pronouns and Coordination

- **Relative Pronoun** (eg. who, which): they function as e.g. the subject or object of the verb embedded in the relative clause (*rc*),
  - [[the [student [who [. . .] knows Sara]$_{rc}$]$_{n}$]$_{np}$ [left]$_{v}$]$_{s}$.

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  - I [[[want to try to write [. . .]] and [hope to see produced [. . .]]] [the movie]$_{np}$]$_{vp}$"

  Again, the interdependent constituents are disconnected from each other.

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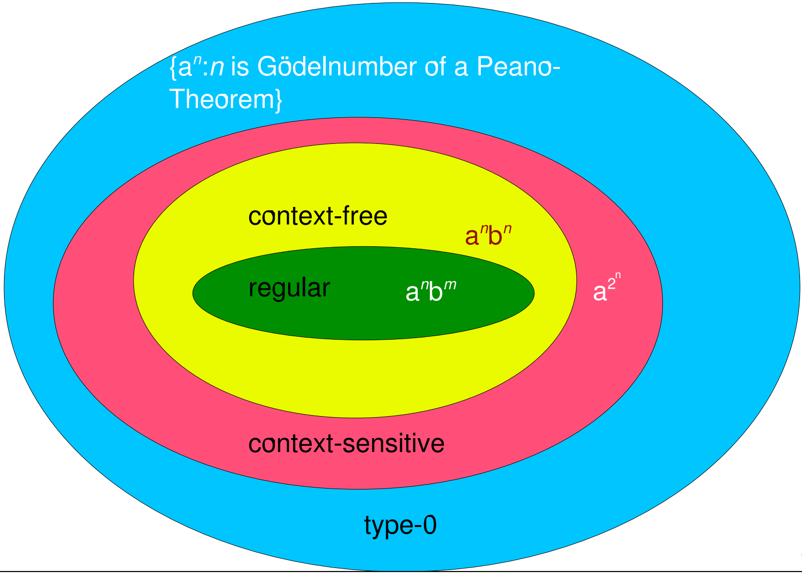
We look at natural language as a formal language and use formal grammars to achieve these goals.

3.1. Chomsky Hierarchy of Languages

The Chomsky Hierarchy



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  5. Joshi (1985) NLs are Mildly Context-sensitive Languages.

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Well known formal grammars are Phrase Structure Grammars (PSG).

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- $s \longrightarrow^* \text{Sara wears the new dress}$
  —which is in the language by repeated applications of rules.

$adj \longrightarrow \text{new}$

$adj \longrightarrow$ new

adj
|
new

$adj \longrightarrow$ new

adj
|
new

$n \longrightarrow$ new

$adj \longrightarrow$ new

```
 adj
  |
 new
```

$n \longrightarrow$ new

```
   n
   |
 dress
```

$adj \rightarrow$ new

adj
|
new

$n \rightarrow$ new

n
|
dress

n $\rightarrow$ adj n

$adj \longrightarrow$ new

$n \longrightarrow$ new

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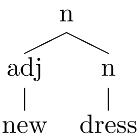

adj → new

n → new

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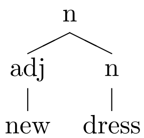

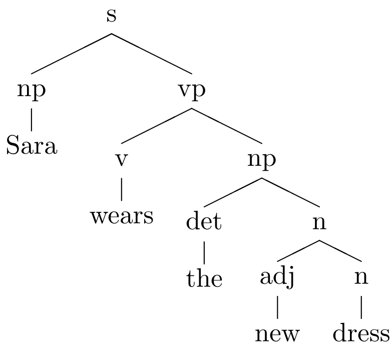

[Sara[wears[the[new dress]$_{n}$]$_{np}$]$_{vp}$]$_{s}$

3.3.3. PSG: Advantages and Disadvantages Advantages

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- PSG as such don't handle long-distance dependencies, since there is no connection among categories occurring in different rewriting rules.

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6. Classical Logic vs. Non-Classical Logics

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Non-classical logics are are either weaker or stronger than classical logics.

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  - Associativity of conjunction $(A \wedge B) \wedge C \Leftrightarrow A \wedge (B \wedge C)$
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    natural language phrases.
  - Examples of modal operators: Possibility and Necessity in the future, $\Diamond$, $\Box$, respectively, or Possibility and Necessity in the past, ⧫, ■

7. Proof Theory: Gentzen Sequent Calculus

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- The logical rules are divided into:
  - **Right rules**: rules introducing the logical constants on the right of the turnstile $(\vdash)$ (instead of introduction rules of ND), and
  - **Left rules**: rules introducing the constants on the left (instead of the elimination rules ND).

