Polarity Items in Resource Logics. A comparison.

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□ 1. Introduction

◦ *Aim.* To show how Resource Logics can treat Polarity Items (PIs).

◦ *Method.* In particular, I compare how this has been done in Multiplicative Linear Logic (MLL) and Multimodal Lambek Calculus (MLC).

◦ *Result.* This comparison shows that:

- the two approaches treat polarity in a similar way;
- MLL is too expressive to deal with NPIs whereas MLC captures the needed fragment of the family of Resource Logics.

□ 2. Polarity Items

Definition

a. *Positive Polarity Items* (PPIs) are expressions which cannot felicitously appear in negative contexts. They are licensed by upward monotonic functions;

b. *Negative Polarity Items* (NPIs) are expressions which can only appear felicitously in negative contexts. They are licensed by downward monotonic functions.

Proposal

Polarity = Resource

- ◦ produced by the *Licenser*
- • consumed by the *Polarity Items*

Linguistic data

Somebody left.	Kim **rarely** says *anything at all.*
Nobody left.	*Kim says *anything at all.*
Nobody left *yet.*	**Nobody rarely** says *anything.*
*Somebody left *yet.*	**Nobody** says *anything.*

Nobody thinks *anyone* left.

*Somebody thinks *anyone* left.

No students who read *anything* about it came to the talk.

*Some students who read *anything* about it came to the talk.

□ 3. Resource Logics

- **What are they?**

  Logics sensitive to the (number and structure of the) resources they reason about.

- **How are they used for linguistic applications?**

  They are employed as parsers: grammaticality means derivability.

The Non Associative Lambek Calculus

- *The Languages*

$\mathsf{CAT} := \mathsf{ATO} \mid \mathsf{CAT}/\mathsf{CAT} \mid \mathsf{CAT}\backslash\mathsf{CAT} \mid \mathsf{CAT} \bullet \mathsf{CAT}$

$\mathsf{STR} := \mathsf{LEX} \mid \mathsf{STR} \circ \mathsf{STR}$

- *The Rules*

Logical Rules

$$\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{(\Gamma \circ \Delta) \vdash A}\,[/E] \qquad \frac{(\Gamma \circ B) \vdash A}{\Gamma \vdash A/B}\,[/I]$$

$$\frac{\Gamma \vdash B \quad \Delta \vdash B\backslash A}{(\Gamma \circ \Delta) \vdash A}\,[\backslash E] \qquad \frac{(B \circ \Gamma) \vdash A}{\Gamma \vdash B\backslash A}\,[\backslash I]$$

$$\frac{\Delta \vdash A \bullet B \quad \Gamma[(A \circ B)] \vdash C}{\Gamma[\Delta] \vdash C}\,[\bullet E] \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A \bullet B}\,[\bullet I]$$

- *Some theorems*

1a. Application: $A \bullet (A\backslash B) \Rightarrow B$

$(A/B) \bullet B \Rightarrow A$

2a. Co-application: $A \Rightarrow B\backslash(B \bullet A)$

$A \Rightarrow (A \bullet B)/B$

3a. Monotonicity $\backslash, /$: *if* $A \Rightarrow B$ *and* $C \Rightarrow D$,

then $(D\backslash A) \Rightarrow (C\backslash B)$

if $A \Rightarrow B$ *and* $C \Rightarrow D$,

then $(A/D) \Rightarrow (B/C)$

The Multimodal Lambek Calculus

- *The Languages*

$$\mathsf{CAT}_{\Diamond\Box^\downarrow} := \mathsf{CAT} \mid \Diamond\mathsf{CAT} \mid \Box^\downarrow\mathsf{CAT}$$

$$\mathsf{STR}_{\Diamond\Box^\downarrow} := \mathsf{STR} \mid \langle\mathsf{STR}\rangle$$

- *The Rules*

$$\frac{\Delta \vdash \Diamond A \quad \Gamma[\langle A\rangle] \vdash B}{\Gamma[\Delta] \vdash B}\ [\Diamond\mathrm{E}] \qquad \frac{\Gamma \vdash A}{\langle\Gamma\rangle \vdash \Diamond A}\ [\Diamond\mathrm{I}]$$

$$\frac{\Gamma \vdash \Box^\downarrow A}{\langle\Gamma\rangle \vdash A}\ [\Box^\downarrow\mathrm{E}] \qquad \frac{\langle\Gamma\rangle \vdash A}{\Gamma \vdash \Box^\downarrow A}\ [\Box^\downarrow\mathrm{I}]$$

