In Situ Binding: A Hybrid Approach

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Abstract

Due to the concatenative interpretation of its composition operator, the base categorial type logic $\mathsf{NL}(\Diamond)$ encounters some difficulties when analyzing non-concatenative material. Hence, modeling linguistic phenomena like anaphora resolution, long distance binding, gapping, discontinuity and quantifier raising becomes a problem in this logic. We propose an extension of $\mathsf{NL}(\Diamond)$ with hybrid operators (nominals and the :-operator), and show that this extension has the needed expressivity to define frame classes leading to a proper modeling of long distance binding. We focus our presentation on the treatment of quantifier phrases.

1 Introduction

Any system employed to reason with linguistic signs must be able to properly account for the relation holding between the form and meaning components of natural language expressions. Since Montague's Universal Grammar [22], most literature in formal semantics is based on Frege's principle of compositionality and takes the meaning of a complex syntactic expression to be a function of the meaning of its constituent parts and of the derivational steps that have put them together. Due to this tight connection between the syntactic and semantic level, a desirable property of natural language analyzes is a parallel treatment of the two components. This simultaneous reasoning becomes particularly challenging when the syntactic and semantic constraints differ. Well known problems of this kind are anaphora resolution, long distance binding, gapping, discontinuity phenomena and quantifier raising.

As shown in [18], gapping, discontinuity phenomena and quantifier raising can receive a uniform formal treatment. They are caused by the occurrence of expressions which take scope wider than they occur syntactically. For example, in the case of the quantifier phrases (QPs) like "John *thinks someone* left," the quantifier *someone* can have either narrow or wide scope with respect to *thinks* giving rise to scope ambiguity (*de dicto* or *de re* readings). Even though at the syntactic level *someone* occupies the subject position of the embedded sentence, semantically it can take scope over a higher sentential clause.

Similar behavior is shown by other binders, like reflexive pronouns [3,12], reciprocal pronouns [7,5], and by the pied-piping phenomenon [20,17]. In all these cases, the binders must enter into structural composition with a discontinuous configuration of resources by scoping over it and binding a variable within such a structure.

The difference between the syntactic and semantic behavior of natural language binders is problematic for any formal linguistic account. Standard formal analyzes solve this problem by marking the syntactic position of the binder, and using the marker at semantic level for both scoping and binding.

In this paper we will assume a logical perspective on the binding problem by using the Categorial Type Logic (CTL) framework to reason with linguistic resources. Thanks to the Curry-Howard correspondence between proofs and lambda-terms, CTL is able to compute form and meaning composition in parallel [23]. However, due to the concatenative interpretation of the composition operator, classical categorial logics encounter some difficulties when analyzing non-concatenative phenomena. Several solutions have been proposed to enrich the logic with further operators or structural postulates [11,12,16,17]. In this paper, we show how "hybridization" can help CTL to solve the problem of long distance binding.

The structure of the paper is as follows. We start in the next section by briefly introducing the basic notions of CTL. In Section 3 we discuss two different linguistic approaches to binding: the one classical of Generative Grammars (based on movement), and the standard approach used by CTL (which models binding "in situ"). In Section 4 we introduce the hybrid logic $\mathsf{HNL}(\Diamond)$ and in Section 5 we show how in situ binding can be modeled in this logic. The last section contains conclusions and directions for further research.

2 A Crash Course in CTL

The base categorial language is NL (introduced by Lambek in [10]), a non-associative, non-commutative calculus of pure residuation. The language contains the operators $\bullet$ (which stands for linguistic composition), $\backslash$ and $/$ (allowing the definition of functional categories). NL was extended by Moortgat to $\mathsf{NL}(\Diamond)$, by the addition of a pair of residuated unary modalities $\Diamond$ and $\Box^{\downarrow}$ [14]. The unary modalities behave as "markers" used to single out certain categories which require special treatment (e.g., versions of associativity have been used to analyze clitics). The particular reasoning characteristic of these phenomena is then specified by means of "structural postulates," which are nothing more than constraints on the accessibility relations governing the modalities. Nowadays, $\mathsf{NL}(\Diamond)$ is further generalized to multi-mode versions, where indexed families of operators are introduced. In the rest of this section we will formally introduce syntax, semantics and proof theory of the uni-mode logic $\mathsf{NL}(\Diamond)$. The extension to multi-mode versions is standard.

Definition 2.1 Given a set PROP of propositional symbols, the logical language of $\mathsf{NL}(\Diamond)$ is obtained as

$$\mathsf{FORM} ::= A \mid \mathsf{FORM} * \mathsf{FORM} \mid \heartsuit \mathsf{FORM}$$

where $A \in \mathsf{PROP}$, $* \in \{\backslash, \bullet, /\}$ and $\heartsuit \in \{\Diamond, \Box^{\downarrow}\}$.

