Logical Structures in Natural Language: Introduction

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Contents

1. Administrativa

Schedule 6 ECTS, Tot: 36 hrs (18 hrs with RB, 18 hrs with RZ), 6 hrs per week. Mondays and Thursdays (10:30-12:30) and Wednesdays (13:00-15:00)

Period 3rd-26th of April with RB; afterwords with RZ.

Exam Written exercises

Office hours On appointment.

Teaching Material

- Lecture notes: `http://disi.unitn.it/~bernardi/Courses/LSL/16-17.html`

April Logic, Logic applied to Language

May Zoom into a challenging research problem with RZ.

10th of April: shall we start at 10:00?

2. What is Logic?

Lewis Carroll “Through the Looking Glass”:

> “Contrariwise”, continued Tweedledee, “if it was so, it might be; and if it were so, it would be; but as it isn’t, it ain’t. That’s logic.”

Question What’s your answer?

“The Game of Logic” by Lewis Carroll:

> seven words -“Propositions, Attribute, Term, Subject, Predicate, Particular, Universal” – charmingly useful, if any friend should happen to ask if you have ever studied Logic. Mind you bring all seven words into your answer, and your friend will go away deeply impressed – ’a sadder and wiser man’.

2.1. Other answers

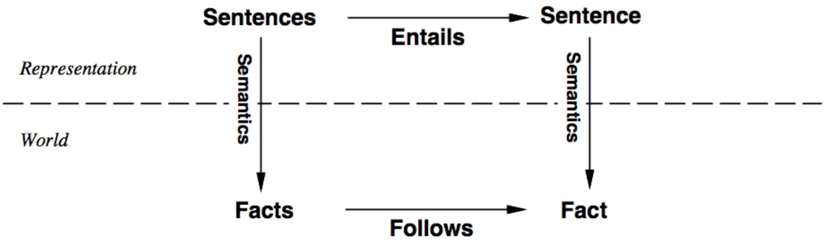
Moshe Vardi's students

- the ability to determine correct answers through a standardized process
- the study of formal inference
- a sequence of verified statements
- reasoning, as opposed to intuition
- the deduction of statements from a set of statements

Wikipedia

> Logic [...] is most often said to be the study of criteria for the evaluation of arguments [..], the task of the logician is: to advance an account of *valid and fallacious inference* to allow one to distinguish logical from flawed arguments.

2.2. Logic in a picture



A logic allows the axiomatization of the domain information, and the drawing of conclusions from that information.

- Syntax
- Semantics
- Logical inference = *reasoning*

2.3. The main concern

Modern Logic teaches us that one claim is a *logical consequence* of another if

> there is no way the latter could be *true* without the former also being true.

It is also used to *disconfirm* a theory

> if a particular claim is a logical consequence of a theory, and we discover that the claim is *false*, then we know the theory itself must be incorrect in some way or another.

Examples of theories: physical theory; economic theory, etc.

Our main concern in this course introduce the main aspects to solve a given problem with a logic approach.

2.4. Logic as "science of reasoning"

- Goal of logic: to make sure that from a set of **premises** it is possible to derive a *correct consequent*

```
If there is no electricity, the light is not on.
The light is on or the candle is on.
There is no electricity.
==========================================
The candle is on
```

```
premise_1
...
premise_n
============
conclusion
```

Notation: $\underbrace{\text{Premise}_1, \ldots, \text{Premise}_n}_{\textit{antecedent}} \models \underbrace{\text{conclusion}}_{\textit{consequent}}$

2.5. Counter-example

A counterexample is an exception to a proposed general rule. For example, consider the proposition

"all students are lazy".

Because this statement makes the claim that a certain property (laziness) holds for all students, even a single example of a diligent student will prove it false. Thus, any hard-working student is a counterexample to "all students are lazy".

Counterexamples are often used to prove the boundaries of possible theorems. By using counterexamples to show that certain conjectures are false.