- *Some Theorems*

Theorems

1b.	Application:	$\Diamond\Box^{\downarrow}A \Rightarrow A$
2b.	Co-application:	$A \Rightarrow \Box^{\downarrow}\Diamond A$

Multiplicative Linear Logic

◦ *The Language*

$$\mathsf{CAT} := \mathsf{ATO} \mid \mathsf{CAT} \otimes \mathsf{CAT} \mid \mathsf{CAT} \multimap \mathsf{CAT}$$

◦ *The Rules*: *Implicit Structural Rules*

$$\frac{\Gamma[(\Delta_2 \circ \Delta_1)] \vdash C}{\Gamma[(\Delta_1 \circ \Delta_2)] \vdash C}\,[\mathrm{Per}] \qquad \frac{\Gamma[\Delta_1 \circ (\Delta_2 \circ \Delta_3)] \vdash C}{\Gamma[(\Delta_1 \circ \Delta_2) \circ \Delta_3] \vdash C}\,[\mathrm{Ass}]$$

□ 4. Polarity Items in Resource Logics

Proposal: MLL (Fry 1999)

$$\text{Polarity} = \text{Resource} := \ell$$

Logical Types

- ∘ Licenser: $(\ell \multimap (A \otimes \ell)) \multimap B$ $\quad A \multimap B$
- • NPI: $(A \otimes \ell) \multimap (B \otimes \ell)$ $\quad A \multimap B$

Theorems

$$\frac{B \Rightarrow B \quad A \Rightarrow (\ell \multimap (A \otimes \ell))}{((\ell \multimap (A \otimes \ell)) \multimap B \Rightarrow A \multimap B} \text{[Mon]}$$

Co-Application: $A \Rightarrow (\ell \multimap (A \otimes \ell))$

Example Nobody left yet

- ○ *nobody*: $(\ell \multimap (vp \otimes \ell)) \multimap s$ GQ: $vp \multimap s$
- • *yet*: $(vp \otimes \ell) \multimap (vp \otimes \ell)$ Adv: $vp \multimap vp$

$$
\begin{prooftree}
\AxiomC{$\text{nobody} \vdash (\ell \multimap (vp \otimes \ell)) \multimap s$}
\AxiomC{$\text{left} \vdash vp$}
\AxiomC{$[x \vdash \ell]^1$}
\RightLabel{$[\otimes \mathrm{I}]$}
\BinaryInfC{$\text{left} \circ x \vdash vp \otimes \ell$}
\AxiomC{$\text{yet} \vdash (vp \otimes \ell) \multimap (vp \otimes \ell)$}
\RightLabel{$[\multimap \mathrm{E}]$}
\BinaryInfC{$(\text{left} \circ x) \circ \text{yet} \vdash vp \otimes \ell$}
\RightLabel{[Ass]}
\UnaryInfC{$\text{left} \circ (x \circ \text{yet}) \vdash vp \otimes \ell$}
\RightLabel{[Per]}
\UnaryInfC{$\text{left} \circ (\text{yet} \circ x) \vdash vp \otimes \ell$}
\RightLabel{[Ass]}
\UnaryInfC{$(\text{left} \circ \text{yet}) \circ x \vdash (vp \otimes \ell)$}
\RightLabel{$[\multimap \mathrm{I}]^1$}
\UnaryInfC{$\text{left} \circ \text{yet} \vdash \ell \multimap (vp \otimes \ell)$}
\RightLabel{$[\multimap \mathrm{E}]$}
\BinaryInfC{$\text{nobody} \circ (\text{left} \circ \text{yet}) \vdash s$}
\end{prooftree}
$$

Disadvantages

1. The logic is too expressive.

   Syntactic control is stipulated by a formal grammar, viz. Lexical Functional Grammar (LFG).

   We want to avoid this two-level formalism.

2. Polarity is treated as a ‘phantom’ resource.

   It is concatenated with a linguistic category.