Definition 2.2 A model for $\mathsf{NL}(\Diamond)$ is a tuple $M = \langle W, R^{\bullet}, R^{\Diamond}, V\rangle$ where W is a non-empty set, $R^{\bullet} \subseteq W^3$, $R^{\Diamond} \subseteq W^2$, and V is a valuation $V : \mathsf{PROP} \to \mathcal{P}(W)$. The $R^{\bullet}$ relation governs the residuated triple $(\backslash, \bullet, /)$, the $R^{\Diamond}$ relation governs the residuated pair $(\Diamond, \Box^{\downarrow})$. Given a model $M = \langle W, R^{\bullet}, R^{\Diamond}, V\rangle$ and $x, y \in W$, the *satisfiability relation* is inductively defined as follows:

$$\begin{aligned}
M, x \models A &\text{ iff } x \in V(A) \text{ where } A \in \mathsf{PROP}.\\
M, x \models \Diamond A &\text{ iff } \exists y[R^{\Diamond}xy \;\&\; M, y \models A].\\
M, y \models \Box^{\downarrow} A &\text{ iff } \forall x[R^{\Diamond}xy \to M, x \models A].\\
M, x \models A \bullet B &\text{ iff } \exists y \exists z[R^{\bullet}xyz \;\&\; M, y \models A \;\&\; M, z \models B].\\
M, y \models A/B &\text{ iff } \forall x \forall z[(R^{\bullet}xyz \;\&\; M, z \models B) \to M, x \models A].\\
M, z \models B\backslash A &\text{ iff } \forall x \forall y[(R^{\bullet}xyz \;\&\; M, y \models B) \to M, x \models A].
\end{aligned}$$

A model M satisfies a formula ϕ ($M \models \phi$) if for all elements w in the model we have $M, w \models \phi$. M satisfies an implication $\phi_1 \to \phi_2$ with $\phi_1, \phi_2 \in \mathsf{FORM}$ if $M \models \phi_1$ implies $M \models \phi_2$.

One of the standard proof theoretic presentation of CTL is in terms of a Gentzen-style sequent calculus. By taking advantage of the Curry-Howard correspondence between this kind of calculus and lambda terms [8], CTL inherits its important ability to compute form and meaning composition of natural language structures in parallel.

The Gentzen system is stated in terms of sequents: pairs $\Gamma \vdash A$ where Γ is a structural term and A is a logic formula. The set STRUCT of structural terms used in $\mathsf{NL}(\Diamond)$ is

$$\mathsf{STRUCT} \quad ::= \quad \mathsf{FORM} \mid (\mathsf{STRUCT} \circ \mathsf{STRUCT}) \mid \langle \mathsf{STRUCT} \rangle.$$

The logic rules in the Gentzen system for $\mathsf{NL}(\Diamond)$ are given in Figure 1 below. The rules also specify the way to construct the semantic lambda terms corresponding to each category. In the figure, A, B, C are formulas, Γ, Δ are structural terms and the notation $\Gamma[\varphi]$ is used to single out a particular instance of the substructure φ in Γ. In $t|A$, t is the lambda term corresponding to the category A. $[v/s]$ stands for the substitution of v for s. If z is a pair $\langle t, u\rangle$ then $(z)_0 = t$ and $(z)_1 = u$.

$$\frac{}{x|A \vdash x|A}\,[\mathrm{Ax}]$$

$$\frac{\Delta \vdash t|B \quad \Gamma[x|A] \vdash u|C}{\Gamma[(y|A/B \circ \Delta)] \vdash u[y(t)/x]|C}\,[/\mathrm{L}] \qquad \frac{\Gamma \circ x|B \vdash t|A}{\Gamma \vdash \lambda x.t|A/B}\,[/\mathrm{R}]$$

$$\frac{\Delta \vdash t|B \quad \Gamma[x|A] \vdash u|C}{\Gamma[(\Delta \circ y|B\backslash A)] \vdash u[y(t)/x]|C}\,[\backslash\mathrm{L}] \qquad \frac{x|B \circ \Gamma \vdash t|A}{\Gamma \vdash \lambda x.t|B\backslash A}\,[\backslash\mathrm{R}]$$

$$\frac{\Gamma[(x|A \circ y|B)] \vdash t|C}{\Gamma[z|(A \bullet B)] \vdash t[(z)_0/x, (z)_1/y]|C}\,[\bullet\mathrm{L}] \qquad \frac{\Gamma \vdash t|A \quad \Delta \vdash u|B}{(\Gamma \circ \Delta) \vdash \langle t, u\rangle|(A \bullet B)}\,[\bullet\mathrm{R}]$$

$$\frac{\Gamma[y|A] \vdash t|B}{\Gamma[\langle x|\Box^{\downarrow}A\rangle] \vdash t[x/y]|B}\,[\Box^{\downarrow}\mathrm{L}] \qquad \frac{\langle\Gamma\rangle \vdash t|A}{\Gamma \vdash t|\Box^{\downarrow}A}\,[\Box^{\downarrow}\mathrm{R}]$$

$$\frac{\Gamma[\langle y|A\rangle] \vdash t|B}{\Gamma[x|\Diamond A] \vdash t[x/y]|B}\,[\Diamond\mathrm{L}] \qquad \frac{\Gamma \vdash t|A}{\langle\Gamma\rangle \vdash t|\Diamond A}\,[\Diamond\mathrm{R}]$$

Fig. 1. Gentzen sequent calculus for $\mathsf{NL}(\Diamond)$.

Notice that the unary operators do not modify the semantic interpretation of the category.

The base Gentzen system for $\mathsf{NL}(\Diamond)$ can be extended with structural postulates. In [9] the following result was proved. Define weak Sahlqvist structural rules as rules of the form

$$\frac{\Gamma[\Sigma'[\Phi_1, \ldots, \Phi_m]] \vdash C}{\Gamma[\Sigma[\Delta_1, \ldots, \Delta_n]] \vdash C}$$

subject to the following conditions:

1. both Σ and Σ' contains only the structural operators $\circ$, $\langle\cdot\rangle$;
2. Σ' contains at least one structural operator;
3. there is no repetition of variables in $\Delta_1, \ldots, \Delta_n$;
4. all variables in $\Phi_1, \ldots, \Phi_m$ occur in $\Delta_1, \ldots, \Delta_n$.