3. Important Questions in Logic

- **Expressive Power** of representation language

$\leadsto$ able to *represent* the problem

- **Correctness** of entailment procedure

$\leadsto$ *no false* conclusions are drawn

- **Completeness** of entailment procedure

$\leadsto$ *all correct* conclusions are drawn

- **Decidability** of entailment problem

$\leadsto$ there exists a (terminating) algorithm to compute entailment

- **Complexity**

$\leadsto$ resources needed for computing the solution

3.1. Correctness (Soundness) and Completeness

1. Assume my system has to check whether a mushroom is “poisonous”:
    - For all mushroom x if x is poisonous, it’s recognized by the system. [Complete System]
    - For all mushroom x recognized by the system, x is poisonous. [Sound System]

Incomplete An incomplete system could miss to recognize as “poisonous” mushrooms that are poisonous. Consequence: I die.

Unsound An unsound system could happen to recognize as “poisonous” mushrooms that are not poisonous. Consequence: I eat one mushroom less, than I could.

Hence, soundness is important, but Completeness is vital!

2. Assume my system has to check whether a mushroom is “eatable”.

   Incomplete An incomplete system could miss to recognize as “eatable” mushrooms that are eatable. I eat one mushroom less, than I could.

   Unsound An unsound system could happen to recognize as “eatable” mushrooms that are not eatable. Consequence: I die.

   Hence, completeness is important, but Soundness is vital!

4. What is a Logic?

Clearly distinguish the definitions of:

- the *formal language*
  - Syntax
  - Semantics
  - Expressive Power
- the *reasoning problem* (e.g., entailment)
  - Decidability
  - Computational Complexity
- the *problem solving procedure*
  - Soundness and Completeness
  - Complexity

4.1. The ideal Logic

- Expressive
- With decidable reasoning problems
- With sound and complete reasoning procedures
- With efficient reasoning procedures

4.2. Many Logics

- Propositional Logic
- First Order Logic
- Modal Logic
- Temporal Logic
- Relevant Logic
- ...

4.3. Types of Logics

- Logics are characterized by what they commit to as "primitives"
- Ontological commitment: what exists—facts? objects? time? beliefs?
- Epistemological commitment: what states of knowledge?

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief 0…1
Fuzzy logic	degree of truth	degree of belief 0…1

5. Background

1. Sets ?
2. Functions ?

See Heim & Kratzer , Ch. 1

6. Set: basic notions

Element To express that x is an element (or member) of the set A we use $x \in A$.

Elements can be sets, for instance the set $A = \{1, 2, \{3, 4\}\}$ has as elements the numbers 1 and 2 and the set $\{3, 4\}$.

Subset If every member of set B is also a member of set A, then B is said to be a subset of A.

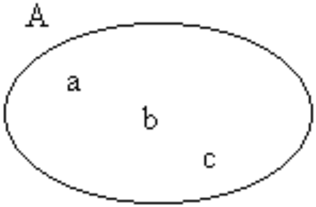
If every member of A is a member of B, we speak of *improper* subset ($B \subseteq A$). We speak of a *proper* subset when at least one element of A is not part of the set B ($B \subset A$).

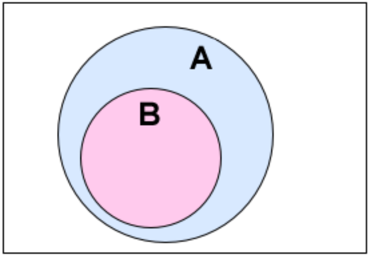
Membership vs. subset The inclusion of a set B in a set A as subset ($\subseteq$) should not be confused with the inclusion of B in A as a member ($\in$).