7.1. Classical Logic: Logical Rules

$$A, \Gamma \vdash A, \Delta$$

$$\frac{A, B, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash \Delta, A \quad \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \wedge B}\ (\wedge R)$$

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The , stands for "and" when occurring on the right of the turnstile ($\vdash$) and for "or" when occurring on its left. Think of **Tableaux** ($\mathrm{T} \vdash \mathrm{F}$ —axioms contradictions.)

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Hidden in Classical Logic there are the following structural rules.

Weakening	$\dfrac{\Lambda \vdash \Sigma}{B, \Lambda \vdash \Sigma}$	$\dfrac{\Lambda \vdash \Sigma}{\Lambda \vdash B, \Sigma}$	Axiom: $\Gamma, A \vdash \Delta, A$
Contraction	$\dfrac{A, A, \Gamma \vdash \Sigma}{A, \Gamma \vdash \Sigma}$	$\dfrac{\Gamma \vdash A, A, \Sigma}{\Gamma \vdash A, \Sigma}$	structures are sets: nr. does not count
Permutation	$\dfrac{A, B, \Gamma \vdash \Sigma}{B, A, \Gamma \vdash \Sigma}$	$\dfrac{\Gamma \vdash A, B, \Sigma}{\Gamma \vdash B, A, \Sigma}$	structures are sets: order doesn't count

Furthermore, the comma is associative: $(A, (B, C)) = ((A, B), C)$.

8. Conditional and Logical Consequence

In particular, the separation of structural rules from logical rules helped highlighting the **crucial role** played by conditional and the residuation **condition below** that captures the tied connection between conditional and logical consequence, i.e. the core of logic.

$$p, q \vdash r \quad \text{iff} \quad p \vdash q \rightarrow r$$

It says that r follows from p together with q just when $q \rightarrow r$ follows from p alone.

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$$p, q \vdash r \text{ iff } p \vdash q \to r$$

It says that r follows from p together with q just when $q \to r$ follows from p alone.

However, there is one extra factor in the equation. Not only is there the **turnstile** ($\vdash$), for logical consequence, and the **conditional** ($\to$), encoding consequence inside the language of propositions, there is also the **comma**, indicating the combination of premises. The behaviour of premise combination is also important in determining the behaviour of the conditional.

As the comma's behaviour varies, so does the conditional.

9. The "comma"

The comma's behavior varies accordingly to the structural rules allowed. Hence the latter play an important role.

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- **fine tune logics** on the base of the object of investigation.

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They are logics sensitive to the number and order of occurrence of assumptions. By this reason, they are sometimes called "**Resource Sensitive Logics**".

In this light, structures are not seen anymore as sets but, for instance, as lists so to distinguish the order and quantity of formulas occurrences.

11.1. Substructural Logics: Examples

The most well-known substructural logics are:

1. Relevant Logic: rejects Weakening.
2. BCK logic: rejects Contraction.
3. Linear logic: rejects Weakening and Contraction.
4. Lambek Calculus: rejects all the substructural rules of Classical Logic.

11.1.1. Relevant Logics: Intuition

Aim Understand the notions of consequence and conditionality where the **conclusion of a valid argument is relevant to the premises**, and where the consequent of a true conditionals is relevant to the antecedent.

We want "**if** A **then** B" to mean that "B follows from A", i.e. we used A in the deduction of B, hence A **is relevant to derive** B. For instance, we do not want that from false everything falls:

> "The moon is made of green cheese. Therefore, either it is raining in Ecuador now or it is not."

this argument commits "fallacies of relevance".

Similarly, we do not want that everything follows from contradiction:

> "It is raining in Ecuador and it is not raining. Therefore, I am teaching in Australia."

Relevant Logic: Rejected Tautologies The following formulas are tautologies of Classical Logic and Intuitionistic Logic:

$$A \to (B \to B) \quad \neg(B \to B) \to A$$

there is no such connection of relevance: The consequent of the main conditional needs not have anything to do with the antecedent.

