Natural language reasoning: a proof-theoretic perspective

PhD Proposal

Raffaella Bernardi

University of Chieti, Italy

currently at: UiL-OTS, University of Utrecht

Introduction

Our aim:

- To build a (fragment of) Natural Logic,
  i.e. a system of reasoning based directly on linguistic form.

Our domain of investigation:

- monotonic reasoning
- sentence and discourse level
- negation

Problems to deal with:

- dynamic properties of natural language interpretation
- incremental nature of natural language processing

Our tools:

- a Grammar Logic: Multimodal Categorial Grammar
- a Proof System: Proof Nets

Natural reasoning: Monotonic inference

Focusing the attention on monotonic reasoning means to deal with inference such as:

$$\frac{\text{A } \textbf{nice boy} \text{ loves Mary.}}{\text{A } \textbf{boy} \text{ loves Mary.}}$$

$$\frac{\text{No boy } \textbf{loves} \text{ Mary.}}{\text{No boy } \textbf{passionally loves} \text{ Mary.}}$$

Formally:

1. 'A' is an $\uparrow$MON function in its first argument:
   i.e. $\forall x, y$ if $x \leq y$, then $f(x) \leq f(y)$,
   e.g. 'nice boy' $\leq$ 'boy', then 'a nice boy' $\leq$ 'a boy'.

2. 'No' is a $\downarrow$MON function in its second argument:
   i.e. $\forall x, y$ if $x \leq y$, then $f(y) \leq f(x)$,
   e.g. 'passionally loves' $\leq$ 'loves',
   then 'No boy loves' $\leq$ 'No boy passionally loves'.

We have implemented an algorithm able to deal with these monotonic inferences.

It starts from the lexical entries labeled with monotonicity markers and propagates them through the sentence's derivation via the inference rules for the type logical connectives.

Lexical items with internal markers:

- no $\in$ ((e,t)$^-$, ((e,t)$^-$, t))
- a $\in$ ((e,t)$^+$, ((e,t)$^+$, t))

where the '+' and the '-' stand, respectively, for 'upward' and 'downward' monotonic functions.

Structural Marking:

$$\frac{X \in (a,b)^+ \quad Y \in a}{XY \in b}$$

Internal Marking:

$$\frac{X \in (a^*,b) \quad Y \in a^*}{XY \in b}$$

Examples:

1) A boy loves Mary

1. $\exists x_3((\overset{+}{\mathbf{boy}}\ \overset{+}{x}_3) \land (\overset{+}{(\overset{+}{\mathbf{love}}\ \overset{+}{\mathbf{m}})}\ \overset{+}{x}_3))$

2) No boy loves Mary

2. $\exists x_3((\overset{+}{\mathbf{boy}}\ \overset{+}{x}_3) \land \neg(\overset{+}{(\overset{+}{\mathbf{love}}\ \overset{+}{\mathbf{m}})}\ \overset{+}{x}_3))$

Reasoning with Discourse

Natural reasoning mostly works at discourse level.

Working on this level means to account for:

1. the **dynamic properties**: the meaning of a sentence and the value assigned to variables can *pass on* within and across sentence boundaries, creating semantic links.

   The links will be called 'internal' and 'external', respectively;

2. the **incremental properties**: the discourse is analysed in a left-right manner.

We start from the dynamic properties, considering what this means with respect to the classical connectives.

Dynamic properties of classical connectives

Linguistic examples:

[$\wedge$] [A man]$_i$ entered and he$_i$ smiled. He$_i$ was happy.

[$\vee$] $*$[A man]$_i$ walks in the park or he$_i$ hates it.

[$\neg$] [No man]$_i$ who$_i$ walks in the park hates it. *He$_i$ whistles.

Formally:

	Externally dynamic	*Internally dynamic*
$\wedge$	Yes	Yes
$\vee$	No	No
$\neg$	No	Yes
$\exists$	Yes	Yes

Multimodal Categorial Grammar

Vocabulary:

- Composition operations: $/$, $\bullet$, $\backslash$
- Control: $\Diamond$, $\Box$

Rules:

- Logical inferences: Introduction and Elimination rules
- Structural inferences

In order to account for the 'and' vs 'or' distinction we make use of the $\Box$ operator of MMCG.

Marking each sentence with a $\Box$ we control the possible semantic links among sentences.