In the example above $B = \{1, 2\}$ is subset of A ($\{1, 2\} \subset \{1, 2, \{3, 4\}\}$ but is not a member of it ($\{1, 2\} \notin \{1, 2, \{3, 4\}\}$ whereas $\{3, 4\}$ is a member of A ($\{3, 4\} \in \{1, 2, \{3, 4\}\}$ but it's not a subset of it $\{3, 4\} \not\subset \{1, 2, \{3, 4\}\}$.

Empty Set Among the subsets, there is always the empty set $\{\}$ o $\emptyset$.

6.1. Sets: Venn diagrams





Alternative notation for $A = \{1,2,3,4,5,6\}$:

$$A = \{x|x \text{ a positive real number smaller than 7}\}$$

we read it: the set of all the x such that x is a positive real number smaller than 7.

6.2. Sets: Operations

Union Union ($\cup$) of two sets A and B is the set of all things that are members of either A or B: $A \cup B =_{def} \{x | x \in A \text{ or } x \in B\}$

Example: $A = \{1,2,3\}$ and $B = \{2,3,4\}$: $A \cup B = \{1,2,3,4\}$.

Intersection Intersection of two sets A e B is the set of all things that are members of both A and B: $A \cap B =_{def} \{x | x \in A \text{ and } x \in B\}$

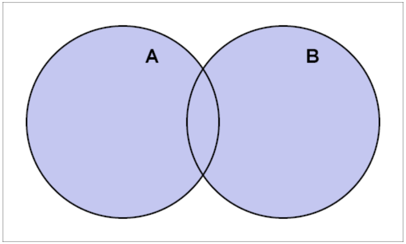
Example: $A = \{1,2,3\}$ and $B = \{2,3,4\}$: $A \cap B = \{2,3\}$.

Difference Given two sets A and B, the relative complement of A with respect to B is the set of only those elements of B that are not members of A: $B \setminus A$ (alternatively, $B - A$.) Formally, $B \setminus A =_{def} B - A = \{x \in B \text{ and } x \notin A\}$

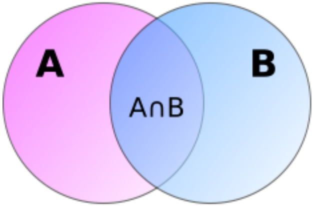
Example: $\{1,2,3,4,5\} \setminus \{3\} = \{1,2,4,5\}$

Cartesian Product the cartesian product of two sets A and B is the set of ordered pairs $\langle a,b \rangle$ such that a in A and b in B. Formally: $A \times B =_{def} \{(a,b) : a \in A \text{ e } b \in B\}$.

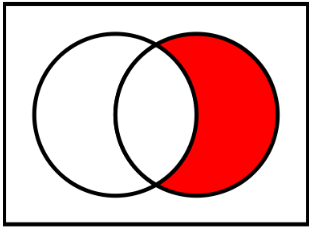
6.3. Sets: Venn operations



Union



Intersection



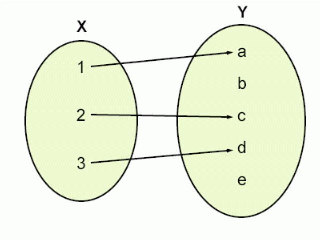
Difference

How would you represent two disjoint sets? Which is the result of their intersection?

Exercises

7. Functions

A function f is an operation that assigns to each element x of the set X one and only one element y of the set Y. $f : X \to Y$



- X is the domain of f and Y is the range (co-domain).
- x is the argument of the function, whereas y or $f(x)$ is the value assigned to it.

7.1. Functions: definitions

A function can be defined in various ways, the most straightforward one is to simply list the function's elements. Functions are sets of ordered pairs (e.g., $< a, b >$). Eg.:

$$\mathbf{F} := \{< a,b >, < c,b >, < d,e >\}$$

is equivalent to:

$$\mathbf{F} := \begin{bmatrix} a & \rightarrow & b \\ c & \rightarrow & b \\ d & \rightarrow & e \end{bmatrix}$$

and to:

Let F be that function f with domain $\{a, c, d\}$ such that $f(a) = f(c) = b$ and $f(d) = e$.