Theorem 2.3 (Sahlqvist Completeness) *If P is a weak Sahlqvist structural rule, then $\mathsf{NL}(\Diamond) + P$ is a sound and complete calculus for the class of models satisfying the first-order condition on the accessibility relations corresponding to P.*

For example, if certain linguistic phenomena requires associative composition, we can introduce a new mode $_a$ for $\bullet$ and require that whenever $A \bullet_a (B \bullet_a C)$

is the case $(A \bullet_a B) \bullet_a C$ is also the case (i.e., $A \bullet_a (B \bullet_a C) \to (A \bullet_a B) \bullet_a C$ is a tautology of the logic). To obtain this we would extend the Gentzen calculus with the rule

$$\frac{\Gamma[\Delta_1 \circ_a (\Delta_2 \circ_a \Delta_3)] \vdash D}{\Gamma[(\Delta_1 \circ_a \Delta_2) \circ_a \Delta_3] \vdash D} \text{[ASS]}.$$

By Theorem 2.3, the extended calculus is complete for the intended class of models. Again, structural rules do not modify semantic terms.

3 Linguistic Theories of Binding

In modern linguistic literature the binding problem has been addressed mainly by means of two different approaches. We briefly describe them below by looking at QPs and we refer the reader to [6] for an in-depth comparison.

Generative Grammar: Binding as Movement. Generative Grammars (GG) produce different logical forms (LFs) for an ambiguous well-formed clause. Due to the different constituent command relations at the level of LFs, the surface structure (s-structure) receives different meanings. Within this framework, the standard theory of quantifier scope (e.g. [19]) is based on two central assumptions: (i) Quantifier scope is determined by the constituent command relation holding at the level of LFs; (ii) QPs are assigned scope by undergoing *movement* to their scope positions in the derivation of the LF representations.

More specifically, the movement operation takes an s-structure and returns LF representations that are unambiguous with respect to quantifier scope relations. While moving the QP to a position higher than the one where it occurs syntactically, a variable (trace) is introduced which is bound by the moved constituent (the quantifier). The proper link between the binder and the bound variable is achieved by co-indexing trace and binder.

CTL: "In Situ" Binding. As we already mentioned, discrepancies between syntactic and semantic composition can be difficult to describe and analyze in CTL. To tackle this problem in the case of problematic binders, Moortgat provides in [12] a three-place operator $q(A, B, C)$ which captures their behavior "in situ." The intuitive interpretation is the following. Syntactically, the q-category occupies the position of an expression of category A within a structural context of category B and while doing so it turns it into one of category C. Consider the example below involving the (binary) generalized quantifier *more-than*:

John bought *more* books *than* Mary sold records.

The structure "more books" receives category $q(np, s, s/s_{\text{than}})$, because:

1. it syntactically behaves as "apples" in "John bought apples" and "apples" has category np,
2. it can only do so inside a structural context of category s: "red apples" is grammatical but "red more books" is not,

3. by replacing the np it creates a requirement for the "than"-part of the generalized quantifier, turning the s category of the sentence into s/s_{than}.

The following inference rule captures this behavior

$$\frac{\Delta[A] \vdash B}{\Delta[q(A,B,C)] \vdash C}\ .$$

In addition, the q-operator should also reflect the binding and scoping behavior of QPs. Let z be the lambda term associated to $q(A,B,C)$, then z should be able to take scope over any structure t it might occur in and bind the term x it's taking the place of. As is standard, we will assign a higher order type to the QPs (i.e., semantically $q(A,B,C)$ has type $(\mathsf{type}(A) \rightarrow \mathsf{type}(B)) \rightarrow \mathsf{type}(C))$, so that binding x actually means applying z to the function $\lambda x.t$. In a rule:

$$\frac{\Delta[x|A] \vdash t|B}{\Delta[z|q(A,B,C)] \vdash z(\lambda x.t)|C}\ .$$

Finally, all this can happen with the q-category deeply embedded in a structure, so that we have to enable the inference rule to work on arbitrary contexts. Formally, let Γ, Δ be structures, z, u, y, t lambda terms and x a new variable,

$$\frac{\Delta[x|A] \vdash t|B \quad \Gamma[y|C] \vdash u|D}{\Gamma[\Delta[z|q(A,B,C)]] \vdash u[z(\lambda x.t)/y]|D}\ [qL].$$

Notice how this rule of use captures the behavior we described above: the q-operator binds a variable x of category A in the binding domain corresponding to Δ of category B (left branch) while turning it into one of category C (right branch).

As an example, let's consider the sentence "John thinks someone left." In this framework the category assigned to the unary QP "someone" is $q(np,s,s)$ whose behavior is illustrated by the derivation below (we show only the relevant lambda terms),

$$\dfrac{\dfrac{\dfrac{np \vdash np \quad s \vdash s}{np \bullet np\backslash s \vdash s}[\backslash L] \quad \dfrac{np \vdash np \quad s \vdash s}{np \bullet np\backslash s \vdash s}[\backslash L]}{np \bullet ((np\backslash s)/s \bullet (x|np \bullet np\backslash s)) \vdash t|s}[/L] \quad y|s \vdash y|s}{\underbrace{np}_{\text{john}} \bullet (\underbrace{(np\backslash s)/s}_{\text{thinks}} \bullet (\underbrace{\mathtt{someone}|q(np,s,s)}_{\text{someone}} \bullet \underbrace{np\backslash s}_{\text{left}})) \vdash y[\mathtt{someone}(\lambda x.t)/y]|s}[qL]$$

where t is $\mathtt{think}(\mathtt{left}(x))(\mathtt{john})$.

