Logica & Linguaggio: Linguaggio

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Contents

1. Aristotele, Stoici e Frege

- Gli aristotelici erano interessati ai rapporti tra i termini delle premesse e conclusioni di un ragionamento (Sillogismo: “Tutti gli A sono B”, “Tutti i B sono C”, dunque “Tutti gli A sono C”).
- Gli stoici consideravano centrale per la logica la relazione condizionale “Se ... allora”.
- Svolta del’900 è la sintesi di queste due tradizioni con l’introduzione dei quantificatori ad opera di Frege.

Frege Matematico, logico e filosofo tedesco. Cercava di sviluppare l’ideografia (un linguaggio formale) per superare i limiti del linguaggio naturale (ambiguo). A tal fine studiò il linguaggio naturale e portò alla luce importanti suoi aspetti tanto da essere considerato anche fodatore della filosofia del linguaggio.

1.1. Frege: Espressioni sature/insature

Frege generalizza il concetto di funzione: al posto di argomenti e valori possono esserci elementi qualsiasi. Possiamo quindi scrivere e.s. “Donna(x)”, se sostituiamo alla variabile una costante, e.s. “r” per “raffaella” otteniamo “Donna(r)=vero”.

Espressioni sature vs. insature Distingue le espressioni in sature (e.s. una frase) ed insature (e.s. un concetto).

“Cesare conquistò la Gallia”. “Cesare” è una parte finita in sé stessa (argomento della funzione) e “( ) conquistò la Gallia” è una espressione insatura, ha bisogno di essere completata da un argomento.

Aristotele aveva posto l’attenzione sulla struttura soggetto/predicato, al suo posto Frege introduce la distinzione argomento/funzione.

Funzioni di primo e secondo livello Le funzioni si differenziano quindi dagli oggetti, si differenziano inoltre tra funzioni che hanno come argomento altre funzioni e funzioni che hanno come argomento oggetti. Si definiscono di primo livello se l’argomento è un oggetto. Di secondo livello se l’argomento è una funzione.

1.2. Frege: Concetti, Quantificatori

- Concetto (termine che può fungere da predicato): è una funzione il cui valore, con un argomento x (oggetto, e.s. nome proprio), è sempre un valore di verità. Ciò vale sia per le proprietà che per le relazioni.
- Quantificatori: funzioni di secondo livello. Introduce i simboli $\forall, \exists$.

La logica di primo ordine, introdotta da Frege, contiene la sillogistica aristotelica:

$$\forall x(Greco\ x \rightarrow Uomo\ x)$$

$$\forall x(Uomo\ x \rightarrow Mortale\ x)$$

Dunque, $\forall x(Greco\ x \rightarrow Mortale\ x)$

Grazie ai simboli introddotti da Frege si possono rappresentare frasi con più di un quantificatore.

1.3. Frege: Forma logica vs. forma grammaticale

"C'è un numero naturale più grande di ciascun numero naturale."

1. $\forall x \exists y PGrande(y,x)$
2. $\exists y \forall x PGrande(y,x)$

1. è vero, mentre 2. è falso.

la differenza dell'interpretazione della frase è data dalla differenza del campo d'azione dei quantificatori. Frege distingue così tra:

- Forma grammaticale (soggetto-predicato)
- Forma logica (funzione-argomento)

2. Tarski

Costrusice una semantica formale in maniera rigorosa e precisa. Wittgeinstein aveva considerato le condizioni di verità per gli enunciati composti con i connettivi logici ma non aveva considerato le condizioni di verità per gli enunciati semplici e quantificati. Tarski fornisce una definizione precisa per tutti questi enunciati introducendo le nozioni di:

- modello
- dominio
- funzione di interpretazione
- soddisfazione
- assegnazione

pone le basi della Teoria dei Modelli.

