Logica & Linguaggio, PL: Tableaux

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  We can conclude ψ is *falsifiable*:

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  Falsiability is a weaker property then unsatisifiability.

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3. Alberi di refutazione (tableaux

Le tavole di verità non sono l'algoritmo più efficiente. Esistono altre procedure più veloci. Gli alberi di *refutazione* (tablaux) sono uno di questi:

> Si formi una lista di formule con tutte le premesse e la negazione della conclusione. Se si arriva a trovare un'interpretazione per la quale tale lista contiene tutte formule vere, allora quell'interpretazione mostra che esiste un controesempio: l'argomentazione non è valida (non è una conseguenza logica). Se non si riesce a trovare nessuna interpretazione che renda vera tale lista, allora la conclusione non è stata refutata, dunque l'argomentazione è valida.

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  Can you make a stronger claim?

  No this is already a strong result, there is no need to look at $\neg\psi$.