The q-rule provides a solution to long distance binding in terms of a sequent calculus. However, it is a rule of use and not a logical rule, since the q-connective is not *primitive*. The challenge is to show how it can be synthesized in terms of the primitive operators of CTL (or some extension thereof).

To get a better grasp on the problem we are analyzing, we can decompose it into two sub-problems: 1) How do QPs reach the high level structure over which they take scope (right branch of the q-rule)? and 2) How do they bind the proper variables within such a structure (left branch)? We will refer to these two sub-problems as the *scoping* and *binding* problem, respectively.

In GG the first question was dealt with by means of the movement operation and the second by means of co-indexing traces and binders. In CTL a solution involving "discontinuous" operators (i.e., modalities which are able to access deeply embedded arguments) has been proposed in [11,17], while different definitions of the q-operator in terms of primitive operators of the base logic $\mathsf{NL}(\Diamond)$ and structural postulates are provided in [13,16]. We will briefly review the latter approach as it will be the basis for our own proposal.

In Moortgat's solutions the issues of scoping and binding are dealt with by means of two complementary sets of structural postulates (variations of the mixed associativity and mixed commutativity postulates). The first one makes accessible to the QP the structural position from where it can take scope over the higher sentential clause. The second set of postulates reconstructs the structure of the whole expression containing the hypothetical np and brings the latter back to the position originally occupied by the quantifier. In both cases, the postulates build a series of internal structural points which are copies of the original ones. Both the movement of the QP and its replacement with the np are reduced to structural recomposition inferred step by step.

Moortgat's decompositions shed light on the proof theoretical behavior of the q-operator. However, they do not seem to be the proper solution to the "in situ" binding problem: in the first solution the category assigned to the quantifier is fairly natural $(\Diamond(s/(\Box^{\downarrow}np\backslash s))$ [1], but the postulates required are difficult to motivate (using what Moortgat calls a "place holder" to mark the position where the QP should end); in the second solution the postulates have been simplified at the expense of assigning the category $(s/(np\backslash s)) \bullet (np\backslash np)$ to the QP (the identity $np\backslash np$ is called an "annihilator" by Moortgat).

What seems to be missing in the CTL framework is a way to refer to structural positions so that we can ensure that both the QP and np hold at the same point. In [13,16], this lack in the logical setting is overcome by means of the ad-hoc use of the place-holder or the annihilator. We will now show how a "hybridized" version of CTL provides us instead with the required expressivity to directly enforce this condition.

4 Hybrid CTL

Standard modal languages (like the basic modal language or CTL itself) lack a means to directly refer to elements in the model. These languages can't express (in the object language) that a formula should be true at a particular point. As an example, the formula $\Diamond p$ says that p is true in *some* accessible point (in first-order terms $\exists y(Rxy \wedge Py)$), but we cannot *force* that point to

[1] The assigned category is simply the result of the translation of the standard semantic type $(e \rightarrow t) \rightarrow t$ of QPs into the CTL language with the addition of unary modalities to control the application of a special set of structural postulates.

be a particular point i ($\exists y(Rxy \wedge Py \wedge y = i)$, or simply $Rxi \wedge Pi$). Hybridization solves this expressivity shortcoming by adding a way to assign names to points, together with different mechanisms to access named points and a limited form of equality [2]. Interestingly, in many cases hybridization leads not only to enhanced expressive power, but also to a better behaved system allowing for straightforward axiomatization of different frame classes and a generally improved model theory. Hybridization for the basic modal logic has been by now extensively investigated [2,1].

To hybridize $\mathsf{NL}(\Diamond)$ we only need two simple changes. We first extend the vocabulary to include names (technically, they are called "nominals") and the :-operator to assign formulas to named points ($i{:}p$ is called an :-formula and is read "p is true at the point named i"). Formally, given a set $\mathsf{PROP} = \{p, q, r, \ldots\}$ of propositional symbols and a set $\mathsf{NOM} = \{i, j, k, \ldots\}$ of nominals, the language of $\mathsf{HNL}(\Diamond)$ (hybrid $\mathsf{NL}(\Diamond)$) is defined recursively as

$$\mathsf{FORM} ::= \top \mid A \mid i \mid \mathsf{FORM} * \mathsf{FORM} \mid \heartsuit\mathsf{FORM} \mid i{:}\mathsf{FORM}$$

where $A \in \mathsf{PROP}$, $i \in \mathsf{NOM}$, $* \in \{\backslash, \bullet, /\}$ and $\heartsuit \in \{\Diamond, \Box^{\downarrow}\}$. This is the basic system with only one mode for each of the residuated operators. Again, multi-mode extensions are standard.

The second step to complete hybridization is to adjust the models to ensure that nominals actually *name* single points. Hybrid Kripke models are tuples of the form $\langle W, R^{\bullet}, R^{\Diamond}, V, \cdot^{*}\rangle$ where $\langle W, R^{\bullet}, R^{\Diamond}, V\rangle$ is a standard Kripke model for $\mathsf{NL}(\Diamond)$ and $\cdot^{*} : \mathsf{NOM} \to W$ is a naming function assigning to each element of NOM a corresponding element in W. The semantic of the standard CTL operators is then defined as always, the hybrid part is as below. Given a hybrid model $M = \langle W, R^{\bullet}, R^{\Diamond}, V, \cdot^{*}\rangle$ and $w \in W$,

$$\begin{array}{lcl} M, w \models i & \text{iff} & i^{*} = w, \\ M, w \models i{:}\phi & \text{iff} & M, i^{*} \models \phi. \end{array}$$

For example, the formula $\Diamond j$ will be true in the point w if w can access the point named j (e.g., $R^{\Diamond}wj^{*}$). Similarly $(j \bullet k)$ is true in w if $R^{\bullet}wj^{*}k^{*}$. But we can go one step further: formulas like $i{:}\Diamond j$ and $i{:}(j \bullet k)$ actually force the corresponding named points to stand in a particular accessibility relation ($R^{\Diamond}i^{*}j^{*}$ and $R^{\bullet}i^{*}j^{*}k^{*}$, respectively), and this is independent of where the formula is evaluated.