3. Linguaggio Formale

Abbiamo visto che

- di un linguaggio formale si definisce:
  - la sintassi
  - la semantica
- che sintassi e semantica sono collegate (dal "sse" nella definizione di funzione di valutazione).
- che il significato di una formula è dato dal significato delle sue parti.

4. Natural Language

Which levels does NL have?

4.1. Syntax

Syntax "setting out things together", in our case things are words. The main question addressed here is "*How do words compose together to form a grammatical sentence (s) (or fragments of it)?*"

- *Part of Speech*: words are said to belong to *classes*/categories. The main categories are nouns (n), verbs (v), adjectives (adj), articles (art) and adverbs (adv).
- *Constituents*: Groups of categories may form a single *unit or phrase* called constituent. The main phrases are noun phrases (np), verb phrases (vp), prepositional phrases (pp). Noun phrases for instance are: "she"; "Michael"; "Rajeev Goré"; "the house"; "a young two-year child".

Tests like substitution help decide whether words form constituents. Another possible test is coordination.

4.2. Sentence Structures: English

The structure of a sentence can be represented in several ways, the most common are the following notations: (i) brackets or (ii) trees. For instance, "John ate the cat" is a sentence (s) consisting of noun phrase (np) and a verb phrase (vp). The noun phrase is composed of a verb (v) "ate" and an np, which consists of an article (art) "the" and a common noun (n) "cat".

$$[\text{John}_{np}\ [\text{ate}_{v}\ [\text{the}_{art}\ \text{cat}_{n}]_{np}]_{vp}]_{s}$$

There are Formal Grammars that given a linguistic string produce the parse tree/syntactic structure.

4.3. Dependencies

Dependency: Categories are interdependent, for example

Ryanair services $[\text{Pescara}]_{np}$	Ryanair flies $[\text{to Pescara}]_{pp}$
*Ryanair services $[\text{to Pescara}]_{pp}$	*Ryanair flies $[\text{Pescara}]_{np}$

the verbs services and flies determine which category can/must be juxtaposed. If their constraints are not satisfied the structure is ungrammatical.

4.4. Long-distance Dependencies

Interdependent constituents need not be juxtaposed, but may form long-distance dependencies, manifested by gaps

- What cities does Ryanair service [...]?

The constituent what cities depends on the verb service, but is at the front of the sentence rather than at the object position.

Such distance can be large,

- Which flight do you want me to book [...]?
- Which flight do you want me to have the travel agent book [...]?
- Which flight do you want me to have the travel agent nearby my office book [...]?

4.5. Relative Pronouns and Coordination

- *Relative Pronoun* (eg. who, which): they function as e.g. the subject or object of the verb embedded in the relative clause (*rc*),
  - [[the [student [who [...] knows Sara]$_{rc}$]$_{n}$]$_{np}$ [left]$_{v}$]$_{s}$.
  - [[the [book [which Sara wrote [...]]$_{rc}$]$_{n}$]$_{np}$ [is interesting]$_{v}$]$_{s}$.

- *Coordination*: Expressions of the same syntactic category can be coordinated via "and", "or", "but" to form more complex phrases of the same category. For instance, a coordinated verb phrase can consist of two other verb phrases separated by a conjunction:
  - There are no flights [[leaving Denver]$_{vp}$ and [arriving in San Francisco]$_{vp}$]$_{vp}$

  The conjuncted expressions belong to traditional constituent classes, *vp*. However, we could also have
  - I [[[want to try to write [...]] and [hope to see produced [...]]] [the movie]$_{np}$]$_{vp}$"

  Again, the interdependent constituents are disconnected from each other.

5. Semantics

Semantics: it's the study of the meaning of words and phrases.

Lexical Semantics Words are seen as having a systematic structure that governs what they mean, how they *relate to actual entities* and how they can be used. Studies on this topic result into e.g. Dictionary or Ontologies like WordNET.

Formal Semantics Meaning is based on references (the objects in the world) and logical language is use to represent this reference based meaning.