   We want to represent the latter as having a special ‘property’.

Replacing binary with unary operators.

Co-Application

$A \Rightarrow (\ell \multimap (A \otimes \ell))$ *Binary*

$\rightsquigarrow$

$A \Rightarrow \Box^{\downarrow}\Diamond A$ *Unary*

Monotonicity

$$\frac{B \Rightarrow B \quad A \Rightarrow (\ell \multimap (A \otimes \ell))}{((\ell \multimap (A \otimes \ell)) \multimap B \Rightarrow A \multimap B} \text{ [Mon]}$$

$\rightsquigarrow$

$$\frac{B \Rightarrow B \quad A \Rightarrow \Box^{\downarrow}\Diamond A}{B/\Box^{\downarrow}\Diamond A \Rightarrow B/A} \text{ [Mon]}$$

Proposal: MLC (Bernardi 1999)

$$\text{Polarity} = \text{Resource} := \Box^{\downarrow}\Diamond$$

Logical Types

- ◦ Licenser: $B/\Box^{\downarrow}\Diamond A$
- • NPI: $\Box^{\downarrow}\Diamond A\backslash\Box^{\downarrow}\Diamond B$

Example Nobody left yet

$$\dfrac{\text{nobody} \vdash s/\Box^{\downarrow}\Diamond vp \qquad \dfrac{\dfrac{\dfrac{\text{left} \vdash vp}{\langle\text{left}\rangle \vdash \Diamond vp}[\Diamond I]}{\text{left} \vdash \Box^{\downarrow}\Diamond vp}[\Box^{\downarrow} I] \qquad \text{yet} \vdash \Box^{\downarrow}\Diamond vp\backslash\Box^{\downarrow}\Diamond vp}{\text{left} \circ \text{yet} \vdash \Box^{\downarrow}\Diamond vp}[\backslash E]}{\text{nobody} \circ (\text{left} \circ \text{yet}) \vdash s}[/E]$$

Advantages

1. We avoid the two-level formalism.

   MLC has the right expressiveness for both semantic and syntactic composition.

2. Licensers and NPIs are treated as categories having a particular 'property'.

□ 5. Open Questions

- NPIs can occur in embedded sentences. This requires some structural reasoning.
  - How can MLC be extended to account for these cases?
- GQs differ in their scope possibility.
  - How does NPIs' distribution relate to this fact?
- NPIs can be licensed by pragmatic implicatures which are at a discourse level.
  - How can MLC be extended to account for 'reasoning' at discourse level?

□ 6. Conclusion

We have shown that:

- NPI behavior can be derived in Resource Logics (RL).
- MLL is a too powerful fragment of the family of RL.
- MLC has the right expressiveness.

Structural Reasoning

When NPIs occur in embedded sentences some structural reasoning is needed.

Examples:

> **Nobody** thinks *anyone* left.
>
> **No** students who knew *anything* about it came to the talk.

Structural Properties

$$\frac{\Gamma[\langle \Delta_1 \circ \Delta_2 \rangle] \vdash C}{\Gamma[(\langle \Delta_1 \rangle \circ \langle \Delta_2 \rangle)] \vdash C}\ [\text{Dis}]$$