7.2. n-argument functions

When the domain of a function f is the cartesian product of two or more sets and hence the function acts over pairs (or n-tuples) of elements of a set, then the value of the pair (x,y) is denoted as $f(x,y)$ (Heim & Kratzer's notation: $x\ f\ y$). In this case, the function is called function of two (or more) arguments.

Example: the multiplication function assigns to two natural numbers their product: $f(x,y) = x \times y$ (or $\times(x,y)$.) This function can be defined formally as having domain $N \times N$, the set of all natural numbers pairs. Note:

$$f : X \times Y \to Z \text{ is equivalent to } f : X \to (Y \to Z)$$

8. Basic Notions

Meaning The meaning of a sentence is its truth value (truth, false).

Principle of Compositionality The meaning (hence being true or false) of a complex sentence depends on the meaning of the sentence it is composed of.

Reasoning $\text{Premise}_1, \ldots, \text{Premise}_n \models$ conclusion iff the conclusion is true in all the situations in which the premises are true.

8.1. Meaning: Interpretation function

An interpretation function assigns to a sentence its meaning. In other words, it assigns to a formula its truth value.

$$\underbrace{\text{Spring starts on the 21st of March}}_{p}$$

Let's consider two situations, in the first one we are in north hemisphere and in the second one we are in the south hemisphere.

North Hemisphere: p it's true.

South Hemishpere: p it's false.

The function V_1 (of North Hemisphere) $V_1(p) = true$ vs. the function V_2 (of the South Hemisphere) $V_2(p) = false$

$$V_i : \{p, q, \ldots\} \rightarrow \{true, false\}$$

8.2. Compositionality: truth-functional connective

1. $\underbrace{\text{Spring starts on the 21st of March}}_{p}$ `True`
2. $\underbrace{\text{In 2017 Easter will be in June}}_{q}$ `False`
3. Spring starts on the 21st of March and in 2017 Easter will be in June `False`
4. In 2017 Easter will not be in June `True`
5. In 2017 Easter will be in June because Spring starts on the 21st of March.

- (p and q) is true iff p is true and q is true. V(p and q)=true iff V(p)=true and V(q)=true.
- (not q) is true iff q is false. V(not q)=truth iff V(q)=false.
- (q because p) does not dependent on the truth/falsity of p and q.

"and" is a truth-functional connective: The truth of the sentence composed by it ("p and q") depends on the truth of the sentences that it connects. The same holds for "not". Instead this does not hold for "because".

8.3. Reasoning

$$\underbrace{\{P_1 \ldots P_n\}}_{\text{Premises}} \models \alpha$$

The set of premises *implies* the sentence α iff

α is true in all the situations (for all the interpretation functions) in which the premises are true.

We denote the set of all the situations in which α is truth by $W(\alpha)$, we've:

$$W(\text{Premises}) \subseteq W(\alpha)$$

the set of the interpretations in which the premises are true is a sub-set of the set of the interpretations in which α is true.

9. Why Logic and Language?

- to understand natural language: its syntax, semantics and the interaction between syntax and semantics.
- to exploit logical representations to verify inferences automatically
- to exploit logical representations to verify if within a text there are contradictions.
- To exploit logical representations and their inference systems to support Question Answering systems in finding good answers to users.
- …

10. Agenda

I need to know your background!

What do you know of PL? FoL? Lambda calculus?

1. Part I: Logic
   - Propositional Logic from the 2nd till ??? of March
   - First order Logic from the ?? till the ?? of March
2. Part II: Logic & Language
   - Lambda Calculus: from the ?? till the ??? of March

11. Goals

You will learn to:

- formalize a problem in PL and FoL
- use reasoning tools (truth tables and tableaux) to prove logical consequences.
- familiarise with the logical approach to natural language.

Please, send me a mail – I'd like to have your email address in case of last minute note.