Distributional Semantics Meaning based on use/context.

We look at FS. Jacopo DS. Pianta/Vieu LS.

5.1. Formal Semantics

We will exploit:

- Set theory to represent the meaning of words and phrases.
- The relation between a set and its characteristic function.
- Lambda-Terms (and FOL) to represent functions capturing linguistic expressions.
- The principle of Compositionality [Frege]
- The connection between Syntax and Semantics [Montague]

5.2. Pioneers

Gottlob Frege Frege aims to avoid having to use natural language.

- Linguistics expressions can be divided into complete vs. not-complete.
- Proper name and sentences are complete (entity and truth value)
- A concept is not-complete, it's a one-argument function
- A transitive verb is not-complete, it's a two-argument function
- A quantifier phrase is not-complete, it's a higher order functions.

Richard Montague Montague aims to define a model-theoretic semantics for natural language. He treats natural language as a formal language:

- Syntax-Semantics go in parallel.
- It's possible to define an algorithm to compose the meaning representation of the sentence out of the meaning representation of its single words.

6. Formal Semantics: Main questions

The main questions are:

1. What does a given sentence mean?
2. How is its meaning built?
3. How do we infer some piece of information out of another?

The first and last question are closely connected.

In fact, since we are ultimately interested in understanding, explaining and accounting for the entailment relation holding among sentences, we can think of *the meaning of a sentence as its truth value*, as logicians teach us.

7. Logical Approach

To tackle these questions we will use Logic, since using Logic helps us answering the above questions at once.

1. Logics have a precise semantics in terms of *models* —so if we can translate/represent a natural language sentence S into a logical formula ϕ, then we have a precise grasp on at least part of the meaning of S.

2. Important *inference problems* have been studied for the best known logics, and often good *computational implementations* exist. So translating into a logic gives us a handle on inference.

When we look at these problems from a computational perspective, i.e. we bring in the implementation aspect too, we move from Formal Semantics to *Computational Semantics*.

7.1. Example

Let our model be based on the set of entities $D_e = \{\texttt{lori}, \texttt{ale}, \texttt{sara}, \texttt{pim}\}$ which represent *Lori, Ale, Sara* and *Pim*, respectively. Assume that they all know themselves, plus *Ale* and *Lori* know each other, but they do not know *Sara* or *Pim*; *Sara* does know *Lori* but not *Ale* or *Pim*. The first three are students whereas *Pim* is a professor, and both *Lori* and *Pim* are tall. This is easily expressed set theoretically. Let $[\![\texttt{w}]\!]$ (it's like I of Logic) indicate the interpretation of w:

$$
\begin{array}{lcl}
[\![\texttt{sara}]\!] & = & \text{sara}; \\
[\![\texttt{pim}]\!] & = & \text{pim}; \\
[\![\texttt{lori}]\!] & = & \text{lori}; \\
[\![\texttt{know}]\!] & = & \{\langle\text{lori, ale}\rangle, \langle\text{ale,lori}\rangle, \langle\text{sara, lori}\rangle, \\
 & & \langle\text{lori, lori}\rangle, \langle\text{ale, ale}\rangle, \langle\text{sara, sara}\rangle, \langle\text{pim, pim}\rangle\}; \\
[\![\texttt{student}]\!] & = & \{\text{lori, ale, sara}\}; \\
[\![\texttt{professor}]\!] & = & \{\text{pim}\}; \\
[\![\texttt{tall}]\!] & = & \{\text{lori, pim}\}.
\end{array}
$$

which is nothing else to say that, for example, the relation *know* is the *set of pairs* $\langle\alpha,\beta\rangle$ where α knows β; or that 'student' is the set of all those elements which are a student.

7.2. Exercises: Relations vs. Functions

Think of which function you can assign to the words in the model considered before and repeated here:

Sara, Pim, Lori, know, student, professor, tall,

7.3. Summing up

Summarizing, when trying to formalize natural language semantics, at least two sorts of objects are needed to start with: the set of *truth values* t, and the one of *entities* e.