$$
\begin{prooftree}
\AxiomC{$\mathrm{nobody} \vdash s/\Box^{\downarrow}\Diamond(np\backslash s)$}
\AxiomC{$\mathrm{anyone} \vdash \Box^{\downarrow}\Diamond s/\Box^{\downarrow}\Diamond(np\backslash s)$}
\AxiomC{$\mathrm{left} \vdash np\backslash s$}
\RightLabel{$\Diamond I$}
\UnaryInfC{$\langle\mathrm{left}\rangle \vdash \Diamond(np\backslash s)$}
\RightLabel{$\Box^{\downarrow} I$}
\UnaryInfC{$\mathrm{left} \vdash \Box^{\downarrow}\Diamond(np\backslash s)$}
\RightLabel{$/E$}
\BinaryInfC{$\mathrm{anyone} \circ \mathrm{left} \vdash \Box^{\downarrow}\Diamond s$}
\RightLabel{$\Box^{\downarrow} E$}
\UnaryInfC{$\langle\mathrm{anyone} \circ \mathrm{left}\rangle \vdash \Diamond s$}
\AxiomC{$\mathrm{thinks} \vdash (np\backslash s)/s$}
\AxiomC{$\mathrm{r}_1 \vdash s$}
\RightLabel{$/E$}
\BinaryInfC{$\mathrm{thinks} \circ \mathrm{r}_1 \vdash np\backslash s$}
\RightLabel{$\Diamond I$}
\UnaryInfC{$\langle\mathrm{thinks} \circ \mathrm{r}_1\rangle \vdash \Diamond(np\backslash s)$}
\RightLabel{Dis_1}
\UnaryInfC{$\langle\mathrm{thinks}\rangle \circ \langle\mathrm{r}_1\rangle \vdash \Diamond(np\backslash s)$}
\RightLabel{$\Diamond E$}
\BinaryInfC{$\langle\mathrm{thinks}\rangle \circ \langle\mathrm{anyone} \circ \mathrm{left}\rangle \vdash \Diamond(np\backslash s)$}
\RightLabel{Dis_1}
\UnaryInfC{$\langle\mathrm{thinks} \circ (\mathrm{anyone} \circ \mathrm{left})\rangle \vdash \Diamond(np\backslash s)$}
\RightLabel{$\Box^{\downarrow} I$}
\UnaryInfC{$\mathrm{thinks} \circ (\mathrm{anyone} \circ \mathrm{left}) \vdash \Box^{\downarrow}\Diamond(np\backslash s)$}
\RightLabel{$/E$}
\BinaryInfC{$\mathrm{nobody} \circ (\mathrm{thinks} \circ (\mathrm{anyone} \circ \mathrm{left})) \vdash s$}
\end{prooftree}
$$

$$\mathrm{nobody}(\lambda z.\mathrm{think}(\lambda y.\mathrm{anyone}(z, \mathrm{leave}(z))))$$

$$
\begin{prooftree}
\AxiomC{$\mathrm{read} \vdash tv$}
\RightLabel{$\Diamond$ I}
\UnaryInfC{$\langle \mathrm{read} \rangle \vdash \Diamond tv$}
\RightLabel{$\Box^{\downarrow}$ I}
\UnaryInfC{$\mathrm{read} \vdash \Box^{\downarrow}\Diamond tv$}
\AxiomC{$\mathrm{anything} \vdash \Box^{\downarrow}\Diamond tv \backslash \Box^{\downarrow}\Diamond vp$}
\BinaryInfC{$\mathrm{read} \circ \mathrm{anything} \vdash \Box^{\downarrow}\Diamond vp$}
\RightLabel{$\Box^{\downarrow}$ E}
\UnaryInfC{$\langle \mathrm{read} \circ \mathrm{anything} \rangle \vdash \Diamond vp$}
\AxiomC{$\mathrm{student} \vdash n$}
\AxiomC{$\mathrm{who} \vdash (n \backslash n)/vp$}
\AxiomC{$\mathrm{s}_2 \vdash vp$}
\RightLabel{$/$E}
\BinaryInfC{$\mathrm{who} \circ \mathrm{s}_2 \vdash n \backslash n$}
\RightLabel{$\backslash$E}
\BinaryInfC{$\mathrm{student} \circ (\mathrm{who} \circ \mathrm{s}_2) \vdash n$}
\RightLabel{$\Diamond$ I}
\UnaryInfC{$\langle \mathrm{student} \circ (\mathrm{who} \circ \mathrm{s}_2) \rangle \vdash \Diamond n$}
\RightLabel{Dis_2}
\UnaryInfC{$\langle \mathrm{student} \langle \circ \langle \mathrm{who} \circ \mathrm{s}_2 \rangle \vdash \Diamond n$}
\RightLabel{Dis_2}
\UnaryInfC{$\langle \mathrm{student} \rangle \circ (\langle \mathrm{who} \rangle \circ \langle \mathrm{s}_2 \rangle) \vdash \Diamond n$}
\RightLabel{$\Diamond$ E}
\BinaryInfC{$\langle \mathrm{student} \langle \circ (\langle \mathrm{who} \rangle \circ \langle \mathrm{read} \circ \mathrm{anything} \rangle) \vdash \Diamond n$}
\RightLabel{Dis_1}
\UnaryInfC{$\langle \mathrm{student} \rangle \circ \langle \mathrm{who} \circ (\mathrm{read} \circ \mathrm{anything}) \rangle \vdash \Diamond n$}
\RightLabel{Dis_1}
\UnaryInfC{$\langle \mathrm{student} \circ (\mathrm{who} \circ (\mathrm{read} \circ \mathrm{anything})) \rangle \vdash \Diamond n$}
\RightLabel{$\Box^{\downarrow}$ I}
\UnaryInfC{$\mathrm{student} \circ (\mathrm{who} \circ (\mathrm{read} \circ \mathrm{anything})) \vdash \Box^{\downarrow}\Diamond n$}
\end{prooftree}
$$