Lexical entries:

- and $\vdash \lambda \mathbf{X}.\lambda \mathbf{Y}.(\mathbf{X} \wedge \mathbf{Y})$: s$\backslash$(s/s)
- or $\vdash \lambda \mathbf{X}.\lambda \mathbf{Y}.({}^{\vee}\mathbf{X} \vee {}^{\vee}\mathbf{Y})$: $\Box$s$\backslash$(s/$\Box$s)
- a $\vdash \lambda \mathbf{X}.\lambda \mathbf{Y}.{}^{\wedge}(\exists\, \mathbf{z} \wedge \mathbf{X}(\mathbf{z}) \wedge {}^{\vee}\mathbf{Y}(\mathbf{z}))$: ($\Box$s/(np$\backslash\Box$s))/n
- nobody $\vdash \lambda \mathbf{Y}.{}^{\wedge} \neg(\exists \mathbf{z} \wedge {}^{\vee}\mathbf{Y}(\mathbf{z}))$: ($\Box$s/(np$\backslash\Box$s))
- she $\vdash \mathbf{x}$: $\Box$np

Structural inferences:

In order to allow the links across sentences to *pass on* only in case of the 'and' connection, we introduce the following postulate.

[Pd] $\Diamond((A \circ \mathrm{B}) \circ C) \rightarrow ((\Diamond A \circ \mathrm{B}) \circ \Diamond C)$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{(\mathrm{a} \circ \mathrm{girl}) \circ \mathrm{walks} \vdash \Box^{\downarrow} s}}{\langle (\mathrm{a} \circ \mathrm{girl}) \circ \mathrm{walks} \rangle \vdash s}\,[\Box^{\downarrow}E] \quad \mathrm{and} \vdash s\backslash(s/s)}{\langle (\mathrm{a} \circ \mathrm{girl}) \circ \mathrm{walks} \rangle \circ \mathrm{and} \vdash s/s}\,[\backslash E] \quad \dfrac{\dfrac{\dfrac{\dfrac{\mathrm{she} \vdash \Box^{\downarrow} np}{\langle \mathrm{she} \rangle \vdash np}\,[\Box^{\downarrow}E] \quad \mathrm{whistles} \vdash np\backslash\Box^{\downarrow} s}{\langle \mathrm{she} \rangle \circ \mathrm{whistles} \vdash \Box^{\downarrow} s}\,[\backslash E]}{\langle \langle \mathrm{she} \rangle \circ \mathrm{whistles} \rangle \vdash s}\,[\Box^{\downarrow}E]}{}}{(\langle (\mathrm{a} \circ \mathrm{girl}) \circ \mathrm{walks} \rangle \circ \mathrm{and}) \circ \langle \langle \mathrm{she} \rangle \circ \mathrm{whistles} \rangle \vdash s}\,[/E]}{\langle (((\mathrm{a} \circ \mathrm{girl}) \circ \mathrm{walks}) \circ \mathrm{and}) \circ (\langle \mathrm{she} \rangle \circ \mathrm{whistles}) \rangle \vdash s}\,[Pd]}{(((\mathrm{a} \circ \mathrm{girl}) \circ \mathrm{walks}) \circ \mathrm{and}) \circ (\langle \mathrm{she} \rangle \circ \mathrm{whistles}) \vdash \Box^{\downarrow} s}\,[\Box^{\downarrow}I]
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\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{{}^{\wedge}(\exists \mathbf{x1} \wedge \mathbf{girl}(\mathbf{x1}) \wedge \mathbf{walk}(\mathbf{x1})) : \Box^{\downarrow} s} \quad \lambda x_4.\lambda y_4.(x_4 \wedge y_4) : s\backslash(s/s)}{\exists \mathbf{x1} \wedge \mathbf{girl}(\mathbf{x1}) \wedge \mathbf{walk}(\mathbf{x1}) : s}\,[\Box^{\downarrow}E] \quad \dfrac{\dfrac{\dfrac{\dfrac{\mathbf{x1} : \Box^{\downarrow} np}{{}^{\vee}\mathbf{x1} : np}\,[\Box^{\downarrow}E] \quad \lambda z_4.{}^{\wedge}\mathbf{whistle}(z_4) : np\backslash\Box^{\downarrow} s}{{}^{\wedge}\mathbf{whistle}({}^{\vee}\mathbf{x1}) : \Box^{\downarrow} s}\,[\backslash E]}{\mathbf{whistle}({}^{\vee}\mathbf{x1}) : s}\,[\Box^{\downarrow}E]}{}}{\lambda y_4.(\exists \mathbf{x1} \wedge \mathbf{girl}(\mathbf{x1}) \wedge \mathbf{walk}(\mathbf{x1}) \wedge y_4) : s/s}\,[\backslash E]}{\exists \mathbf{x1} \wedge \mathbf{girl}(\mathbf{x1}) \wedge \mathbf{walk}(\mathbf{x1}) \wedge \mathbf{whistle}({}^{\vee}\mathbf{x1}) : s}\,[/E]}{{}^{\wedge}(\exists \mathbf{x1} \wedge \mathbf{girl}(\mathbf{x1}) \wedge \mathbf{walk}(\mathbf{x1}) \wedge \mathbf{whistle}({}^{\vee}\mathbf{x1})) : \Box^{\downarrow} s}\,[\Box^{\downarrow}I]
$$

