Exercises on Tableaux for FoL

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1 Model

(a) Let $\mathcal{L} = \langle P, D, G \rangle$ be a FOL language with P e D unary symbols and G a binary symbol. Given the model with the following domain $D = \{0, 1, 2\}$ and interpretation:

$$\begin{aligned} \mathcal{I}(P) &= \{0, 2\}, \\ \mathcal{I}(D) &= \{1\}, \\ \mathcal{I}(G) &= \{(0, 1), (0, 2), (1, 2)\}, \end{aligned}$$

verify whether in $\mathcal{M}$ the following formulas are true and explain your answer

$$\begin{aligned} &(1)\ \forall x(\exists y((P(x) \lor D(x)) \to G(x, y))) \\ &(2)\ \exists x(\forall y(\neg G(x, y))) \end{aligned}$$

(b) Let $\mathcal{L} = \langle P, S \rangle$ be a FOL language with S and P binary and unary symbols, respectively.Given the model with the following domain $D = \{0, 1\}$, and interpretation:

$$\begin{aligned} \mathcal{I}(P) &= \{1\}, \\ \mathcal{I}(S) &= \{(0, 1), (1, 1)\}, \end{aligned}$$

verify whether in $\mathcal{M}$ the following formulas are true and explain the answer.

$$\begin{aligned} &\forall x(\exists y(S(x, y) \to P(y))) \\ &\exists x(\forall y(S(x, y) \land P(y))) \end{aligned}$$

2 Tautology

Check whether the following formula is a tautology:

1. $\forall x(F(x) \to F(a))$

2. $\neg(\exists x Fx \land \forall x \neg Fx)$

3 Valid Entailment

Check whether the following entailments are valid:

1. $\forall x \forall y (Fxy \to \neg Fyx) \models \neg \exists Fxx$
2. $\exists x \forall y Lxy \models \forall x \exists y Lyx$
3. $\exists x \exists y Lxy \models \exists x Lxx$
4. $\forall x \exists y Lxy \models Laa$

4 Solutions

4.1 Tautology

1.

1	$\neg(\forall x F(x) \rightarrow F(a))$	
2	$\exists x \neg(F(x) \rightarrow F(a))$	from 1
3	$\neg(F(b) \rightarrow F(a))$	from 2
4	$F(b)$	from 3
5	$\neg F(a)$	from 3

2.

1.	$\exists x Fx \wedge \forall x \neg Fx$	
2.	$\exists x Fx$	from 1
3.	$\forall x \neg Fx$	from 1
4.	Fa	from 2
5.	$\neg Fa$	from 3

4.2 Valid Entailment

1.

1.		from
2.		from
3.		from
4.		from

2.

1.	$\exists x \forall y Lxy$	Prem
2.	$\neg(\forall x \exists y Lyx)$	from negation of Conc
3.	$\forall y Lay$	from 1
4.	$\exists x \neg \exists y Lyx$	from 2
5.	$\neg y Lyb$	from 4
6.	$\forall y \neg Lyb$	from 5
7.	Lab	from
8.	$\neg Lab$	from
	X	Contradiction Lab and $\neg Lab$

The entailment is valid.

3.

1.	$\exists x \exists y Lxy$	Prem
2.	$\neg(\exists x Lxx)$	negation of Conc
3.	$\exists y Lay$	from Prem
4.	Lab	from 3
5.	$\forall x \neg Lxx$	from 2
6.	$\neg Laa$	from 5
7.	$\neg Lbb$	from 5

The entailment is not valid. The couter-example is given by the model with $D = \{a, b\}$ and $\mathcal{I}(L) = \{(a, b)\}$

4.

1.	$\forall x \exists y Lxy$	Prem
2.	$\neg Laa$	negation of Conc
3.	$\exists y Lay$	from 1
4.	Lab	from

The entailment is not valid. The counter-example is given by the model with $D = \{a, b\}$ and $\mathcal{I}(L) = \{a, b\}$