By using nominals and the :-operator together with the modal operators $\Diamond$ and $\bullet$ we have now at our disposal a very flexible language to specify classes of models. As an example, we can enforce a condition like (1) on the accessibility relation $R^{\bullet}$ by ensuring that (2) is a tautology of the system:

$$(1)\ \ \forall ijk(R^{\bullet}ijk \to R^{\bullet}kji) \qquad\qquad (2)\ \ i{:}(j \bullet k) \to k{:}(j \bullet i).$$

[2] The term "hybrid" refers exactly to the ability that the extended language has to freely mix propositions and terms in the object language.

That (1) implies (2) is easy to check. Now, suppose (2) is satisfied in a frame $F = \langle W, R^{\bullet}, R^{\Diamond} \rangle$ but (1) isn't. Then there are elements w_1, w_2, w_3 in the domain of F such that $R^{\bullet}w_1w_2w_3$ but not $R^{\bullet}w_3w_2w_3$. Let M be any model based on F such that $i^* = w_1$, $j^* = w_2$ and $k^* = w_3$. Then clearly $M \not\models i{:}(j \bullet k) \rightarrow k{:}(j \bullet i)$.

It's not difficult to check that (1) cannot be expressed in $\mathsf{NL}(\Diamond)$. Similarly, conditions involving equality of certain elements in the model can now be expressed as the formula $i{:}j$ is true iff i and j denote the same point in a model. We will take advantage of this enhanced expressivity at the frame level in Section 5 when modeling the behavior of in situ quantifiers.

Even though hybridization results also in extended expressivity at the *formula level*, we won't make use of this new expressivity in our solution (i.e., the lexical categories assigned in the lexicon are all standard $\mathsf{NL}(\Diamond)$ formulas).

Hybrid Proof Theory. In Figure 2 we introduce a Gentzen sequent calculus for $\mathsf{HNL}(\Diamond)$ (drawing from ideas already present in [9,21]). The calculus uses sequents with single-formula right sides as is standard in CTL, but it uses multi-formula left sides instead of structures. Structural operators are no longer required as their role is internalized into the logics by means of the hybrid machinery: all formulas in a proof are prefixed by :-operators (see [9,21]) receiving in this way unique labels. The structural relation between the different labels is explicitly given in the sequent. Such calculus are usually called ":-driven" or "labeled."

The rules in Figure 2 can be proved sound and complete with respect to the following notion of validity. A hybrid model M satisfies a sequent $\Gamma \vdash A$ for $\Gamma \cup \{A\}$ a set of :-formulas in FORM, if $M \models \Gamma$ implies $M \models A$. A sequent is valid if it is satisfied in every model.

Recipes for semantic construction (i.e., lambda terms) can be added to the rules in Figure 2 in the usual way. The rules governing the hybrid operators do not modify the semantic representation.

5 "In Situ" Binding in $\mathsf{HNL}(\Diamond)$

Let's start by having a closer look at the behavior of QPs in CTL. Following the Curry-Howard correspondence and our discussion in Section 3, a QP is (modulo directionality) of category $s/(np\backslash s)$ though *syntactically* it behaves as *a simple np*. In other words, the higher order category assigned to QPs (required to obtain a proper semantical binding) has to be such that it can be "lowered" to np once its semantical role has been performed. Two things are required to obtain this lowering: the embedded np in $s/(np\backslash s)$ should take the place of the whole category and the s categories should disappear.

As we will now show in detail, $\mathsf{HNL}(\Diamond)$ allows us to enforce these requirements easily. No special markers (like the place-holder and annihilator of [13,16]) are required to indicate which points should be identified: each

$$\frac{}{i{:}A \vdash i{:}A}\ [\mathrm{Ax}]$$

$$\frac{\Gamma \vdash k{:}B \quad \Delta, i{:}A \vdash l{:}C}{\Gamma, \Delta, j{:}A/B, i{:}j \bullet k \vdash l{:}C}\ [/\mathrm{L}] \qquad \frac{\Gamma, k{:}B, i{:}j \bullet k \vdash i{:}A}{\Gamma \vdash j{:}A/B}\ [/\mathrm{R}]\ i, j \text{ new}$$

$$\frac{\Delta \vdash j{:}B \quad \Gamma, i{:}A \vdash l{:}C}{\Gamma, \Delta, k{:}B\backslash A, i{:}j \bullet k \vdash l{:}C}\ [\backslash\mathrm{L}] \qquad \frac{\Gamma, j{:}B, i{:}j \bullet k \vdash i{:}A}{\Gamma \vdash k{:}B\backslash A}\ [\backslash\mathrm{R}]\ i, j \text{ new}$$

$$\frac{i{:}j \bullet k, j{:}A, k{:}B, \Gamma \vdash l{:}C}{i{:}(A \bullet B), \Gamma \vdash l{:}C}\ [\bullet\mathrm{L}]\ j, k \text{ new} \qquad \frac{\Gamma \vdash j{:}A \quad \Delta \vdash k{:}B}{\Gamma, \Delta, i{:}j \bullet k \vdash i{:}A \bullet B}\ [\bullet\mathrm{R}]$$