1. *A girl walks and she whistles.*

$^{\wedge}(\exists \mathbf{x1} \wedge \mathbf{girl}(\mathbf{x1}) \wedge \mathbf{walk}(\mathbf{x1}) \wedge \mathbf{whistle}({}^{\vee}\mathbf{x1}))$

2. *Nobody walks and she whistles.*

$^{\wedge}(\neg(\exists \mathbf{x1} \wedge \mathbf{walk}(\mathbf{x1})) \wedge \mathbf{whistle}({}^{\vee}\mathbf{x1}))$

$$\dfrac{\dfrac{\begin{matrix}\vdots \\ (\mathrm{a} \circ \mathrm{girl}) \circ \mathrm{walks} \vdash \Box^{\downarrow} s\end{matrix} \quad \mathrm{or} \vdash \Box^{\downarrow} s \backslash (s / \Box^{\downarrow} s)}{((\mathrm{a} \circ \mathrm{girl}) \circ \mathrm{walks}) \circ \mathrm{or} \vdash s / \Box^{\downarrow} s}[\backslash E] \quad \dfrac{\dfrac{\mathrm{she} \vdash \Box^{\downarrow} np}{\langle \mathrm{she} \rangle \vdash np}[\Box^{\downarrow} E] \quad \mathrm{whistles} \vdash np \backslash \Box^{\downarrow} s}{\langle \mathrm{she} \rangle \circ \mathrm{whistles} \vdash \Box^{\downarrow} s}[\backslash E]}{(((\mathrm{a} \circ \mathrm{girl}) \circ \mathrm{walks}) \circ \mathrm{or}) \circ (\langle \mathrm{she} \rangle \circ \mathrm{whistles}) \vdash s}[/E]$$

$$\dfrac{\dfrac{\begin{matrix}\vdots \\ {}^{\wedge}(\exists \mathbf{x1} \wedge \mathbf{girl}(\mathbf{x1}) \wedge \mathbf{walk}(\mathbf{x1})) : \Box^{\downarrow} s\end{matrix} \quad \lambda x_4.\lambda y_4.({}^{\vee} x_4 \vee {}^{\vee} y_4) : \Box^{\downarrow} s \backslash (s / \Box^{\downarrow} s)}{\lambda y_4.((\exists \mathbf{x1} \wedge \mathbf{girl}(\mathbf{x1}) \wedge \mathbf{walk}(\mathbf{x1})) \vee {}^{\vee} y_4) : s / \Box^{\downarrow} s}[\backslash E] \quad \dfrac{\dfrac{\mathbf{x1} : \Box^{\downarrow} np}{{}^{\vee}\mathbf{x1} : np}[\Box^{\downarrow} E] \quad \lambda z_4.{}^{\wedge}\mathbf{whistle}(z_4) : np \backslash \Box^{\downarrow} s}{{}^{\wedge}\mathbf{whistle}({}^{\vee}\mathbf{x1}) : \Box^{\downarrow} s}[\backslash E]}{(\exists \mathbf{x1} \wedge \mathbf{girl}(\mathbf{x1}) \wedge \mathbf{walk}(\mathbf{x1})) \vee \mathbf{whistle}({}^{\vee}\mathbf{x1}) : s}[/E]$$

1. *A girl walks or she whistles.*

$\exists \mathbf{x1} \wedge \mathbf{girl}(\mathbf{x1}) \wedge \mathbf{walk}(\mathbf{x1}) \vee \mathbf{whistle}({}^{\vee}\mathbf{x1})$

2. *Nobody walks or she whistles.*

$\neg(\exists \mathbf{x1} \wedge \mathbf{walk}(\mathbf{x1})) \vee \mathbf{whistle}({}^{\vee}\mathbf{x1})$

Open question: Which is the right interpretation of the $\Box$ operator, or, more precisely, which is the semantic domain of $\Box$A?

