# Logica & Linguaggio: Calcolo di Lambda III

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# 1. Recall: Formal Semantics Main questions

The main questions are:

1. What does a given sentence mean?
2. How is its meaning built?
3. How do we infer some piece of information out of another?

## 1.1. Building Meaning Representations

To build a meaning representation we need to fulfill three tasks:

**Task 1** Specify a reasonable **syntax** for the natural language fragment of interest.

**Task 2** Specify semantic representations for the **lexical items**.

**Task 3** Specify the **translation** of constituents **compositionally**. That is, we need to specify the translation of such expressions in terms of the translation of their parts, parts here referring to the substructure given to us by the syntax.

Moreover, when interested in Computational Semantics, all three tasks need to be carried out in a way that leads to computational implementation naturally.

# 2. Determiners

Which is the lambda term representing quantifiers like "nobody", "everybody", "a man" or "every student" or a determiners like "a", "every" or "no" ? We know how to represent in FOL the following sentences

- "Nobody left"
  $\neg\exists x.\texttt{left}(x)$
- "Everybody left"
  $\forall x.\texttt{left}(x)$
- "Every student left"
  $\forall x.\texttt{Student}(x) \to \texttt{left}(x)$
- "A student left"
  $\exists x.\texttt{Student}(x) \wedge \texttt{left}(x)$
- "No student left"
  $\neg\exists x.\texttt{Student}(x) \wedge \texttt{left}(x)$

But how do we reach these meaning representations starting from the lexicon?

## 2.1. Determiners (cont'd)

Let's start representing "a man" as $\exists x.man(x)$. Applying the rules we have seen so far, we obtain that the representation of "A man loves Mary" is:

$$love(\exists x.man(x), mary)$$

which is clearly wrong.

Notice that $\exists x.man(x)$ just isn't the meaning of "a man". If anything, it translates the complete sentence "There is a man".

FOL does not give us the possibility to express its meaning representation. We will see now that instead lambda terms provide us with the proper expressivity.

## 2.2. Quantified NP

a) Every male student of Filosofia & Informatica attends the LoLa course.

b) No male student of Filosofia & Informatica attends the Logic course.

a) means that if Enrico constitute the set of the male students of Filosofia & Informatica, then it is true that he attends the LoLa course.

b) means that for none of the individual members of the set of male students of Filosofia & Informatica it is true that he attends the Logic course.

What is the interpretation of "every male student" and of "no male student"?

Individual constants used to denote specific individuals cannot be used to denote quantified expressions like "every man", "no student", "some friends".

Quantified-NPs like "every man", "no student", "some friends" are called non-referential.

## 2.3. Generalized Quantifiers

A Generalized Quantifier (GQ) is a set of properties, i.e. **a set of sets-of-individuals**.

For instance, "every man" denotes the set of properties that every man has. The property of "walking" is in this set iff every man walks. For instance,

$$\begin{array}{lcl} [\![\texttt{man}]\!] & = & \{a, b, c\}; \\ [\![\texttt{fat}]\!] & = & \{a, b, c, d\}; \\ [\![\texttt{dog}]\!] & = & \{d\}; \\ [\![\texttt{run}]\!] & = & \{a, b\}; \\ [\![\texttt{jump}]\!] & = & \{b, c, d\}; \\ [\![\texttt{laugh}]\!] & = & \{b, d\}; \end{array}$$

Which is the interpretation of "every man"? What do we know that holds for every man?

"Every man is man", "Every man is fat"

$$[\![\texttt{every man}]\!] = \{X | [\![\texttt{man}]\!] \subseteq X\} = \{\{a, b, c\}, \{a, b, c, d\}\} = \{[\![\texttt{man}]\!], [\![\texttt{fat}]\!]\}$$

## 2.4. Generalized Quantifiers

$$
\begin{array}{lcl}
[\![\texttt{no man}]\!] & = & \{X \subseteq E \mid [\![\texttt{man}]\!] \cap X = \emptyset\}. \\
[\![\texttt{some man}]\!] & = & \{X \subseteq E \mid [\![\texttt{man}]\!] \cap X \neq \emptyset\}. \\
[\![\texttt{every man}]\!] & = & \{X \subseteq E \mid [\![\texttt{man}]\!] \subseteq X\}. \\
[\![\texttt{man which VP}]\!] & = & [\![\texttt{man}]\!] \cap [\![\texttt{VP}]\!].
\end{array}
$$

Therefore, determiners are as below:

$$
\begin{array}{lcl}
[\![\texttt{no N}]\!] & = & \{X \subseteq E \mid [\![\texttt{N}]\!] \cap X = \emptyset\}. \\
[\![\texttt{some N}]\!] & = & \{X \subseteq E \mid [\![\texttt{N}]\!] \cap X \neq \emptyset\}. \\
[\![\texttt{every N}]\!] & = & \{X \subseteq E \mid [\![\texttt{N}]\!] \subseteq X\}. \\
[\![\texttt{N which VP}]\!] & = & [\![\texttt{N}]\!] \cap [\![\texttt{VP}]\!].
\end{array}
$$

Generalized quantifiers have attracted the attention of many researchers working on the interaction between logic and linguistics.

Which is the lambda term representing quantifiers like “nobody”, “everybody”, “a man”or “every student” or a determiners like “a”, “every” or “no” ?

## 2.5. Determiners (Cont'd)

Let's start from what we have, namely "man" and "loves Mary":
$\lambda y.man(y)$, $\lambda x.love(x, mary)$.
Hence, the term representing "a" is:

$$\lambda X.\lambda Y.\exists z.X(z) \wedge Y(z)$$

Try to obtain the meaning representation for "a man", and the "a man loves Mary".
By $\beta$-conversion twice we obtain that "a man" is $\lambda Y.\exists z.\mathtt{Man}(z) \wedge Y(z)$, and then $\exists z.\mathtt{Man}(z) \wedge love(z, mary)$