$$
\dfrac{\text{nobody} \vdash s/\Box^{\downarrow}\Diamond vp \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{c}\vdots\\ \text{did} \circ \text{anything} \vdash \Box^{\downarrow}\Diamond vp\end{array}}{\langle \text{did} \circ \text{anything}\rangle \vdash \Diamond vp}\,\Box^{\downarrow}E \qquad \dfrac{\dfrac{\dfrac{\dfrac{\text{thinks} \vdash vp/s \qquad \dfrac{\text{sue} \vdash np \quad \text{p}_3 \vdash np\backslash s}{\text{sue} \circ \text{p}_3 \vdash s}\,\backslash E}{\text{thinks} \circ (\text{sue} \circ \text{p}_3) \vdash vp}\,/E}{\langle \text{thinks} \circ (\text{sue} \circ \text{p}_3)\rangle \vdash \Diamond vp}\,\Diamond I}{\langle \text{thinks}\rangle \circ \langle \text{sue} \circ \text{p}_3\rangle \vdash \Diamond vp}\,\text{Dis}_2}{\langle \text{thinks}\rangle \circ (\langle \text{sue}\rangle \circ \langle \text{p}_3\rangle) \vdash \Diamond vp}\,\text{Dis}_2}{\langle \text{thinks}\rangle \circ (\langle \text{sue}\rangle \circ \langle \text{did} \circ \text{anything}\rangle) \vdash \Diamond vp}\,\Diamond E}{\langle \text{thinks}\rangle \circ \langle \text{sue} \circ (\text{did} \circ \text{anything})\rangle \vdash \Diamond vp}\,\text{Dis}_1}{\langle \text{thinks} \circ (\text{sue} \circ (\text{did} \circ \text{anything}))\rangle \vdash \Diamond vp}\,\text{Dis}_1}{\text{thinks} \circ (\text{sue} \circ (\text{did} \circ \text{anything})) \vdash \Box^{\downarrow}\Diamond vp}\,\Box^{\downarrow}I}{\text{nobody} \circ (\text{thinks} \circ (\text{sue} \circ (\text{did} \circ \text{anything}))) \vdash s}\,/E
$$

Questions. (Moortgat)

- Is the distribution law really necessary?
- Is it possible to control the polarity distribution directly from the lexicon?

An attempt

$$\text{nobody} \vdash s/\Box^{\downarrow}\Diamond(np\backslash s) \quad \rightsquigarrow \quad s/(np\backslash\Box^{\downarrow}\Diamond s)$$

$$\text{anyone} \vdash \Box^{\downarrow}\Diamond s/\Box^{\downarrow}\Diamond(np\backslash s) \quad \rightsquigarrow \quad \Box^{\downarrow}\Diamond s/(np\backslash\Box^{\downarrow}\Diamond s)$$

$$\dfrac{\text{nobody} \vdash s/(np\backslash\Box^{\downarrow}\Diamond s) \qquad \begin{array}{c}\text{anyone} \vdash \Box^{\downarrow}\Diamond s/(np\backslash\Box^{\downarrow}\Diamond s)\\ \vdots \\ \text{thinks} \circ (\text{anyone} \circ \text{left}) \vdash np\backslash\Box^{\downarrow}\Diamond s\end{array}}{\text{nobody} \circ (\text{thinks} \circ (\text{anyone} \circ \text{left})) \vdash s}$$

$$\text{nobody}(\lambda y.\text{anyone}(\lambda z.\text{think}(y, \text{leave}(z))))$$