What we are going to do

Our practical aims:

- To build a (fragment of the) Natural Logic
- To illustrate this fragment with a Natural Language Query System,

  i.e. a system able to answer to questions about the information written in a data base.

  Moreover this system will be able to give as answers the conclusions inferred by the Natural Logic from the data base.

Examples *of the Natural Logic's inference and the Query System's answers:*

$$\frac{\text{I haven't seen anybody. They were away.}}{\text{Everybody was away.}}$$

Data base:

'I haven't see anybody. They were away.'

Q Who was away?
A `Everybody.`

Q Was everybody away?
A `Yes.`

Conclusion

We have **already** good evidences which show that:

the inferences derived by the Natural Logic are obtained through a combination of both syntactic and semantic information extracted from the discourse.

The project is **expected** to:

highlight the strong link we believe exists between syntax and semantics in natural reasoning with natural language.

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\dfrac{\lambda y_2.\lambda z_2.\forall x_3((\overset{+}{y_2}\ \overset{-}{x_3}) \rightarrow (\overset{+}{z_2}\ \overset{+}{x_3})) : (s/_0(np\backslash_0 s))/_0 n \quad \mathbf{boy} : n}{\lambda z_2.\forall x_3((\overset{+}{\mathbf{boy}}\ \overset{-}{x_3}) \rightarrow (\overset{+}{z_2}\ \overset{+}{x_3})) : s/_0(np\backslash_0 s)}\,[/E]
\quad
\dfrac{\lambda y_3.(\overset{+}{\mathbf{love}}\ \overset{+}{y_3})^+ : (np\backslash_0 s)/_0 np \quad \mathbf{m} : np}{(\overset{+}{\mathbf{love}}\ \overset{+}{\mathbf{m}})^+ : np\backslash_0 s}\,[/E]
}{
\forall x_3((\overset{+}{\mathbf{boy}}\ \overset{-}{x_3}) \rightarrow ((\overset{\overset{+}{+}}{\mathbf{love}}\ \overset{+}{\mathbf{m}})\ \overset{+}{x_3})) : s
}\,[/E]
$$

$$
\mathbf{1.}\ \forall x_3((\overset{+}{\mathbf{boy}}\ \overset{-}{x_3}) \rightarrow ((\overset{\overset{+}{+}}{\mathbf{love}}\ \overset{+}{\mathbf{m}})\ \overset{+}{x_3}))
$$

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\dfrac{
\dfrac{\lambda y_2.\lambda z_2.\exists x_3((\overset{+}{y_2}\ \overset{+}{x_3}) \wedge \neg(\overset{+}{z_2}\ \overset{-}{x_3})) : (s/_0(np\backslash_0 s))/_0 n \quad \mathbf{boy} : n}{\lambda z_2.\exists x_3((\overset{+}{\mathbf{boy}}\ \overset{+}{x_3}) \wedge \neg(\overset{+}{z_2}\ \overset{-}{x_3})) : s/_0(np\backslash_0 s)}[/E]
\quad
\dfrac{\lambda y_3.(\overset{+}{\mathbf{love}}\ \overset{+}{y_3})^+ : (np\backslash_0 s)/_0 np \quad \mathbf{m} : np}{(\overset{+}{\mathbf{love}}\ \overset{+}{\mathbf{m}})^+ : np\backslash_0 s}[/E]
}{
\exists x_3((\overset{+}{\mathbf{boy}}\ \overset{+}{x_3}) \wedge \neg(\overset{+}{(\overset{+}{\mathbf{love}}\ \overset{+}{\mathbf{m}})}\ \overset{+}{x_3})) : s
}[/E]
$$

2. $\exists x_3((\overset{+}{\mathbf{boy}}\ \overset{+}{x_3}) \wedge \neg(\overset{+}{(\overset{+}{\mathbf{love}}\ \overset{+}{\mathbf{m}})}\ \overset{+}{x_3}))$

Structural control modalities

$$\frac{\Delta \vdash u : \Diamond A \quad \Gamma[< x : A >] \vdash t : B}{\Gamma[\Delta] \vdash t[^{\vee} u/x] : B} \Diamond E \qquad \frac{\Gamma \vdash t : A}{< \Gamma > \vdash^{\wedge} t : \Diamond A} \Diamond I$$

$$\frac{\Gamma \vdash t : \Box A}{< \Gamma > \vdash^{\vee} t : A} \Box E \qquad \frac{< \Gamma > \vdash t : A}{\Gamma \vdash^{\wedge} t : \Box A} \Box I$$