$$\frac{u{:}\Diamond v, v{:}A, \Gamma \vdash l{:}B}{u{:}\Diamond A, \Gamma \vdash l{:}B}\ [\Diamond\mathrm{L}]\ v \text{ new} \qquad \frac{\Gamma \vdash v{:}A}{u{:}\Diamond v, \Gamma \vdash u{:}\Diamond A}\ [\Diamond\mathrm{R}]$$

$$\frac{\Gamma, v{:}A \vdash l{:}B}{\Gamma, v{:}\Diamond u, u{:}\Box^{\downarrow} A \vdash l{:}B}\ [\Box^{\downarrow}\mathrm{L}] \qquad \frac{\Gamma, v{:}\Diamond u \vdash v{:}B}{\Gamma \vdash u{:}\Box^{\downarrow} B}\ [\Box^{\downarrow}\mathrm{R}]\ v \text{ new}$$

$$\frac{v{:}A, \Gamma \vdash l{:}C}{u{:}v{:}A, \Gamma \vdash l{:}C}\ [{:}\mathrm{L}] \qquad \frac{\Gamma \vdash v{:}C}{\Gamma \vdash u{:}v{:}C}\ [{:}\mathrm{R}]$$

$$\frac{\Gamma, A[i/j], i{:}j \vdash l{:}C}{\Gamma, A, i{:}j \vdash l{:}C}\ [\mathrm{EqL}] \qquad \frac{\Gamma, i{:}j \vdash l{:}C[i/j]}{\Gamma, i{:}j \vdash l{:}C}\ [\mathrm{EqR}]$$

$$\frac{\Gamma, j{:}i \vdash l{:}C}{\Gamma, i{:}j \vdash l{:}C}\ [\mathrm{SL}] \quad \frac{\Gamma \vdash j{:}i}{\Gamma \vdash i{:}j}\ [\mathrm{SR}] \quad \frac{\Gamma \vdash l{:}C}{\Gamma, A \vdash l{:}C}\ [\mathrm{Thin}]\ A \text{ of the form } i{:}j,\ i{:}\Diamond j,\ i{:}j \bullet k$$

Fig. 2. Labeled Gentzen sequent calculus for HNL($\Diamond$)

point in the structure of a proof is labeled, and the logical language allows us to express identity of points when necessary. Thus, the HNL($\Diamond$) solution to the QP puzzle avoids the unpleasant use of traces and double representation levels of GG but takes its idea of linking the QP and hypothetical *np* by means of indexes. On the other hand, it enjoys the advantages of CTL with respect to GG while avoiding the use of ad-hoc tools.

QPs in HNL($\Diamond$). We now explain step by step how the proper category for the QPs can be defined in HNL($\Diamond$).

The first step is to "color" the category assigned to the QP so that the special reasoning driven by it can be applied. We introduce a new mode $_q$ for the unary and binary modalities and use the prefix $\Diamond_q \Box_q^{\downarrow}$ to mark the QP category, obtaining $\Diamond_q \Box_q^{\downarrow}(s/_q(np\backslash^q s))$. As $\Diamond\Box^{\downarrow} A \to A$ is a theorem of NL($\Diamond$) we ensure that the prefix can be eliminated when required. Such "coloring" is a standard device in CTL (see, e.g., the account of wh-extraction in [15]).

Secondly, by means of a set of interaction mode postulates (Figure 3) we make the scoping domain accessible by the QP (these postulates play the role of the variations of mixed associativity and mixed commutativity used

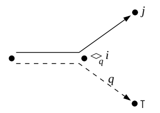
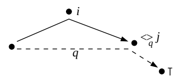
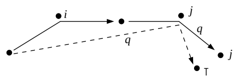
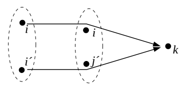
[P1] Triggering Left: $(\Diamond_q i \bullet j) \to \Diamond_q i \bullet_q \top$		
FO Condition:	$\forall xyij(R^{\bullet}xyj \wedge R^{\Diamond}_q yi \to \exists vw(R^q xvw \wedge R^{\Diamond}_q vi))$	
Structural Rule:	$\dfrac{\Gamma, m{:}n \bullet j, n{:}\Diamond_q i, m{:}n \bullet_q k \vdash C}{\Gamma, m{:}n \bullet j, n{:}\Diamond_q i \vdash C}$ k new	
[P1′] Triggering Right: $(i \bullet \Diamond_q j) \to \Diamond_q j \bullet_q \top$		
FO Condition:	$\forall xyij(R^{\bullet}xiy \wedge R^{\Diamond}_q yj \to \exists vw(R^q xvw \wedge R^{\Diamond}_q vj))$	
Structural Rule:	$\dfrac{\Gamma, m{:}i \bullet n, n{:}\Diamond_q j, m{:}n \bullet_q k \vdash C}{\Gamma, m{:}i \bullet n, n{:}\Diamond_q j, \vdash C}$ k new	
[P2] Scoping: $i \bullet (j \bullet_q k) \to j \bullet_q \top$		
FO Condition:	$\forall xiyjk(R^{\bullet}xiy \wedge R^q yjk \to \exists z(R^q xjz))$	
Structural Rule:	$\dfrac{\Gamma, m{:}i \bullet n, n{:}j \bullet_q k, m{:}j \bullet_q l \vdash C}{\Gamma, m{:}i \bullet n, n{:}j \bullet_q k \vdash C}$ l new	
[P3] Binding: $(i{:}j \bullet_q k) \bullet (i'{:}j' \bullet_q k) \to i{:}i' \bullet j{:}j'$		
FO Condition	$\forall ii'jj'k(R^q ijk \wedge R^q i'j'k \to i = i' \wedge j = j')$	
Structural Rule:	$\dfrac{\Gamma, i{:}j \bullet_q k, i'{:}j' \bullet_q k, i{:}i', j{:}j' \vdash C}{\Gamma, i{:}j \bullet_q k, i'{:}j' \bullet_q k \vdash C}$	