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Similarly, in Classical Logic the following formula holds:

$$B \to (\neg B \to A)$$

Relevant Logics: Rejected Structural Rule Proofs:

$$\dfrac{\dfrac{\dfrac{B \vdash B}{A, B \vdash B}\ (\text{Weak})}{A \vdash B \rightarrow B}\ (\rightarrow \mathrm{R})}{\vdash A \rightarrow (B \rightarrow B)}\ (\rightarrow \mathrm{R})$$

$$\dfrac{\dfrac{\dfrac{\dfrac{B \vdash B}{B \vdash A, B}\ (\text{Weak})}{B, \neg B \vdash A}\ (\neg \mathrm{L})}{B \vdash \neg B \rightarrow A}\ (\rightarrow \mathrm{R})}{\vdash B \rightarrow (\neg B \rightarrow A)}\ (\rightarrow \mathrm{R})$$

The proofs are based on weakening. Hence, this structural rule is dropped and axioms are of the form $A \vdash A$, "irrelevant" information cannot be brought in the proof.

Notice, rejecting $B \rightarrow (\neg B \rightarrow A)$ as tautology reduces to reject $\neg(B \wedge \neg B)$, too.

11.1.2. Linear Logic: Intuition

Aim understand and model processes and **resource use**. The idea in this account of deduction is that resources must be used (so premise combination satisfies the relevance criterion) and they do not extend indefinitely. Premises cannot be re-used.

Hence, we want “**if** A **then** B” to mean that **the resource** B **is produced if** A **is consumed**.

In Linear logic, duplicated hypotheses ’count’ differently from single occurrences

> I have a quart of milk from which I can make a pound of butter. If I decide to make butter out of all of my milk, I cannot then conclude that I have both milk and butter!

Linear Logic: Rejected Structural Rules For instance, the proof of the argument below shows that Classical logic is "insensitive" to resources.

$$\{p, p \rightarrow q, p \rightarrow r, (q \wedge r) \rightarrow t, s\} \vdash t$$

s is not used to prove t, p is used twice.

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Hence, in Linear Logic both Contraction and Weakening are dropped:

- The rejection of Contraction means that resources cannot be duplicated (see the p.)
- The rejection of Weakening means that resources cannot be thrown away (see the s.)

Structures are seen as **multisets** (the order of occurrence doesn't count, the quantity does).

Linear Logic: Structural Control Contraction and Weakening are reintroduced by marking formulas for which they can hold with unary operators "!" and "?". For instance,

$$\frac{X \vdash Y}{X, !A \vdash Y} \quad \frac{X \vdash Y}{X \vdash ?A, Y}$$

The use of these "markers" introduces the idea of structural rules that are **controlled rather then globally available**.

11.1.3. Lambek Calculus: Intuition

Aim to model natural language assembly, capture the core of natural languages and their diversity.

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- the multiplicity of linguistics material is important, since linguistic elements must generally be used once and only once during an analysis. Thus, we cannot ignore or waste linguistic material (a), nor can we indiscriminately duplicate it (b).

  a) *The coach smiled the ball. ≠ The coach smiled.

  b) *The coach smiled smiled. ≠ The coach smiled.

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  a) *The coach smiled the ball. $\neq$ The coach smiled.

  b) *The coach smiled smiled. $\neq$ The coach smiled.

- natural language structures are neither commutative (c) nor associative (d)

  c) $[[[\text{the}]_{art}\ [\text{coach}]_n]_{np}\ [\text{smiled}]_v]_s \neq^* [[\text{smiled}]_v[[\text{the}]_{art}[\text{coach}]_v]_{np}]_s$

  d) $[[[\text{the}]_{art}\ [\text{coach}]_n]_{np}\ [\text{smiled}]_v]_s \neq [[\text{the}]_{art}\ [[\text{coach}]_n\ [\text{smiled}]_v]]_s$

- No contraction, no weakening, no associativity, no commutativity. Hence,

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- Implication is split into:
  - right implication ($A\backslash B$ — if A then B $A \rightarrow B$)
  - left implication (B/A — B if A, $B \leftarrow A$)

$$\dfrac{\dfrac{\dfrac{B \vdash B \quad A \vdash A}{A/B, B \vdash A}\,(/L)}{B, A/B \vdash A}\,(\text{Exc})}{A/B \vdash B\backslash A}\,(\backslash R)$$

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- structures are lists
- conjunction is seen as fusion
- And no negation.

Lambek Calculus: Structural Control As in other domains, in Linguistic as well, there is the need of locally controlling structural reasoning and account for the different compositional relations linguistic phenomena may exhibit. For instance,

- E.g. Adjectives: "Vestito nuovo" vs. "Nuovo vestito." Need to some sort of commutativity.