Moreover, we spoke of more complex objects as well, namely functions. More specifically, we saw that the kind of functions we need are *truth-valued functions* (or boolean functions).

Furthermore, we have illustrated how one can move back and forwards between a *set relational and a functional perspective*. The former can be more handy and intuitive when reasoning about entailment relations among expressions; the latter is more useful when looking for lexicon assignments.

References: Keenen 85.

8. Formal Semantics: What

What does a given sentence mean?

The meaning of a sentence is its truth value. Hence, this question can be rephrased in "Which is the meaning representation of a given sentence to be evaluated as true or false?"

- *Meaning Representations*: Predicate-Argument Structures are a suitable meaning representation for natural language sentences. E.g.

  the meaning representation of "Vincent loves Mia" is $\texttt{loves}(\texttt{vicent}, \texttt{mia})$

  whereas the meaning representation of "A student loves Mia" is $\exists x.\texttt{student}(x) \wedge \texttt{loves}(x, \texttt{mia})$.

- *Interpretation*: a sentence is taken to be a proposition and its meaning is the truth value of its meaning representations. E.g.

  $[\![\exists x.\texttt{student}(x) \wedge \texttt{left}(x)]\!] = 1$ iff standard FOL (First Order Logic) definitions are satisfied.

9. Formal Semantics: How

How is the meaning of a sentence built?

To answer this question, we can look back at the example of “Vincent loves Mia”. We see that:

- “Vincent” contributes the constant `vincent`
- “Mia” contributes the constant `mia`
- “loves” contributes the relation symbol `loves`

This observation can bring us to conclude that the *words* making up a sentence contribute all the bits and pieces needed to build the sentence’s meaning representation.

In brief, *meaning flows from the lexicon*.

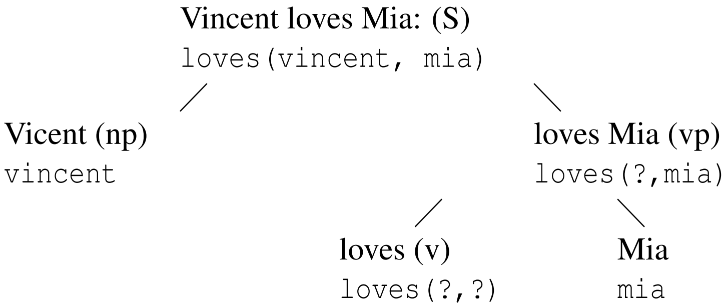
9.1. Formal Semantics: How (cont'd)

But,

1. Why the meaning representation of "Vincent loves Mia" is not `love(mia,vincent)`?
2. What does "a" contribute to in "A student loves Mia"?

As for 1., the missing ingredient is the *syntactic structure*! [Vincent [loves$_v$ Mia$_{np}$]$_{vp}$]$_s$.
We will come back to 2. next time.

9.2. Formal Semantics: How (Cont'd)



Briefly, *syntactic structure guiding gluing*.

9.3. Compositionality

The question to answer is: “How can we specify in which way the bit and pieces combine?”

1. Meaning (representation) ultimately flows from the lexicon.
2. Meaning (representation) is obtained by making use of syntactic information.
3. The meaning of the whole is function of the meaning of its parts, where “parts” refer to substructures given us by the syntax.

9.4. Ambiguity

A single linguistic sentence can legitimately have different meaning representations assigned to it.

For instance, “John saw a man with the telescope”

a. John [saw [a man [with the telescope]$_{pp}$]$_{np}$]$_{vp}$ $\exists x.\mathtt{Man}(x) \wedge \mathtt{Saw}(j,x) \wedge \mathtt{Has}(x,t)$

b. John [[saw [a man]$_{np}$]$_{vp}$ [with the telescope]$_{pp}$]$_{vp}$ $\exists x.\mathtt{Man}(x) \wedge \mathtt{Saw}(j,x) \wedge \mathtt{Has}(j,t)$

Different parse trees result into different meaning representations!