Fig. 3. Structural rules

in [13,16]). The structural reasoning governed by the postulates is triggered by the category assigned to the QP. The [P1] and [P1′] apply first (depending on the QP category being a left or right argument of the composition operator $\bullet$). Then, [P2] gives access to distant structures till the proper scoping domain is reached. Note how the unary modalities are the anchorage for the structural changes, and the binary mode $\bullet_q$ allows the shifting of its first argument's structural position. Finally, the "injectivity" condition [P3] ensures the correct placing of the np and s categories as we explained above, by forcing equality of the appropriate points in the model. This is the core axiom in our solution, and the point where the additional expressivity provided by the hybrid operators is crucial.

The logical system obtained by the addition of the postulates in Figure 3 to HNL($\Diamond$) can be proved sound and complete with respect to the appropriate class of models (i.e., the class of models where the accessibility relation satisfy the first-order conditions given in the figure) in a standard way.

We will now exemplify the HNL($\Diamond$) analysis by showing the system at work on our sample sentence, *John thinks someone left.* The (partial) model given in Figure 4 shows the structure created during the proof and might help the reader to better understand the sequent derivation (notice how nodes 1 and 13, and 6 and 12 are identified by the structural rule [P3]).

The proof consists of five main steps: (i) Naming, (ii) Marking, (iii) Triggering, (iv) Scoping and (v) Binding. In the first one, we simply build the accessibility relations and name the points where the input formulas hold or fail. The marking step frees the quantifier category from the unary modalities and signals with $\Diamond_q$ the point where it holds.

$$\textit{Naming}\quad \dfrac{1{:}2 \bullet 3, 3{:}4 \bullet 5, 5{:}6 \bullet 7, 2{:}np, 4{:}(np\backslash s)/s, 6{:}\Diamond_q\Box_q^{\downarrow}(s/_q(np\backslash^q s)), 7{:}np\backslash s \vdash 1{:}s}{1{:}(\underbrace{np}_{\text{john}} \bullet(\underbrace{(np\backslash s)/s}_{\text{thinks}} \bullet(\underbrace{\Diamond_q\Box_q^{\downarrow}(s/_q(np\backslash^q s)}_{\text{someone}} \bullet \underbrace{np\backslash s}_{\text{left}})))) \vdash 1{:}s}\,[\bullet\text{L} \times 4]$$

Let $\Gamma = 2{:}np, 4{:}(np\backslash s)/s, 7{:}np\backslash s$

$$\textit{Marking}\quad \dfrac{\dfrac{1{:}2 \bullet 3, 3{:}4 \bullet 5, 5{:}6 \bullet 7, 6{:}\Diamond_q 8, \Gamma, 6{:}s/_q(np\backslash^q s) \vdash 1{:}s}{1{:}2 \bullet 3, 3{:}4 \bullet 5, 5{:}6 \bullet 7, 6{:}\Diamond_q 8, \Gamma, 8{:}\Box_q^{\downarrow}(s/_q(np\backslash^q s)) \vdash 1{:}s}\,[\Box_q^{\downarrow}\text{L}]}{1{:}2 \bullet 3, 3{:}4 \bullet 5, 5{:}6 \bullet 7, \Gamma, 6{:}\Diamond_q\Box_q^{\downarrow}(s/_q(np\backslash^q s)) \vdash 1{:}s}\,[\Diamond_q\text{L}]$$

At this point structural reasoning is applied to make the scoping domain accessible from the quantifier's position, so that the QP category can be unfolded by means of $[/_q\text{L}]$.

$$\textit{Triggering}\quad \dfrac{1{:}2 \bullet 3, 3{:}4 \bullet 5, 5{:}6 \bullet 7, 6{:}\Diamond_q 8, 5{:}6 \bullet_q 9, \Gamma, 6{:}s/_q(np\backslash^q s) \vdash 1{:}s}{1{:}2 \bullet 3, 3{:}4 \bullet 5, 5{:}6 \bullet 7, 6{:}\Diamond_q 8, \Gamma, 6{:}s/_q(np\backslash^q s) \vdash 1{:}s}\,[\text{P1}]$$

Let $\Gamma' = \Gamma \cup \{5{:}6 \bullet 7, 6{:}\Diamond_q 8\}$

$$\textit{Scoping}\quad \dfrac{\dfrac{\dfrac{1{:}2 \bullet 3, 3{:}4 \bullet 5, 5{:}6 \bullet_q 9, 3{:}6 \bullet_q 10, 1{:}6 \bullet_q 11, \Gamma' \vdash 11{:}np\backslash^q s \quad 1{:}s \vdash 1{:}s}{1{:}2 \bullet 3, 3{:}4 \bullet 5, 5{:}6 \bullet_q 9, 3{:}6 \bullet_q 10, 1{:}6 \bullet_q 11, \Gamma', 6{:}s/_q(np\backslash^q s) \vdash 1{:}s}\,[/_q\text{L}]}{1{:}2 \bullet 3, 3{:}4 \bullet 5, 5{:}6 \bullet_q 9, 3{:}6 \bullet_q 10, \Gamma', 6{:}s/_q(np\backslash^q s) \vdash 1{:}s}\,[\text{P2}]}{1{:}2 \bullet 3, 3{:}4 \bullet 5, 5{:}6 \bullet_q 9, \Gamma', 6{:}s/_q(np\backslash^q s) \vdash 1{:}s}\,[\text{P2}]$$