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But differently from linear logic, the control is expressed by means of unary or binary logical operators living within the same algebraic structure (residuation).

11.2. Remarks: Residuation

Let $\langle A, \leq_1 \rangle$ and $\langle B, \leq_2 \rangle$ be two partially ordered sets. A pair of functions (f, g) such that $f : A \to B$ and $g : B \to A$ forms a residuated pair if

$$[RES_1] \quad \forall x \in A, y \in B \left(\begin{array}{c} f(x) \leq_2 y \\ \text{iff} \\ x \leq_1 g(y) \end{array} \right)$$

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$$[RES_1] \quad \forall x \in A, y \in B \begin{pmatrix} f(x) \leq_2 y \\ \text{iff} \\ x \leq_1 g(y) \end{pmatrix}$$

Let $\langle C, \leq_3 \rangle$ be a third partially ordered set, a triple of functions (f, g, h) such that $f : A \times B \to C$, $g : A \times C \to B$, $h : C \times B \to A$ forms a residuated triple if

$$[RES_2] \quad \forall x \in A, y \in B, z \in C \begin{pmatrix} x \leq_1 h(z, y) \\ \text{iff} \\ f(x, y) \leq_3 z \\ \text{iff} \\ y \leq_2 g(x, z) \end{pmatrix}$$

Similarly, one could have dual-residuated operators where h and g are in the right side of $\leq$ and f on its right.

11.2.1. Residuation: Intuition

For instance, in Maths:

$$[RES_2] \quad \begin{pmatrix} 2 \leq \frac{9}{4} \\ \text{iff} \\ 2 \times 4 \leq 9 \\ \text{iff} \\ 4 \leq \frac{9}{2} \end{pmatrix}$$

$$p, q \vdash r \quad \text{iff} \quad p \vdash q \longrightarrow r$$

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$$p, q \vdash r \quad \text{iff} \quad p \vdash q \longrightarrow r$$

For instance, in linguistics:

$$[RES_2] \quad \begin{pmatrix} np : mary \leq \frac{s}{iv:walks} \\ \text{iff} \\ np : mary \times iv : walks \leq s \\ \text{iff} \\ walks : iv \leq \frac{s}{np:mary} \end{pmatrix}$$

11.2.2. Residuation: Tonicity and Composition

Saying that (f, g) is a residuated pair is equivalent to the conditions $i)$ and $ii)$,

$i)$ Tonicity: $f(+)$ and $g(+)$.

they preserve the order of their arguments, i.e. $f(x) \leq f(y)$ if $x \leq y$.

$$ii)\text{Composition}: \quad \forall y \in B, x \in A \begin{pmatrix} f(g(y)) \leq_2 y \\ \text{and} \\ x \leq_1 g(f(x)) \end{pmatrix}$$

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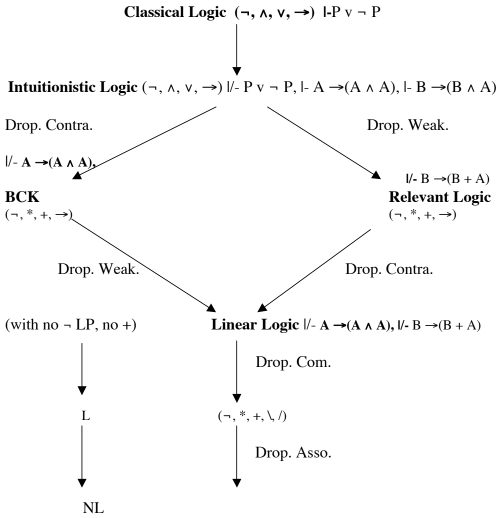
Similarly, saying that (f, g, h) is a residuated triple is equivalent to requiring

i)Tonicity: $f(+,+), g(-,+)$ and $h(+,-)$

where $-$ means, it reverses the order of its argument, i.e. $g(y, z) \leq g(x, z)$ if$x \leq y$.

$$ii)\text{Composition}: \quad \forall x \in A, y \in B, z \in C \begin{pmatrix} f(x, g(x, z)) \leq_3 z \\ \text{and} \\ y \leq_2 g(x, f(x, y)) \\ \text{and} \\ f(h(z, y), y) \leq_3 z \\ \text{and} \\ x \leq_1 h(f(x, y), y) \end{pmatrix}$$

12. Conclusion: Substructural Logic



Plus, the + has its own residuated operators.. the co-implications.

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