10. How far can we go with FOL?

FOL can capture the *what* (partially) and cannot capture the *how*, i.e.

Problems with the "what":

order
- Swimming is healthy. $Healthy(Swim)$: wrong!
  (property of property)
- John has all the properties of Santa Clause $\forall P(P(s) \rightarrow P(j))$: wrong!
  (quantification over properties)
- Red has something in common with green. $\exists P(P(red) \wedge P(green)$: wrong!
  (quant. over properties of properties)

adj.
- There was a red book on the table. $\exists x(Book(x) \wedge Red(x) \wedge On\_the\_table(x))$.
- There was a small elephant in the zoo.
  $\exists x(Elephant(x) \wedge Small(x) \wedge In\_the\_zoo(x))$.: wrong!

adv.
- Milly swam slowly. (modifier of the verb rather than of individuals!)
- Milly swam terribly slow (modifier of a modifier).

10.1. FOL: How?

Problems with the how:

Constituents: it cannot capture the meanings of constituents.

Assembly: it cannot account for meaning representation assembly.

11. Building Meaning Representations

To build a meaning representation we need to fulfill three tasks:

Task 1 Specify a reasonable *syntax* for the natural language fragment of interest.

Task 2 Specify semantic representations for the *lexical items*.

Task 3 Specify the *translation* of constituents *compositionally*. That is, we need to specify the translation of such expressions in terms of the translation of their parts, parts here referring to the substructure given to us by the syntax.

Moreover, when interested in Computational Semantics, all three tasks need to be carried out in a way that leads to computational implementation naturally.

12. From sets to functions

A set and *its characteristic function* amount to the same thing:

> if f_X is a function from Y to $\{F, T\}$, then $X = \{y \mid f_X(y) = T\}$. In other words, the assertion '$y \in X$' and '$f_X(y) = T$' are equivalent.

$$[\![student]\!] = \{t, a, f, j\}$$

`student` can be seen as a function from entities to truth values

`student`: $D_e \to D_t$

Functions can be represented by lambda-terms.

12.1. Function and lambda terms

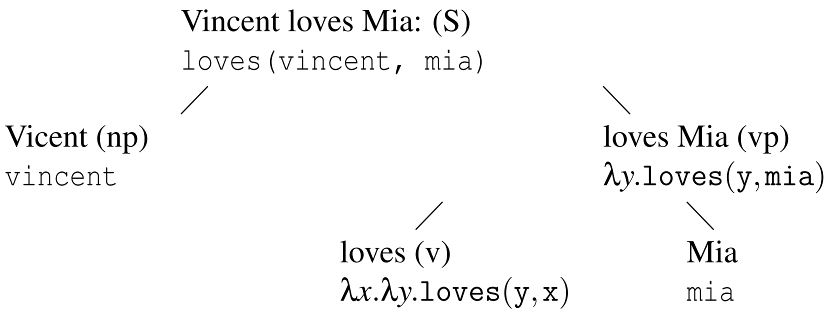
Recall: Function $f : X \to Y$. And $f(x) = y$ e.g. $SUM(x,2)$ if $x = 5$, $SUM(5,2) = 7$.

- $\lambda x.x$
- $\lambda x.(x+2)$
- $\underbrace{(\lambda x.(x+2))}_{function} \underbrace{5}_{argument}$
- $\underbrace{(\lambda x.(x+2))}_{function} \underbrace{5}_{argument} = 5+2$

`student`: $D_e \to D_t : \lambda x.\texttt{student}(x)$

Lambda calculus was introduced by Alonzo Church in the 1930s as part of an investigation into the foundations of mathematics.

12.2. Formal Semantics: How



syntactic structure guiding gluing and the linguistic composition amounts to function application. More tomorrow.