The last step to be computed is the binding of the hypothetical np. As described by the q-rule this operation is properly performed when the np occupies the same structural position of the binder. This result is achieved by means of [P3] as shown below. Let $\Gamma'' = \Gamma \cup \{1{:}2 \bullet 3, 3{:}4 \bullet 5, 5{:}6 \bullet_q 9, 3{:}6 \bullet_q 10\}$

$$\textit{Binding}\quad \dfrac{\dfrac{\dfrac{1{:}6 \bullet_q 11, 13{:}12 \bullet_q 11, 1{:}13, 6{:}12, \Gamma'', 6{:}np \vdash 1{:}s}{1{:}6 \bullet_q 11, 13{:}12 \bullet_q 11, 1{:}13, 6{:}12, \Gamma'', 12{:}np \vdash 13{:}s}\,[\text{EqL}]}{1{:}6 \bullet_q 11, 13{:}12 \bullet_q 11, \Gamma'', 12 : np \vdash 13 : s}\,[\text{P3}]}{1{:}6 \bullet_q 11, \Gamma'' \vdash 11{:}np\backslash^q s}\,[\backslash^q\text{R}]$$

The proof continues by applying the logical rules as in the left branch of the example in Section 3 and reaches the same axioms. Therefore, the labeling of the proof by lambda terms would give the same semantics obtained by the use of the q-rule as discussed there.

As for empirical coverage, given that the solution we propose satisfies the requirements encoded in the q-rule, it accounts for all the phenomena covered by the latter (see [5] for and in-depth discussion). In particular, multiple semantic representations of ambiguous sentences are properly obtained: a different reading is obtained whenever the q-operator reaches a suitable scoping domain either because it is already in a proper position (e.g. `(think(someone (left))(john)` in our sample sentence), or because it is moved by means of the [P1] or [P1′], and [P2] structural postulates as shown above. Moreover, the use of the identification of the structural points where the q-category and

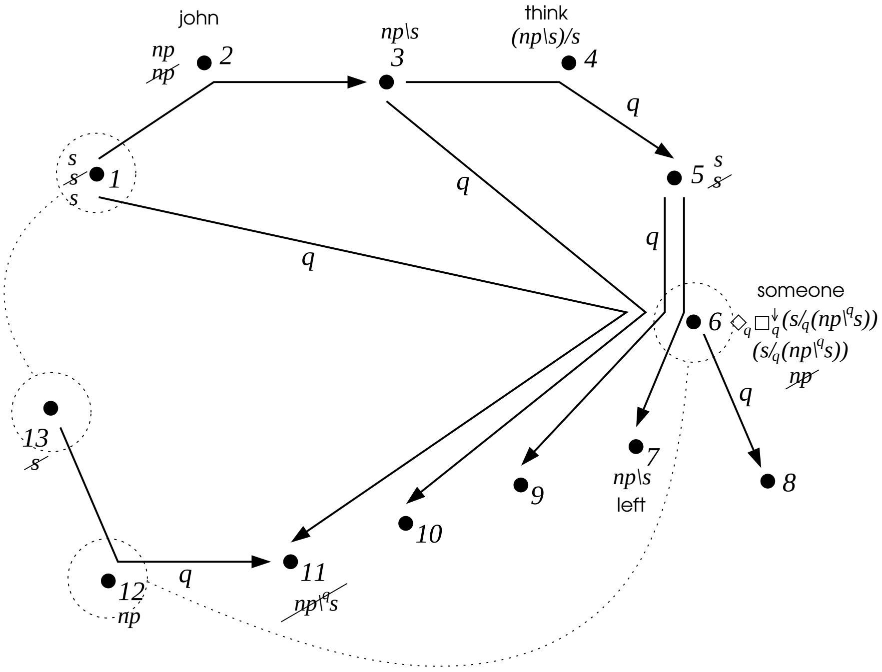


Fig. 4. (Partial) Model corresponding to the example derivation.

hypothetical np-category hold (i.e., the application of [P3]) guarantees that the variables are properly bound also when multiple quantifiers occur.

In addition, the proposed solution overcomes some of the limitations of the q-rule: (i) on the theoretical level, the analysis is done purely in logical terms; and (ii) on the empirical level it avoids the problems encountered by the q-category when in construction with coordination. For instance, the coordination of QPs with proper names can be achieved in the proposed solution by means of a category which is derivable from both (i.e., $s/(np\backslash s)$). Similarly, one could deal with the coordination of QPs with reflexive pronouns by means of the derivable category $(np\backslash s)/(np\backslash(np\backslash s))$.

6 Conclusion and Further Research

We have shown that the hybrid apparatus allows us to define frame classes useful for modeling in situ binding. Departing from the standard CTL tradition, in this paper we have taken a strong model-theoretic perspective which we believe can shed light on the use and possibilities of structural postulates.

There remain many questions and directions for further research. Hybrid logics usually allow for general completeness results (using "pure axioms", see [4]) and this kind of results might also be possible for $\mathsf{HNL}(\Diamond)$. Moreover, we have not yet explored complexity issues for $\mathsf{HNL}(\Diamond)$. Finally, the whole range of extended expressivity for natural language applications offered by $\mathsf{HNL}(\Diamond)$ should be further explored (e.g. hybrid lexical entries).

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